1 Preface

1.1 Brief history

CVM C++ class library[1] encapsulates the concepts of vector and different kinds of matrices including square, band, symmetric and hermitian ones, in Euclidean space of real and complex numbers. First version of the library was released in 1992. Originally it allowed to simplify a code dealing with matrices and vectors in the following way:

```
rvector a(10), b(20);
rmatrix A(20,10);
......
b = A * a;
cout << "Norm of b equals " << b.norm() << endl;
```

By overloading of arithmetic operators and parentheses relative readability of source code was reached.

In 1995[2] Russian office of Intel Corporation began to distribute a CD with last software products from Intel. Among others there was a freely (at that time) distributed library named "Intel BLAS Library". More than 20 years BLAS library (Basic Linear Algebra Subprograms) is known to experts in numerical algorithms programming in FORTRAN language. This library features common vector-matrix operations for data types `REAL` and `DOUBLE RECISION`. This is important to note that CVM library is also released in two versions:

```
#if defined (CVM_FLOAT)
    typedef float treal;
#else
    typedef double treal;
#endif
```

Both implementations of the BLAS are utilized since version 2.0 of CVM library. For example, operator of addition of two vectors utilizes subroutine `DAXPY`. Later Intel changed the name "BLAS" to "Intel Math Kernel Library". Since version 3.0 LAPACK subroutines were added to the Intel MKL library. This functionality was encapsulated in third release of the CVM library. Version 4.0 featured complex numbers. And since version 5.0 band, symmetric and hermitian matrices are implemented as well.

1.2 Features

The memory management mechanism described below is no longer supported by default. It was a good solution few years ago when memory allocation operator was relatively expensive, but

[1] This document describes version 5.6. Copyright © Sergei Nikolaev, 1992–2007, http://cvmlib.com

[2] I'm not sure actually, may be this was 1994

now standard allocator does this job much faster. However, the algorithm described below may be useful as an error detection helper. You will need to rebuild the library in debug mode with `CVM_USE_POOL_MANAGER` *defined in order to use it*.

The last version of the Intel MKL library (at the moment of writing of this page) for Win32 and Linux is 10.00. Version 5.6 of the CVM library was built and tested with that MKL library implementation only.

Since its third release the CVM library implements nontrivial memory management which should be described in detail. Earlier a memory was being allocated using operator `new` in every constructor, and freed with help of `delete` in every destructor. Let us consider an operation of multiplying of vector `a` by matrix `A` and assignment of result to vector `b`:

```
b = a * A;
```

This harmless code calls two constructors (a constructor allocating memory for output and a copy constructor, returning output to a calling function), as well as two destructors deleting those temporary objects[3]. If sizes of the objects are relatively small, your processor will be longer allocating memory than multiplying[4]. The idea of nontrivial memory management came from Jeff Alger's book [1]. Author suggests some approaches to memory allocation (overloading of operators `new` and `delete` or implementation of a class controlling a pool), and also some ways of a pool compaction (Baker's algorithm and in-place compaction) and references counting (using master or "genius" pointers). I decided to implement a class controlling a pool in the CVM library.

Pool is controlled by a class `MemoryPool`. It allocates a memory by blocks (so-called "outer blocks"). The memory block concept is encapsulated in a class `MemoryBlocks`. Size of an outer block equals to the nearest degree of 2 of a requested number of bytes multiplied by two. This can be illustrated on the following example. Let us suppose that it's required to allocate a memory for storage of a vector consisting of 1000 units:

```
rvector v(1000);
```

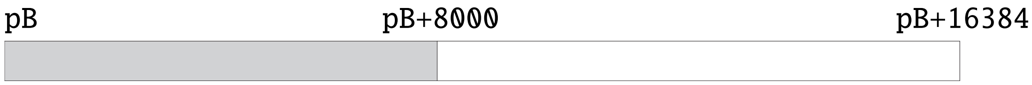
If at the moment of execution of this statement there is no free block of size 8000 or greater in a list of free memory blocks, then the following (simplified) code will be executed:

```
const int nUpBytes = up_value (nBytes);
const int nRest    = nUpBytes - nBytes;
try {
    pB = ::new tbyte[nUpBytes];
}
catch (const std::bad_alloc&) {
    throw (cvmexception (CVM_OUTOFMEMORY));
}
m_lOutBlocks.push_back (pB);
m_blocks.AddPair (pB, nBytes, nRest);
```

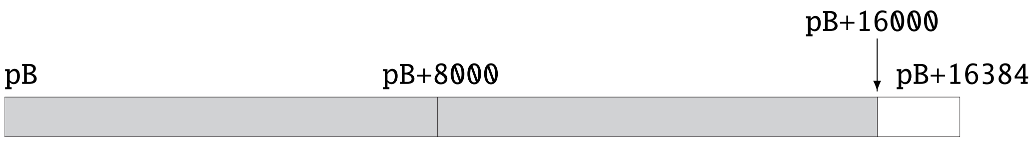
[3] In order to avoid those memory allocations you can use `b.mult(a,A);`

[4] At least under Win32

where variable `nBytes` has value of 8000 (`1000*sizeof(treal)`), and function `up_value` returns the nearest degree of two multiplied by two, i.e. 16384. Thus, the allocated outer block can be represented as follows:

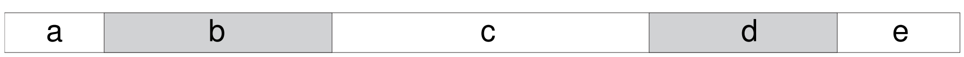


The block used for storage 1000 units of a vector `v` (its start address here is stored in the pointer `pB`) is filled. The remaining block (named as "free block") consists of `nRest=8384` bytes. Further, if application needs one more block of the same size (it happens in most cases while execution of a copy constructor), the memory will be allocated from this free block without calling of operator `new`. The result will be the following:

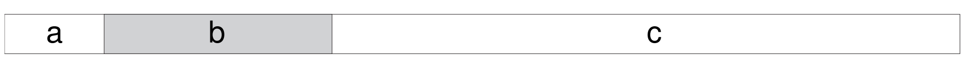


Remaining free block of 384 bytes will be utilized for memory allocation of small objects. In case of creation of an object of size greater than 384 bytes one more outer block will be created, etc. While using of this scheme sooner or later memory begins to be like a sieve of free and occupied blocks. To avoid this chaos, I have applied the logic of blocks releasing, which acts like an algorithm of free space compaction. Difference of this logic from the algorithms of compaction described in [1] is that occupied blocks are never being moved.

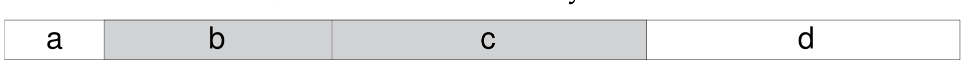
Let's say one of outer blocks looks like



And the block `d` is going to be released. In this case neighbor blocks `c` and `e` will be checked and if one of them (or both, as in this case) will appear as free, then it will be joined with the block being released. The result of release of the block `d` will be the following:



And if the following object to be created will not find a room in the block `a`, but find a room in the block `c`, the result of memory allocation will be the following:



So, there is no any chaos.

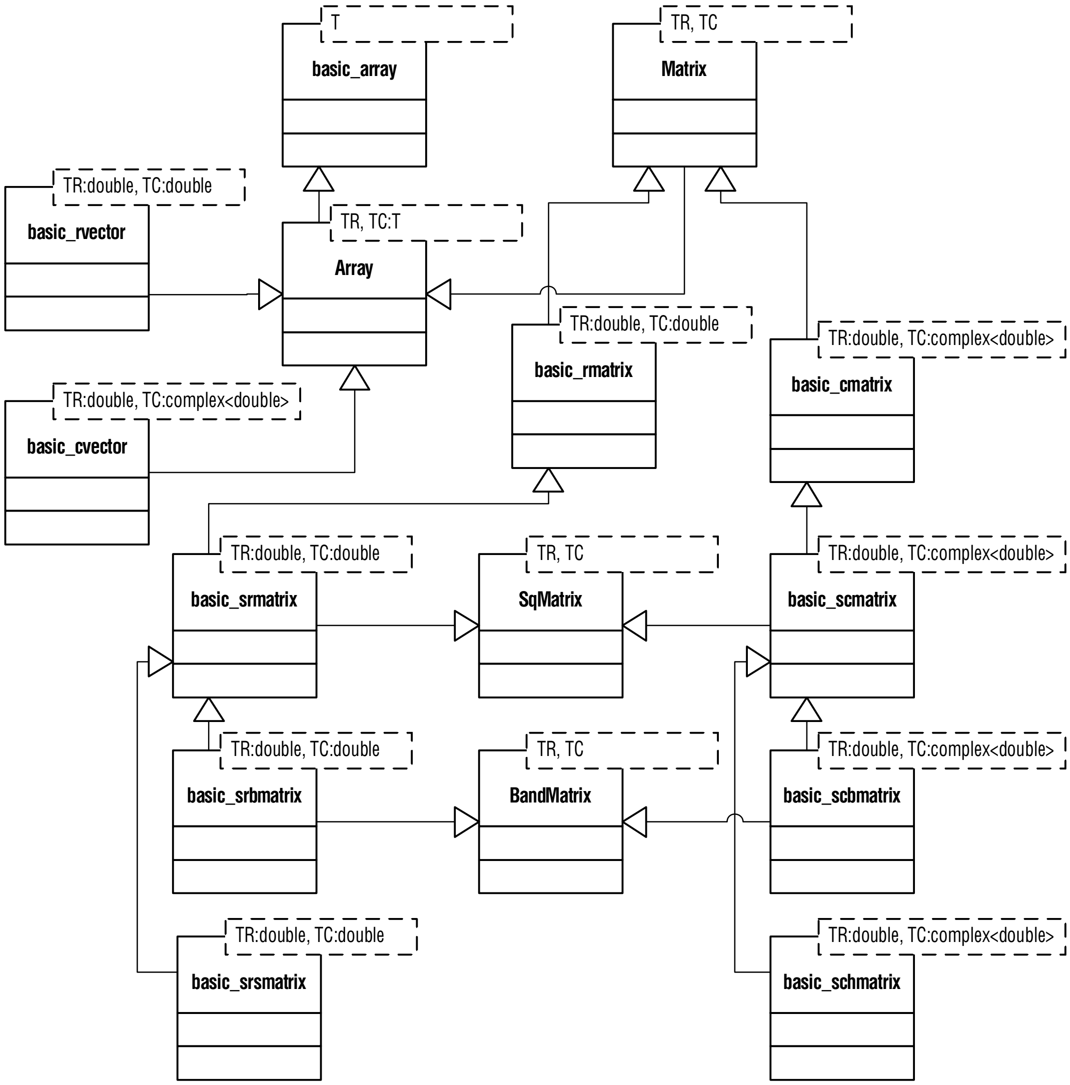
1.2.1 Allocator

Since version 5.1 the library uses `std::allocator`. However, you can rebuild the whole library using you own customized allocator. Just add `-DCVM_ALLOCATOR=MyAllocator` to your compiler command line or define

```
#define CVM_ALLOCATOR MyAllocator
#include <cvm.h>
```

1.3 Object Model

Object model of the CVM library is shown on the following picture



Base class's names are beginning with capital letters. They implement common interfaces and are *not* designed to be instantiated. This is the list of end-user classes:

- `basic_array` is an abstract array of elements of any type. It's used mostly as an array of integers like `basic_array<int>`, but you can use it in your projects within any other types as well.
- `basic_rvector` is a class encapsulating the concept of vector in the space of real numbers.
- `basic_cvector` is a class encapsulating the concept of vector in the space of complex

numbers.
- basic_rmatrix is a class encapsulating the concept of matrix in the space of real numbers.
- basic_cmatrix is a class encapsulating the concept of matrix in the space of complex numbers.
- basic_srmatrix is a class encapsulating the concept of square matrix in the space of real numbers.
- basic_scmatrix is a class encapsulating the concept of square matrix in the space of complex numbers.
- basic_srbmatrix is a class encapsulating the concept of square band matrix in the space of real numbers. Packed storage is used.
- basic_scbmatrix is a class encapsulating the concept of square band matrix in the space of complex numbers. Packed storage is used.
- basic_srsmatrix is a class encapsulating the concept of symmetric matrix in the space of real numbers.
- basic_schmatrix is a class encapsulating the concept of hermitian matrix in the space of complex numbers.

You don't need to use those class names directly unless you want to typedef your own ones. Otherwise you should use the following pre-defined classes (CVMAllocator is omitted here for simplicity):

```
typedef basic_array    <int>             iarray;
typedef basic_rvector  <treal>           rvector;
typedef basic_rmatrix  <treal>           rmatrix;
typedef basic_srmatrix <treal>           srmatrix;
typedef basic_cvector  <treal, tcomplex> cvector;
typedef basic_cmatrix  <treal, tcomplex> cmatrix;
typedef basic_scmatrix <treal, tcomplex> scmatrix;
typedef basic_srbmatrix<treal>           srbmatrix;
typedef basic_scbmatrix<treal, tcomplex> scbmatrix;
typedef basic_srsmatrix<treal>           srsmatrix;
typedef basic_schmatrix<treal, tcomplex> schmatrix;
```

The rest of this manual describes them in details.

1.4 Installation

1.4.1 Directory Structure

The CVM library distribution has the following directory structure.
- .*.sln MS Visual Studio 2005 solution files.
- ftn contains FORTRAN and project files for Intel Fortran 9.1 compiler and GNU gfortran. This source code is the part of CVM library, it contains some numerical algorithms implementation.

- `lib` is the place for libraries to be built.
- `test` contains regression test code and projects.
- `src` contains source code of the library along with `cvm.h` header file.

1.4.2 Usage Notes

Here are definitions and data types used in the library.

`CVM_ACML`	define this macro to link against AMD ACML library
`CVM_FLOAT`	define this macro in order to build a float version
`CVM_NO_NAMESPACE`	define this macro if you don't want to use namespace
`treal`	is `typedef`'ed as `float` if `CVM_FLOAT` is defined and as `double` otherwise (by default)
`tcomplex`	is `typedef`'ed as `std::complex<treal>`
`CVM_ALLOCATOR`	assign this macro to a name of your own allocator in order to rebuild CVM class library

In order to use the library just include its header file:

```
#include <cvm.h>
```

You should also link your project with one of `cvm*.lib` for Microsoft's C++ compilers and `cvm*.so` for GNU C++ compilers (debug versions are `*_debug.lib` and `*_debug.so` respectively).

1.4.3 Installation – Win32

If you don't want to rebuild the library just download an appropriate version of `cvm*.dll` and `cvm*.lib` files from binaries section. If you want to rebuild the whole library you'll need Intel Fortran 9.1 and Intel C++ 9.1 compilers (or higher) along with MS Visual Studio 2005. You'll also need the Intel MKL 9.0 (or higher) library. You will also need STL library coming with MS VC++ or, more preferable, the STLport library. Open `.\cvmlib.sln` solution and choose the library version you want to build.

1.4.4 Installation – Unix

Use the Makefile provided in the root directory:

```
make [release|debug] [IFORT=1] [ICC=1] [MKL=1] [MKL_PATH=/opt/...]
     [ACML=1] [ACML_PATH=/opt/...] [EM64T=1]
     [CVM_FLOAT=1] [ICCT=1] [STATIC_ONLY=1] [IFORT_PATH=/opt/...]
```

Here

- `release|debug` is a target (by default it builds both)
- `IFORT=1` instructs to use Intel Fortran compiler (by default it's `gfortran`)

- `ICC=1` instructs to use Intel C++ compiler (by default it's `g++`)
- `MKL=1` instructs to use Intel MKL library (by default it uses native BLAS and LAPACK libraries)
- `MKL_PATH=path` specifies the directory where the MKL is installed to. By default it's equal to `/opt/intel/mkl/10.0.011/lib/` (please make sure that this path contains `32` and `em64t` subdirectories inside).
- `ACML=1` instructs to use AMD ACML library (overrides `MKL=1`).
- `ACML_PATH=path` specifies the directory where the ACML is installed to. By default it's equal to `/opt/acml4.0.0/` (overrides `MKL_PATH=path`).
- `EM64T=1` instructs to build EM64T version of the library. So far, this feature is compatible with the MKL and Intel's compilers only.
- `CVM_FLOAT=1` instructs to build `float` version of the library.
- `ICCT=1` instructs to use Intel C++ compiler for building the regression test utility (by default it's `g++`)
- `STATIC_ONLY=1` instructs to build static libraries only. Both .so and .a will be built otherwise.
- `IFORT_PATH=path` specifies the directory where the Intel Fortran is installed to. It's required only when you build ACML version within Intel Fortran and Intel C++ compilers. By default it's equal to `/opt/intel/fc/10.1.008/` for 32 bit and `/opt/intel/fce/10.1.008/` for em64t.

On Unix platforms Intel MKL, AMD ACML and native BLAS/LAPACK libraries as well as both Intel's and GNU compilers are supported.

1.5 Storage

The way of storage of matrices units is the same as the FORTRAN's one. Units are stored by columns, see the following example:

```
cvm::rmatrix m(2,2);
m(1,1) = 1.;        // first  row, first  column
m(1,2) = 2.;        // first  row, second column
m(2,1) = 3.;        // second row, first  column
m(2,2) = 4.;        // second row, second column
double* p = m;
cout << p[0] << " " << p[1] << " " << p[2] << " " << p[3] << endl;
```

Output will be the following:

```
1 3 2 4
```

Since version 5.0 band matrices are introduced. Band storage is utilized for such matrices; it can be described as follows (cited from MKL manual): *an* m *by* n *band matrix with* k_l *sub-diagonals and* k_u *super-diagonals is stored compactly in a two-dimensional array with* $k_l + k_u + 1$ *rows and* n *columns. Columns of the matrix are stored in the corresponding columns* *of the array, and diagonals of the matrix are stored in rows of the array*. This way of storage can be illustrated as follows (referenced elements are shown as "$*$", not referenced as "$-$", zeros are not stored):

$$m = n = 3, k_l = 0, k_u = 0 : \begin{bmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{bmatrix}$$

$$m = n = 4, k_l = 1, k_u = 0 : \begin{bmatrix} * & 0 & 0 & 0 \\ * & * & 0 & 0 \\ 0 & * & * & 0 \\ 0 & 0 & * & * \\ & & & - \end{bmatrix}$$

$$m = n = 6, k_l = 1, k_u = 2 : \begin{bmatrix} - & & & & & \\ - & - & & & & \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ 0 & * & * & * & * & 0 \\ 0 & 0 & * & * & * & * \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \\ & & & & & - \end{bmatrix}$$

CVM library implements square band matrices only, therefore $m = n$ is satisfied for them.

1.6 Indexing

Index numbering in CVM library corresponds to the FORTRAN's one: index of the first unit is equal to 1:

```
cvm::rector v(2);
cvm::rmatrix m (2,3);
v[1] = 1.3;       // first vector unit
v(2) = 2.1;       // second vector unit
m(1) = v;         // first column of matrix
m(1,2) = 3.7;     // element located of the first row
                  // and the second column of matrix
```

1.7 Polymorphism

Major number of CVM Class Library member functions are not declared as virtual, but it doesn't mean that the classes are not polymorphic. Those member functions just wrap virtual ones. For example, the following code

```
void print_solution (const srmatrix& a, const rvector& b)
{
    std::cout << a.solve(b);
}
...
rvector b(3);
srmatrix m(3);
srsmatrix ms(3);
...
print_solution(m, b);
print_solution(ms, b);
```

will use symmetric solver for symmetric matrix `ms`.

1.8 Multi-threading

The library fully supports multi-threading environments. Its Win32 binary files are linked with multi-threaded version of the run-time library. However, it's strongly recommended to use MKL-based version of the library in case of using it in multi-threaded applications.

1.9 Regression test utility

The library is shipped with regression utility utilizing almost all its functions and operators. It's strongly recommended to build it upon installation and verify (see `test` directory for workspace and make files). It has the following syntax:

```
regtest_* [-t<Number of threads to run>] [-r<Number of executions>]
```

Example:

```
D:\cvmlib\lib>regtest_ia32.exe -t2 -r2
TESTS STARTED
TESTS STARTED
ALL TESTS SUCEEDED
ALL TESTS SUCEEDED
TESTS STARTED
TESTS STARTED
ALL TESTS SUCEEDED
ALL TESTS SUCEEDED
TOTAL TIME 1.83e+000 sec.
```

2 CVM Class Library Reference

2.1 basic_array

This class contains array-specific member functions inherited in other classes. It can be utilized as a standalone class too. It also provides STL-compatible functions and type definitions, so itself and derived classes can be used in the same way as `std::vector<T>`. Since version 5.0 the `iarray` class is defined as

```
typedef basic_array<int, CVMAllocator> iarray;
```

```
template <typename T>
class basic_array {
public:
    int basic_array ();
    explicit basic_array (int nSize, bool bZeroMemory = true);
    basic_array (const T* p, int nSize);
    basic_array (const T* first, const T* last);
    basic_array (const basic_array& a);
    int size () const;
    T* get ();
    const T* get () const;
    operator T* ();
    operator const T* () const;
    T& operator () (int i) throw (cvmexception);
    T operator () (int i) const throw (cvmexception);
    T& operator [] (size_type i) throw (cvmexception);
    T operator [] (size_type i) const throw (cvmexception);
    T& operator [] (int i) throw (cvmexception);
    T operator [] (int i) const throw (cvmexception);
    basic_array& operator = (const basic_array& a) throw (cvmexception);
    basic_array& assign (const T* p);
    basic_array& set (T x);
    basic_array& resize (int nNewSize) throw (cvmexception);
    // STL-specific type definitions
    typedef T value_type;
    typedef value_type* pointer;
    typedef const value_type* const_pointer;
    typedef value_type* iterator;
    typedef const value_type* const_iterator;
    typedef value_type& reference;
    typedef const value_type& const_reference;
    typedef size_t size_type;
    typedef ptrdiff_t difference_type;
```

```
    typedef std::reverse_iterator<const_iterator> const_reverse_iterator;
    typedef std::reverse_iterator<iterator> reverse_iterator;
    // STL-specific functions
    iterator begin ();
    const_iterator begin () const;
    iterator end ();
    const_iterator end () const;
    reverse_iterator rbegin ();
    const_reverse_iterator rbegin () const;
    reverse_iterator rend ();
    const_reverse_iterator rend () const;
    size_type max_size () const;
    size_type capacity () const;
    bool empty () const;
    reference front ();
    const_reference front ();
    reference back ();
    const_reference back () const;
    void assign (size_type n, const T& val) throw (cvmexception);
    void assign (const_iterator first,
                 const_iterator last) throw (cvmexception);
    void resize (size_type nNewSize) throw (cvmexception);
    void clear ();
    void swap (basic_array& v);
    reference at (size_type n) throw (cvmexception);
    const_reference at (size_type n) const throw (cvmexception);
    void push_back (const T& x) throw (cvmexception);
    void pop_back () throw (cvmexception);
    iterator insert (iterator position, const T& x) throw (cvmexception);
    iterator erase (iterator position) throw (cvmexception);

    template <typename T>
    friend std::istream& operator >> <> (const std::istream& is,
                                         basic_array<T>& aIn);

    template <typename T>
    friend std::ostream& operator << <> (std::ostream& os,
                                         const basic_array<T>& aOut);
};
```

2.1.1 basic_array()

Default constructor

```
basic_array<T>::basic_array();
```

creates an empty basic_array object. See also basic_array. Example:

```
using namespace cvm;

iarray a;
std::cout << a.size() << std::endl;
a.resize(10);
std::cout << a.size() << std::endl;
```

prints

```
0
10
```

2.1.2 basic_array(int, bool)

Constructor

```
explicit basic_array<T>::basic_array(int nSize, bool bZeroMemory = true);
```

creates a `basic_array` object of size equal to `nSize`. Allocated memory is initialized with zero values by default (you can pass `false` in second argument in order to avoid this initialization). The constructor throws an exception of type `cvmexception` in case of non-positive size passed or memory allocation failure. See also `basic_array`. Example:

```
using namespace cvm;

iarray a(5);
std::cout << a.size() << " " << a[1] << std::endl;
```

prints

```
5 0
```

2.1.3 basic_array(const T*, int)

Constructor

```
basic_array<T>::basic_array (const T* p, int nSize);
```

creates a `basic_array` object of size equal to `nSize` and copies `nSize` elements of an array `p` to it. The constructor throws an exception of type `cvmexception` in case of non-positive size passed or memory allocation failure. See also `basic_array`. Example:

```
using namespace cvm;

const int a[] = {1, 2, 3, 4};
iarray v (a, 3);
std::cout << v;
```

prints

```
1 2 3
```

2.1.4 basic_array(const T*, const T*)

Constructor

```
basic_array<T>::basic_array (const T* first, const T* last);
```

creates a `basic_array` object of size equal to `last-first` and copies all elements in the range of `[first,last)` to it. The constructor throws an exception of type `cvmexception` in case of wrong range passed or memory allocation failure. See also `basic_array`. Example:

```
using namespace cvm;

const int a[] = {1, 2, 3, 4};
const iarray v (a+1, a+3);
std::cout << v << std::endl;
```

prints

```
2 3
```

2.1.5 basic_array(const basic_array&)

Copy constructor

```
basic_array<T>::basic_array (const basic_array& a);
```

creates a basic_array object of size equal to size of vector a and sets every element of created array to a value of appropriate element of an a. See also basic_array. Example:

```
using namespace cvm;

iarray a(5);
a.set(3);
iarray b(a);
std::cout << b;
```

prints

```
3 3 3 3 3
```

2.1.6 size()

Function

```
int basic_array<T>::size () const;
```

returns a number of elements of an array. This function is *inherited* in all classes of the library: rvector, cvector, rmatrix, cmatrix, srmatrix, scmatrix, srbmatrix, scbmatrix, srsmatrix and schmatrix. See also basic_array. Example:

```
using namespace cvm;

rvector v(3);
cmatrix m(10,20);
cout << v.size() << " " << m.size() << endl;
```

prints

```
3 200
```

2.1.7 get(), operator T*()

Functions and operators

```
T* basic_array<T>::get ();
const T* basic_array<T>::get () const;
basic_array<T>::operator T* ();
basic_array<T>::operator const T* () const;
```

return a pointer to the beginning (first element) of an array. These functions and operators are *inherited* in all classes of the library: rvector, cvector, rmatrix, cmatrix, srmatrix, scmatrix, srbmatrix, scbmatrix, srsmatrix and schmatrix. See also basic_array. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

void cprint (const std::complex<double>* p, int size)
{
    for (int i = 0; i < size; ++i)
        std::cout << p[i] << " ";
    std::cout << std::endl;
}
...
iarray a(10);
scmatrix m(3);
a[2] = 1;
m(3,1) = std::complex<double>(1., 2.);
std::cout << a.get()[1] << std::endl;
cprint(m, 3);
```

prints

```
1
(0.00e+00,0.00e+00) (0.00e+00,0.00e+00) (1.00e+00,2.00e+00)
```

2.1.8 Indexing operators

Indexing operators

```
T& basic_array<T>::operator () (int i) throw (cvmexception);
T  basic_array<T>::operator () (int i) const throw (cvmexception);
T& basic_array<T>::operator [] (size_type i) throw (cvmexception);
T  basic_array<T>::operator [] (size_type i) const throw (cvmexception);
T& basic_array<T>::operator [] (int i) throw (cvmexception);
T  basic_array<T>::operator [] (int i) const throw (cvmexception);
```

return a reference (or value for constant versions) to `i`-th element of an array. Please note that *all indexing operators of the library are 1-based*. These operators are *inherited* in `rvector` and `cvector` classes of the library but *overridden* in other ones: `rmatrix`, `cmatrix`, `srmatrix`, `scmatrix`, `srbmatrix`, `scbmatrix`, `srsmatrix` and `schmatrix`. See also `basic_array`. Example:

```
using namespace cvm;

try {
    rvector v (10);
    v[1] = 1.;
    v(2) = 2.;
    std::cout << v;

    double a[] = {1., 2., 3., 4.};
    const rvector vc (a, 4);
    std::cout << vc(1) << " " << vc[2] << std::endl;
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
1 2 0 0 0 0 0 0 0 0
1 2
```

2.1.9 operator = (const basic_array&)

Assignment operator

```
basic_array<T>&
basic_array<T>::operator = (const basic_array& a) throw (cvmexception);
```

sets every element of a calling array to a value of appropriate element of an array a and returns a reference to the object changed. This operator is *overridden* in all classes of the library: rvector, cvector , rmatrix, cmatrix, srmatrix, scmatrix, srbmatrix, scbmatrix, srsmatrix and schmatrix. See also basic_array. The operator throws an exception of type cvmexception in case of different sizes of the arrays. Example:

```
using namespace cvm;

iarray a(5), b(5);
a.set(3);
b = a;
std::cout << b;
```

prints

```
3 3 3 3 3
```

2.1.10 assign(const T*)

Function

```
basic_array<T>& basic_array<T>::assign (const T* p);
```

sets every element of a calling array to a value of appropriate element of an array pointed to by parameter p and returns a reference to the object changed. This function is *overridden* in all classes of the library: rvector, cvector , rmatrix, cmatrix, srmatrix, scmatrix, srbmatrix, scbmatrix, srsmatrix and schmatrix. See also basic_array. Example:

```
using namespace cvm;

const int a[] = {1, 2, 3, 4, 5, 6, 7};
iarray v (5);

v.assign(a);
std::cout << v;
```

prints

```
1 2 3 4 5
```

2.1.11 set(T)

Function

```
basic_array<T>& basic_array<T>::set (T x);
```

sets every element of a calling array to a value of parameter `x` and returns a reference to the object changed. This function is *overridden* in all classes of the library: `rvector`, `cvector` , `rmatrix`, `cmatrix`, `srmatrix`, `scmatrix`, `srbmatrix`, `scbmatrix`, `srsmatrix` and `schmatrix`. See also `basic_array`. Example:

```
using namespace cvm;

iarray a(5);
a.set(3);
std::cout << a;
```

prints

```
3 3 3 3 3
```

2.1.12 resize

Function

```
basic_array<T>&
basic_array<T>::resize (int nNewSize) throw (cvmexception);
```

changes a size of a calling array to be equal to nNewSize and returns a reference to the array changed. The array will be filled with zeroes in case of increasing of its size. This function is *overridden* in all classes of the library: rvector, cvector , rmatrix, cmatrix, srmatrix, scmatrix, srbmatrix, scbmatrix, srsmatrix and schmatrix. The function throws an exception of type cvmexception in case of non-positive size passed or memory allocation failure. See also basic_array. Example:

```
using namespace cvm;

try {
    const int a[] = {1, 2, 3, 4};
    iarray v (a, 3);
    std::cout << v;
    v.resize(2);
    std::cout << v;
    v.resize(4);
    std::cout << v;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1 2 3
1 2
1 2 0 0
```

2.1.13 STL-specific type definitions

Type definitions

```
typedef T value_type;
typedef value_type* pointer;
typedef const value_type* const_pointer;
typedef value_type* iterator;
typedef const value_type* const_iterator;
typedef value_type& reference;
typedef const value_type& const_reference;
typedef size_t size_type;
typedef ptrdiff_t difference_type;
typedef std::reverse_iterator<const_iterator> const_reverse_iterator;
typedef std::reverse_iterator<iterator> reverse_iterator;
```

are provided for every class of the library to be compatible with STL algorithms and methods. See also basic_array. Example:

```
using namespace cvm;

rvector vs1(5);
vs1[1] = 1.; vs1[2] = 2.; vs1[3] = 3.; vs1[4] = 4.; vs1[5] = 5.;
std::cout << vs1;

rvector::iterator it = vs1.begin() + 1;
rvector::iterator ite = vs1.erase(it);
std::cout << vs1;
std::cout << *ite << std::endl;

ite = vs1.insert(ite, 10.);
std::cout << vs1;
std::cout << *ite << std::endl;

vs1.push_back(11.);
std::cout << vs1;

vs1.randomize(0., 2.);
std::cout << vs1;
std::sort(vs1.begin(), vs1.end());
std::cout << vs1;
std::cout << *std::max_element(vs1.begin(), vs1.end()) << std::endl;

std::reverse(vs1.begin(), vs1.end());
std::cout << vs1;
```

prints

```
1.00e+000 2.00e+000 3.00e+000 4.00e+000 5.00e+000
1.00e+000 3.00e+000 4.00e+000 5.00e+000
3.00e+000
1.00e+000 1.00e+001 3.00e+000 4.00e+000 5.00e+000
1.00e+001
1.00e+000 1.00e+001 3.00e+000 4.00e+000 5.00e+000 1.10e+001
1.11e-001 4.96e-001 1.70e+000 1.91e+000 1.19e-001 1.11e+000
1.11e-001 1.19e-001 4.96e-001 1.11e+000 1.70e+000 1.91e+000
1.91e+000
1.91e+000 1.70e+000 1.11e+000 4.96e-001 1.19e-001 1.11e-001
```

2.1.14 STL-specific functions: begin(), end(), rbegin(), rend(), max_size(), capacity(), empty(), front(), back(), assign(), resize(), clear(), swap()

Functions

```
basic_array<T>::iterator basic_array<T>::begin();
basic_array<T>::const_iterator basic_array<T>::begin() const;
basic_array<T>::iterator basic_array<T>::end();
basic_array<T>::const_iterator basic_array<T>::end() const;
basic_array<T>::reverse_iterator basic_array<T>::rbegin();
basic_array<T>::const_reverse_iterator basic_array<T>::rbegin() const;
basic_array<T>::reverse_iterator basic_array<T>::rend();
basic_array<T>::const_reverse_iterator basic_array<T>::rend();
basic_array<T>::size_type basic_array<T>::max_size() const;
basic_array<T>::size_type basic_array<T>::capacity() const;
bool basic_array<T>::empty() const;
basic_array<T>::reference basic_array<T>::front();
basic_array<T>::const_reference basic_array<T>::front();
basic_array<T>::reference basic_array<T>::back();
basic_array<T>::const_reference basic_array<T>::back();
void basic_array<T>::assign (size_type n,
                             const T& val) throw (cvmexception);
void basic_array<T>::assign (const_iterator first,
                             const_iterator last) throw (cvmexception);
void basic_array<T>::resize (size_type nNewSize) throw (cvmexception);
void basic_array<T>::clear();
void basic_array<T>::swap (basic_array& v) throw (cvmexception);
```

are provided for every class of the library to be compatible with STL algorithms and methods. See also basic_array and STL documentation for further details. Example:

```
using namespace cvm;

iarray a(5);
a[1] = 1; a[2] = 2; a[3] = 3; a[4] = 4; a[5] = 5;

for (iarray::reverse_iterator it = a.rbegin(); it != a.rend(); ++it)
{
    std::cout << *it << " ";
}
std::cout << std::endl;
std::cout << a.front() << std::endl;
std::cout << a.back() << std::endl;
```

prints

```
5 4 3 2 1
1
5
```

2.1.15 at()

Functions

```
basic_array<T>::reference
basic_array<T>::at(size_type n) throw (cvmexception);

basic_array<T>::const_reference
basic_array<T>::at(size_type n) throw (cvmexception);
```

return a reference to an `(n-1)`-th element of an array, i.e., unlike indexing operators, these functions are `0`-*based*. They are provided for every class of the library to be compatible with STL algorithms and methods. The functions throw an exception of type `cvmexception` in case of negative parameter passed. See also `basic_array` and STL documentation for further details. Example:

```
using namespace cvm;

iarray a(5);
a[1] = 1; a[2] = 2; a[3] = 3; a[4] = 4; a[5] = 5;
std::cout << a.at(0) << " " << a.at(1) << std::endl;
```

prints

```
1 2
```

2.1.16 push_back(const T&), pop_back()

Functions

```
void basic_array<T>::push_back (const T& x) throw (cvmexception);
void basic_array<T>::pop_back () throw (cvmexception);
```

add and remove an element to/from an array. They are provided for every class of the library to be compatible with STL algorithms and methods. Since CVM doesn't preallocate a memory for extra storage, these functions *will require memory reallocation every time they are being executed* and may slow down your application. Please consider usage of `std::vector<T>` in such cases. The functions throw an exception of type `cvmexception` in case of memory allocation failure. See also `basic_array` and STL documentation for further details. Example:

```
using namespace cvm;

iarray a(5);
a.push_back(88);
std::cout << a;
a.pop_back();
std::cout << a;
```

prints

```
0 0 0 0 0 88
0 0 0 0 0
```

2.1.17 insert (iterator, const T&), erase (iterator)

Functions

```
basic_array<T>::iterator
basic_array<T>::insert (iterator pos, const T& x) throw (cvmexception);

basic_array<T>::iterator
basic_array<T>::erase (iterator pos) throw (cvmexception);
```

insert and remove an element to/from an array at given position `pos`. They are provided for every class of the library to be compatible with STL algorithms and methods. Since CVM doesn't pre-allocate a memory for extra storage, these functions *will require memory reallocation every time they are being executed* and may slow down your application. Please consider usage of `std::vector<T>` in such cases. The functions throw an exception of type `cvmexception` in case of memory allocation failure. See also `basic_array` and STL documentation for further details. Example:

```
using namespace cvm;

iarray a(5);
iarray::iterator pos = a.begin() + 2;
a.insert(pos, 88);
std::cout << a;
pos = a.begin() + 1;
a.erase(pos);
std::cout << a;
```

prints

```
0 0 88 0 0 0
0 88 0 0 0
```

2.1.18 operator >> <> (std::istream& is, basic_array<T>& aIn)

Friend template operator

```
template <typename T>
friend std::istream& operator >> <> (std::istream& is,
                                     basic_array<T>& aIn);
```

fills an object referenced by `aIn` with numbers from `is` stream. The operator is redefined in the class `Array`. See also `basic_array` Example:

```
using namespace cvm;

try {
    std::ofstream os;
    os.open ("in.txt");
    os << 1 << " " << 2 << std::endl << 3;
    os.close ();

    std::ifstream is("in.txt");
    iarray v(5);
    is >> v;

    std::cout << v;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1 2 3 0 0
```

2.1.19 operator << <> (std::ostream& os, const basic_array<T>& aOut)

Friend template operator

```
template <typename T>
friend std::ostream& operator << <> (std::ostream& os,
                                     const basic_array<T>& aOut);
```

outputs an object referenced by aOut into os stream. The operator is redefined in the class Array. See also basic_array Example:

```
using namespace cvm;

iarray v(5);
v(1) = 1;
v(2) = 2;

std::cout << v;
```

prints

```
1 2 0 0 0
```

2.2 Array

This class contains a couple of common for arrays member functions inherited in user classes. This class is not designed to be instantiated.

```
template <typename TR, typename TC>
class Array : public basic_array<TC> {
public:
    int incr () const;
    int indofmax () const;
    int indofmin () const;
    virtual TR norm () const;
    virtual TR norminf () const;
    virtual TR norm1 () const;
    virtual TR norm2 () const;

    <typename TR, typename TC>
    friend std::istream& operator >> <> (std::istream& is,
                                         Array<TR,TC>& aIn);

    <typename TR, typename TC>
    friend std::ostream& operator << <> (std::ostream& os,
                                         const Array<TR,TC>& aOut);
};
```

2.2.1 incr

Function

```
int Array<TR,TC>::incr () const;
```

returns an increment between elements of an array. This function is *inherited* in all classes of the library: rvector, cvector, rmatrix, cmatrix, srmatrix, scmatrix, srbmatrix, scbmatrix, srsmatrix and schmatrix. It always returns 1 for matrices. See also Array. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6.};
rvector v1 (a, 3, 2);
rvector v2(10);

std::cout << v1 << v1.incr () << std::endl;
std::cout << v2.incr () << std::endl;
```

prints

```
1 3 5
2
1
```

2.2.2 indofmax

Function

```
int Array<TR,TC>::indofmax () const;
```

returns a 1-based index of an array's element with maximum absolute value. The function is *inherited*[5] in all classes of the library: rvector, cvector, rmatrix, cmatrix, srmatrix, scmatrix, srbmatrix, scbmatrix, srsmatrix and schmatrix. See also Array. Example:

```
using namespace cvm;

double a[] = {3., 2., -5., -4., 5., -6.};
const rvector v (a, 4);
const rmatrix m (a, 2, 3);

std::cout << v << v.indofmax () << std::endl << std::endl;
std::cout << m << m.indofmax () << std::endl;
```

prints

```
3 2 -5 -4
3

3 -5 5
2 -4 -6
6
```

[5]Calls virtual function inside

2.2.3 indofmin

Function

```
int Array<TR,TC>::indofmin () const;
```

returns a 1-based index of an array's element with minimum absolute value. The function is *inherited*[6] in all classes of the library: rvector, cvector, rmatrix, cmatrix, srmatrix, scmatrix, srbmatrix, scbmatrix, srsmatrix and schmatrix. See also Array. Example:

```
using namespace cvm;

double a[] = {3., 2., -5., 0., 0., -6.};
const rvector v (a, 4);
const rmatrix m (a, 2, 3);

std::cout << v << v.indofmin () << std::endl << std::endl;
std::cout << m << m.indofmin () << std::endl;
```

prints

```
3 2 -5 0
4

3 -5 0
2 0 -6
4
```

[6] Calls virtual function inside

2.2.4 norm

Virtual function

```
virtual TR Array<TR,TC>::norm() const;
```

returns Euclidean norm of an array that for vectors is defined as

$$\|x\|_E = \left(\sum_{i=1}^{n} |x_i|^2\right)^{1/2}$$

and for matrices as

$$\|A\|_E = \left(\sum_{i=1}^{m}\sum_{j=1}^{n} |a_{ij}|^2\right)^{1/2},$$

where A is $m \times n$ matrix. The function is *inherited* in the following classes of the library: rvector, cvector, rmatrix, cmatrix, srmatrix, scmatrix, srsmatrix and schmatrix. It's *redefined* in srbmatrix and scbmatrix. See also rvector::normalize, cvector::normalize, rmatrix::normalize, cmatrix::normalize and Array. Example:

```
using namespace cvm;
std::cout.setf (ios::scientific | ios::showpos);
std::cout.precision (12);

double a[] = {1., 2., 3., -4., 5., -6.};
const rvector v (a, 3);
const rmatrix m (a, 2, 3);

std::cout << v << v.norm () << std::endl << std::endl;
std::cout << m << m.norm () << std::endl;
```

prints

```
+1.000000000000e+000 +2.000000000000e+000 +3.000000000000e+000
+3.741657386774e+000

+1.000000000000e+000 +3.000000000000e+000 +5.000000000000e+000
+2.000000000000e+000 -4.000000000000e+000 -6.000000000000e+000
+9.539392014169e+000
```

2.2.5 norminf

Virtual function

```
virtual TR Array<TR,TC>::norminf () const;
```

returns an infinity norm of an array that for vectors is defined as

$$\|x\|_\infty = \max_{i=1,\dots,n} |x_i|$$

and for matrices as

$$\|A\|_\infty = \max_{i=1,\dots,m} \sum_{j=1}^{n} |a_{ij}|,$$

where A is $m \times n$ matrix. The function is *inherited* in `rvector` and `cvector` classes of the library. It's *redefined* in `Matrix`, `srbmatrix`, `scbmatrix`, `srsmatrix` and `schmatrix`. See also `Matrix::norm1`, `Array`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., -4., 5., -6.};
const rvector v (a, 3);
const rmatrix m (a, 2, 3);

std::cout << v << v.norminf () << std::endl;
std::cout << m << m.norminf () << std::endl;
```

prints

```
1 2 3
3

1 3 5
2 -4 -6
12
```

2.2.6 norm1

Virtual function

```
virtual TR Array<TR,TC>::norm1 () const;
```

returns a 1-norm of an array that for vectors is defined as

$$\|x\|_1 = \sum_{i=1}^{n} |x_i|$$

and for matrices as

$$\|A\|_1 = \max_{j=1,\ldots,n} \sum_{i=1}^{m} |a_{ij}|,$$

where x is a vector of size n and A is an $m \times n$ matrix. The function is *inherited* in rvector and cvector classes. It's *redefined* in rmatrix and cmatrix and inherited thereafter. See also Array. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (12);

double a[] = {1., 2., 3., -4., 5., -6.};
const rvector v (a, 3);
const rmatrix m (a, 2, 3);

std::cout << v << v.norm1 () << std::endl << std::endl;
std::cout << m << m.norm1 () << std::endl;
```

prints

```
+1.000000000000e+000 +2.000000000000e+000 +3.000000000000e+000
+6.000000000000e+000

+1.000000000000e+000 +3.000000000000e+000 +5.000000000000e+000
+2.000000000000e+000 -4.000000000000e+000 -6.000000000000e+000
+1.100000000000e+001
```

2.2.7 norm2

Virtual function

```
virtual TR Array<TR,TC>::norm2 () const;
```

returns a 2-norm of an array that for vectors is defined as

$$\|x\|_2 = \|x\|_E = \left(\sum_{i=1}^{n} |x_i|^2\right)^{1/2}$$

and for matrices as

$$\|A\|_2 = \max_i \sigma_i = \left(\max_{|x|=1} (Ax \cdot Ax)\right)^{1/2},$$

where σ_i is an i-th singular value of $m \times n$ matrix A, $i = 1, \dots, \min(m, n)$. The function is *inherited* in rvector and cvector classes. It's *redefined* in rmatrix and cmatrix and inherited thereafter. See also Array. Example:

```
using namespace cvm;
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (12);

double a[] = {1., 2., 3., -4., 5., -6.};
const rvector v (a, 3);
const rmatrix m (a, 2, 3);

std::cout << v << v.norm2 () << std::endl << std::endl;
std::cout << m << m.norm2 () << std::endl;
```

prints

```
+1.000000000000e+000 +2.000000000000e+000 +3.000000000000e+000
+3.741657386774e+000

+1.000000000000e+000 +3.000000000000e+000 +5.000000000000e+000
+2.000000000000e+000 -4.000000000000e+000 -6.000000000000e+000
+9.319612060784e+000
```

2.2.8 operator >> <> (std::istream& is, Array<TR,TC>& aIn)

Friend template operator

```
template <typename TR, typename TC>
friend std::istream& operator >> <> (std::istream& is,
                                     Array<TR,TC>& aIn);
```

fills an object referenced by parameter aIn with numbers from is stream. See also basic_array::operator >> , Array. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    std::ofstream os;
    os.open ("in.txt");
    os << 1.2 << " " << 2.3 << std::endl << 3.4;
    os.close ();

    std::ifstream is("in.txt");
    rvector v(5);
    is >> v;

    std::cout << v;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1.20e+000 2.30e+000 3.40e+000 0.00e+000 0.00e+000
```

2.2.9 operator << <> (std::ostream& os, const Array<TR,TC>& aOut)

Friend template operator

```
template <typename TR, typename TC>
friend std::ostream& operator << <> (std::ostream& os,
                                     const Array<TR,TC>& aOut);
```

outputs an object referenced by aOut into os stream. The operator is redefined in the class Matrix. See also basic_array::operator << , Array. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

cvector v(3);
v(1) = tcomplex (1., 2.);
v(2) = tcomplex (3., 4.);

std::cout << v;
```

prints

```
(1.00e+000,2.00e+000) (3.00e+000,4.00e+000) (0.00e+000,0.00e+000)
```

2.3 rvector

This is end-user class encapsulating a vector in Euclidean space of real numbers.

```
template <typename TR>
class rvector : public Array<TR,TR> {
public:
    rvector ();
    explicit rvector (int nSize);
    rvector (int nSize, TR d);
    rvector (TR* pD, int nSize, int nIncr = 1);
    rvector (const rvector& v);
    rvector& operator = (const rvector& v) throw (cvmexception);
    rvector& assign (const TR* p, int nIncr = 1);
    rvector& assign (int n, const rvector& v) throw (cvmexception);
    rvector& set (TR x);
    rvector& resize (int nNewSize) throw (cvmexception);
    bool operator == (const rvector& v) const;
    bool operator != (const rvector& v) const;
    rvector& operator << (const rvector& v) throw (cvmexception);
    rvector operator + (const rvector& v) const throw (cvmexception);
    rvector operator - (const rvector& v) const throw (cvmexception);
    rvector& sum (const rvector& v1,
                  const rvector& v2) const throw (cvmexception);
    rvector& diff (const rvector& v1,
                   const rvector& v2) const throw (cvmexception);
    rvector& operator += (const rvector& v) throw (cvmexception);
    rvector& operator -= (const rvector& v) throw (cvmexception);
    rvector operator - () const;
    rvector operator * (TR d) const throw (cvmexception);
    rvector operator / (TR d) const throw (cvmexception);
    rvector& operator *= (TR d);
    rvector& operator /= (TR d) throw (cvmexception);
    rvector& normalize ();
    TR operator * (const rvector& v) const throw (cvmexception);
    rvector operator * (const rmatrix& m) const
                        throw (cvmexception);
    rvector& mult (const rvector& v, const rmatrix& m)
                   throw (cvmexception);
    rvector& mult (const rmatrix& m, const rvector& v)
                   throw (cvmexception);
    rmatrix rank1update (const rvector& v) const;
    rvector& solve (const srmatrix& mA,
```

```
                    const rvector& vB, TR& dErr)
                    throw (cvmexception);
    rvector& solve (const srmatrix& mA,
                    const rvector& vB) throw (cvmexception);
    rvector& solve_lu (const srmatrix& mA, const srmatrix& mLU,
                       const int* pPivots, const rvector& vB, TR& dErr)
                       throw (cvmexception);
    rvector& solve_lu (const srmatrix& mA, const srmatrix& mLU,
                       const int* pPivots, const rvector& vB)
                       throw (cvmexception);
    rvector& svd (const rmatrix& mArg) throw (cvmexception);
    rvector& svd (const cmatrix& mArg) throw (cvmexception);
    rvector& svd (const rmatrix& mArg,
                  srmatrix& mU, srmatrix& mVH) throw (cvmexception);
    rvector& svd (const cmatrix& mArg,
                  scmatrix& mU, scmatrix& mVH) throw (cvmexception);
    rvector& eig (const srsmatrix& mArg) throw (cvmexception);
    rvector& eig (const srsmatrix& mArg,
                  srmatrix& mEigVect) throw (cvmexception);
    rvector& eig (const schmatrix& mArg) throw (cvmexception);
    rvector& eig (const schmatrix& mArg,
                  scmatrix& mEigVect) throw (cvmexception);
    rvector& gemv (bool bLeft, const rmatrix& m, TR dAlpha,
                   const rvector& v, TR dBeta) throw (cvmexception);
    rvector& gbmv (bool bLeft, const srbmatrix& m, TR dAlpha,
                   const rvector& v, TR dBeta) throw (cvmexception);
    rvector& randomize (TR dFrom, TR dTo);
};
```

2.3.1 rvector ()

Constructor

```
rvector::rvector ();
```

creates an empty `rvector` object. See also `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

rvector v;
std::cout << v.size() << std::endl;

v.resize (5);
v(1) = 1.5;
std::cout << v;
```

prints

```
0
1.50e+000 0.00e+000 0.00e+000 0.00e+000 0.00e+0000
```

2.3.2 rvector (int)

Constructor

```
explicit rvector::rvector (int nSize);
```

creates a `rvector` object of size equal to `nSize`. The constructor throws an exception of type `cvmexception` in case of non-positive size passed or memory allocation failure. See also `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

rvector v (5);
std::cout << v;
```

prints

```
0.00e+000 0.00e+000 0.00e+000 0.00e+000 0.00e+000
```

2.3.3 rvector (int, TR)

Constructor

```
rvector::rvector (int nSize, TR d);
```

creates a `rvector` object of size equal to `nSize` and fills it with value of `d`. The constructor throws an exception of type `cvmexception` in case of non-positive size passed or memory allocation failure. See also `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

rvector v (5, 1.5);
std::cout << v;
```

prints

```
1.50e+00 1.50e+00 1.50e+00 1.50e+00 1.50e+00
```

2.3.4 rvector (TR*,int,int)

Constructor

```
rvector::rvector (TR* pD, int nSize, int nIncr = 1);
```

creates a `rvector` object of size equal to `nSize`. Unlike others, this constructor *does not allocate a memory*. It just shares a memory with an array pointed to by `pD` using distance between elements equal to `nIncr`. See also `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

double a[] = {1., 2., 3., 4., 5., 6., 7.,};

rvector v1 (a, 4, 2);

std::cout << v1;
v1(2) = 88.;
std::cout << v1 << std::endl;

for (int i = 0; i < 3; i++) {
    std::cout << a[i] << " ";
}
std::cout << std::endl;

rvector v2 (a, 5);
std::cout << v2;
```

prints

```
1.00e+000 3.00e+000 5.00e+000 7.00e+000
1.00e+000 8.80e+001 5.00e+000 7.00e+000

1.00e+000 2.00e+000 8.80e+001
1.00e+000 2.00e+000 8.80e+001 4.00e+000 5.00e+000
```

2.3.5 rvector (const rvector&)

Copy constructor

```
rvector::rvector (const rvector& v);
```

creates a `rvector` object of size equal to size of vector `v` and sets every element of created vector to a value of appropriate element of `v`. The constructor throws an exception of type `cvmexception` in case of memory allocation failure. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

double a[] = {1., 2., 3., 4., 5., 6., 7.,};
const rvector v (a, 4, 2);
rvector vc (v);

vc(1) = 88.;
std::cout << vc;
std::cout << v;
```

prints

```
8.80e+001 3.00e+000 5.00e+000 7.00e+000
1.00e+000 3.00e+000 5.00e+000 7.00e+000
```

2.3.6 operator = (const rvector&)

Operator

`rvector& rvector::operator = (const rvector& v) throw (cvmexception);`

sets every element of a calling vector to a value of appropriate element of a vector `v` and returns a reference to the vector changed. The operator throws an exception of type `cvmexception` in case of different vector sizes. See also `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    double a[] = {1., 2., 3., 4., 5.};
    const rvector v (a, 5);
    rvector vc(5);

    vc = v;
    std::cout << vc;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1.00e+000 2.00e+000 3.00e+000 4.00e+000 5.00e+000
```

2.3.7 assign(const TR*, int)

Function

```
rvector& rvector::assign (const TR* p, int nIncr = 1);
```

sets every element of a calling vector to a value of every nIncr-th element of an array pointed to by parameter p and returns a reference to the vector changed. See also rvector. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

const double a[] = {1., 2., 3., 4., 5., 6., 7.,};
rvector v (5);
rvector v2 (4);

v.assign(a);
v2.assign(a, 2);
std::cout << v;
std::cout << v2;
```

prints

```
1.00e+000 2.00e+000 3.00e+000 4.00e+000 5.00e+000
1.00e+000 3.00e+000 5.00e+000 7.00e+000
```

2.3.8 assign (int, const rvector&)

Function

```
rvector& rvector::assign (int n, const rvector& v) throw (cvmexception);
```

sets every element of a calling vector's sub-vector beginning with 1-based index `n` to a vector `v` and returns a reference to the vector changed. The function throws an exception of type `cvmexception` if `n` is not positive or `v.size()+n-1` is greater than a calling vector's size. See also `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

rvector v1(5);
rvector v2(2);
v1.set(1.);
v2.set(2.);
v1.assign(3, v2);
std::cout << v1;
```

prints

```
1.00e+00 1.00e+00 2.00e+00 2.00e+00 1.00e+00
```

2.3.9 set(TR)

Function

```
rvector& rvector::set (TR x);
```

sets every element of a calling vector to a value of parameter x and returns a reference to the vector changed. See also rvector. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

rvector v(5);
v.set(3.);
std::cout << v;
```

prints

```
3.00e+000 3.00e+000 3.00e+000 3.00e+000 3.00e+000
```

2.3.10 resize

Function

```
rvector& rvector::resize (int nNewSize) throw (cvmexception);
```

changes a size of a calling vector to be equal to `nNewSize` and returns a reference to the vector changed. In case of increasing of its size, the vector is filled up with zeroes. The function throws an exception of type `cvmexception` in case of negative size passed or memory allocation failure. See also `basic_array::resize` and `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    double a[] = {1., 2., 3., 4.};
    rvector v (a, 3);
    std::cout << v;
    v.resize(2);
    std::cout << v;
    v.resize(4);
    std::cout << v;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1.00e+00 2.00e+00 3.00e+00
1.00e+00 2.00e+00
1.00e+00 2.00e+00 0.00e+00 0.00e+00
```

2.3.11 operator ==

Operator

```
bool rvector::operator == (const rvector& v) const;
```

compares a calling vector with a vector `v` and returns `true` if they have the same sizes and their appropriate elements differ by not more than the smallest normalized positive number. Returns `false` otherwise. See also `rvector`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4.};
rvector v1 (a, 4);
rvector v2 (4);

v2 (1) = 1.; v2 (2) = 2.;
v2 (3) = 3.; v2 (4) = 4.;

cout << (v1 == v2) << endl;
```

prints

```
1
```

2.3.12 `operator !=`

Operator

`bool rvector::operator != (const rvector& v) const;`

compares a calling vector with a vector `v` and returns `true` if they have different sizes or at least of their appropriate elements differs by more than the smallest normalized positive number. Returns `false` otherwise. See also `rvector`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4.};
rvector v1 (a, 4);
rvector v2 (4);

v2 (1) = 1.; v2 (2) = 2.;
v2 (3) = 3.; v2 (4) = 4.;

cout << (v1 != v2) << endl;
```

prints

```
0
```

2.3.13 operator <<

Operator

```
rvector& rvector::operator << (const rvector& v) throw (cvmexception);
```

destroys a calling vector, creates a new one as a copy of v and returns a reference to the vector changed. The operator throws an exception of type cvmexception in case of memory allocation failure. See also rvector. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    rvector v (5);
    rvector vc (3);
    v(1) = 1.;
    v(2) = 2.;
    std::cout << v << vc << std::endl;

    vc << v;
    std::cout << vc;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1.00e+000 2.00e+000 0.00e+000 0.00e+000 0.00e+000
0.00e+000 0.00e+000 0.00e+000

1.00e+000 2.00e+000 0.00e+000 0.00e+000 0.00e+000
```

2.3.14 operator +

Operator

`rvector rvector::operator + (const rvector& v) const throw (cvmexception);`

creates an object of type `rvector` as a sum of a calling vector and vector `v`. It throws an exception of type `cvmexception` in case of different sizes of the operands or memory allocation failure. See also `rvector::sum`, `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    double a[] = {1., 2., 3., 4.};
    const double b[] = {3., 5., 7., 9.};
    const rvector va (a, 4);
    rvector vb (4);
    vb.assign(b);

    std::cout << va + vb;
    std::cout << va + va;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
4.00e+000 7.00e+000 1.00e+001 1.30e+001
2.00e+000 4.00e+000 6.00e+000 8.00e+000
```

2.3.15 operator -

Operator

```
rvector rvector::operator - (const rvector& v) const throw (cvmexception);
```

creates an object of type `rvector` as a difference of a calling vector and vector `v`. It throws an exception of type `cvmexception` in case of different sizes of the operands or memory allocation failure. See also `rvector::diff`, `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    double a[] = {1., 2., 3., 4.};
    const double b[] = {3., 5., 7., 9.};
    const rvector va (a, 4);
    rvector vb (4);
    vb.assign(b);

    std::cout << va - vb;
    std::cout << va - va;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
-2.00e+000 -3.00e+000 -4.00e+000 -5.00e+000
0.00e+000 0.00e+000 0.00e+000 0.00e+000
```

2.3.16 sum

Function

```
rvector& rvector::sum (const rvector& v1, const rvector& v2)
throw (cvmexception);
```

assigns a result of addition of vectors v1 and v2 to a calling vector and returns a reference to the vector changed. It throws an exception of type cvmexception in case of different sizes of the operands. See also rvector::operator + , rvector. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    double a[] = {1., 2., 3., 4.};
    double b[] = {2., 3., 4., 5.};
    rvector va (a, 4);
    rvector vb (b, 4);
    rvector v (4);

    std::cout << v.sum(va, vb);
    std::cout << v.sum(v, va);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
3.00e+00 5.00e+00 7.00e+00 9.00e+00
4.00e+00 7.00e+00 1.00e+01 1.30e+01
```

2.3.17 diff

Function

```
rvector& rvector::diff (const rvector& v1, const rvector& v2)
throw (cvmexception);
```

assigns a result of subtraction of vectors `v1` and `v2` to a calling vector and returns a reference to the vector changed. It throws an exception of type `cvmexception` in case of different sizes of the operands. See also `rvector::operator -` , `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    double a[] = {1., 2., 3., 4.};
    double b[] = {2., 3., 4., 5.};
    rvector va (a, 4);
    rvector vb (b, 4);
    rvector v (4);

    std::cout << v.diff(va, vb);
    std::cout << v.diff(v, va);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
-1.00e+00 -1.00e+00 -1.00e+00 -1.00e+00
-2.00e+00 -3.00e+00 -4.00e+00 -5.00e+00
```

2.3.18 operator +=

Operator

```
rvector& rvector::operator += (const rvector& v) throw (cvmexception);
```

adds a vector v to a calling vector and returns a reference to the vector changed. It throws an exception of type cvmexception in case of different sizes of the operands. See also rvector::operator + , rvector::sum, rvector. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    rvector v1 (4);
    rvector v2 (4);
    v1.set(1.);
    v2.set(2.);

    v1 += v2;
    std::cout << v1;

    // well, you can do this too, but temporary object would be created
    v2 += v2;
    std::cout << v2;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
3.00e+00 3.00e+00 3.00e+00 3.00e+00
4.00e+00 4.00e+00 4.00e+00 4.00e+00
```

2.3.19 operator -=

Operator

```
rvector& rvector::operator -= (const rvector& v) throw (cvmexception);
```

subtracts a vector v from a calling vector and returns a reference to the vector changed. It throws an exception of type cvmexception in case of different sizes of the operands. See also rvector::operator - , rvector::diff, rvector. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    rvector v1 (4);
    rvector v2 (4);
    v1.set(1.);
    v2.set(2.);

    v1 -= v2;
    std::cout << v1;

    // well, you can do this too, but temporary object would be created
    v2 -= v2;
    std::cout << v2;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
-1.00e+00 -1.00e+00 -1.00e+00 -1.00e+00
0.00e+00 0.00e+00 0.00e+00 0.00e+00
```

2.3.20 operator - ()

Operator

```
rvector rvector::operator - () const throw (cvmexception);
```

creates an object of type `rvector` as a calling vector multiplied by -1. It throws an exception of type `cvmexception` in case of memory allocation failure. See also `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

double a[] = {1., 2., 3., 4.};
const rvector v (a, 4);

std::cout << - v;
```

prints

```
-1.00e+00 -2.00e+00 -3.00e+00 -4.00e+00
```

2.3.21 operator * (TR)

Operator

```
rvector rvector::operator * (TR d) const throw (cvmexception);
```

creates an object of type `rvector` as a product of a calling vector and a number `d`. It throws an exception of type `cvmexception` in case of memory allocation failure. See also `rvector::operator *=`, `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

double a[] = {1., 2., 3., 4.};
const rvector v (a, 4);

std::cout << v * 5.;
```

prints

```
5.00e+00 1.00e+01 1.50e+01 2.00e+01
```

2.3.22 operator / (TR)

Operator

```
rvector rvector::operator / (TR d) const throw (cvmexception);
```

creates an object of type `rvector` as a quotient of a calling vector and a number `d`. It throws an exception of type `cvmexception` if `d` has an absolute value equal or less than the smallest normalized positive number. It also throws the exception in case of memory allocation failure. See also `rvector::operator /=`, `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    double a[] = {1., 2., 3., 4.};
    const rvector v (a, 4);

    std::cout << v / 2.;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
5.00e-01 1.00e+00 1.50e+00 2.00e+00
```

2.3.23 operator *=

Operator

```
rvector& rvector::operator *= (TR d);
```

multiplies a calling vector by number `d` and returns a reference to the vector changed. See also `rvector::operator *` , `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

rvector v (4);
v.set(2.);
v *= 2.
std::cout << v;
```

prints

```
4.00e+00 4.00e+00 4.00e+00 4.00e+00
```

2.3.24 operator /=

Operator

```
rvector& rvector::operator /= (TR d) throw (cvmexception);
```

divides a calling vector by number d and returns a reference to the vector changed. It throws an exception of type cvmexception if d has an absolute value equal or less than the smallest normalized positive number. See also rvector::operator / , rvector. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    rvector v (4);
    v.set(3.);
    v /= 2.;
    std::cout << v;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1.50e+00 1.50e+00 1.50e+00 1.50e+00
```

2.3.25 normalize

Function

```
rvector& rvector::normalize ();
```

normalizes a calling vector so its Euclidean norm becomes equal to 1 if it was greater than the smallest normalized positive number before the call (otherwise the function does nothing). See also `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

rvector v(4);
v(1) = 1.;
v(2) = 2.;
v(3) = 3.;
v(4) = 4.;
std::cout << v.normalize();
std::cout << v.norm() << std::endl;
```

prints

```
1.83e-01 3.65e-01 5.48e-01 7.30e-01
1.00e+00
```

2.3.26 operator * (const rvector&)

Operator

```
TR rvector::operator * (const rvector& v) const throw (cvmexception);
```

returns a scalar product of a calling vector and v. It throws an exception of type cvmexception if the operands have different sizes. See also rvector. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    double a[] = {1., 2., 3., 4.};
    rvector v1(4);
    rvector v2(4);
    v1.assign(a);
    v2.assign(a);

    std::cout << v1 * v2 << std::endl;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
3.00e+01
```

2.3.27 operator * (const rmatrix&)

Operator

```
rvector rvector::operator * (const rmatrix& m) const
throw (cvmexception);
```

creates an object of type `rvector` as a product of a calling vector and a matrix `m`. Use `rvector::mult (const rvector&, const rmatrix&)` in order to avoid creation of the object. This operator throws an exception of type `cvmexception` if the calling vector's size differs from a number of rows in the matrix `m`. See also `rvector`, `rmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    rvector v(2);
    rmatrix m(2, 3);
    v.set(2.);
    m.set(1.);

    std::cout << v * m;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
4.00e+00 4.00e+00 4.00e+00
```

2.3.28 mult (const rvector&, const rmatrix&)

Function

```
rvector& rvector::mult (const rvector& v, const rmatrix& m)
throw (cvmexception);
```

sets a calling vector to be equal to a product of a vector v by a matrix m and returns a reference to the object changed. See also rvector::mult (const rmatrix&, const rvector&), rvector. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    rvector v2(2), v3(3);
    rmatrix m(2, 3);
    v2.set(2.);
    m.set(1.);

    std::cout << v3.mult(v2, m);
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
4.00e+00 4.00e+00 4.00e+00
```

2.3.29 mult (const rmatrix&, const rvector&)

Function

```
rvector& rvector::mult (const rmatrix& m, const rvector& v)
throw (cvmexception);
```

sets a calling vector to be equal to a product of a matrix `m` by a vector `v` and returns a reference to the vector changed. See also `rvector::mult (const rvector&, const rmatrix&)`, `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    rvector v2(2), v3(3);
    rmatrix m(2, 3);
    v3.set(2.);
    m.set(1.);

    std::cout << v2.mult(m, v3);
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
6.00e+00 6.00e+00
```

2.3.30 rank1update

Function

```
rmatrix rvector::rank1update (const rvector& v) const;
```

creates an object of type rmatrix as a rank-1 update of a calling vector and a vector v. The rank-1 update operation of a vector-column x of a size m and a vector-row y of a size n is defined as $m \times n$ matrix

$$\begin{pmatrix} x_1y_1 & x_1y_2 & \cdots & x_1y_n \\ x_2y_1 & x_2y_2 & \cdots & x_2y_n \\ \multicolumn{4}{c}{\dotfill} \\ x_my_1 & x_my_2 & \cdots & x_my_n \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix} \begin{pmatrix} y_1 & y_2 & \cdots & y_n \end{pmatrix}.$$

See also rvector. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

rvector v1(3);
rvector v2(2);
v1.set(2.);
v2.set(2.);
std::cout << v1.rank1update (v2);
```

prints

```
4.00e+00 4.00e+00
4.00e+00 4.00e+00
4.00e+00 4.00e+00
```

2.3.31 solve

Functions

```
rvector& rvector::solve (const srmatrix& mA,
                         const rvector& vB, TR& dErr)
                         throw (cvmexception);
rvector& rvector::solve (const srmatrix& mA,
                         const rvector& vB)
                         throw (cvmexception);
```

set a calling vector to be equal to a solution x of a linear equation $Ax = b$ where parameter `mA` is the square matrix A and parameter `vB` is the vector b. Every function returns a reference to the vector changed. The first version also sets output parameter `dErr` to be equal to a norm of computation error. These functions throw exception of type `cvmexception` in case of inappropriate sizes of the objects or when the matrix A is close to cingular. See also `rvector`, `srmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (12);

try {
    double m[] = {1., -1., 1., 2., -2., 1., 3., -2., 1.};
    double b[] = {1., 2., 3.};
    srmatrix ma(m, 3);
    rvector  vb(b, 3);
    rvector  vx(3);
    double   dErr = 0.;

    std::cout << vx.solve (ma, vb, dErr);
    std::cout << dErr << std::endl;

    std::cout << ma * vx - vb;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
+8.000000000000e+00 -8.000000000000e+00 +3.000000000000e+00
+6.661338147751e-15
+0.000000000000e+00 +0.000000000000e+00 +0.000000000000e+00
```

2.3.32 solve_lu

Functions

```
rvector&
rvector::solve_lu (const srmatrix& mA, const srmatrix& mLU,
                   const int* pPivots, const rvector& vB, TR& dErr)
                   throw (cvmexception);

rvector&
rvector::solve_lu (const srmatrix& mA, const srmatrix& mLU,
                   const int* pPivots, const rvector& vB)
                   throw (cvmexception);
```

set a calling vector to be equal to a solution x of a linear equation $Ax = b$ where parameter `mA` is the square matrix A, parameter `mLU` is LU factorization of the matrix A, parameter `pPivots` is an array of pivot numbers created while factorizing the matrix A and parameter `vB` is the vector b. Every function returns a reference to the vector changed. The first version also sets output parameter `dErr` to be equal to a norm of computation error. These functions are useful when you need to solve few linear equations of kind $Ax = b$ with the same matrix A and different vectors b. In such case you save on matrix A factorization since it's needed to be performed just one time. These functions throw exception of type `cvmexception` in case of inappropriate sizes of the objects or when the matrix A is close to cingular. See also `rvector`, `srmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (12);

try {
    double m[] = {1., -1., 1., 2., -2., 1., 3., -2., 1.};
    double b1[] = {1., 2., 3.};
    double b2[] = {0., -1., -2.};
    srmatrix ma(m, 3);
    srmatrix mLU(3);
    rvector  vb1(b1, 3);
    rvector  vb2(b2, 3);
    rvector  vx1(3);
    rvector  vx2(3);
    iarray   nPivots(3);
    double   dErr = 0.;

    mLU.low_up(ma, nPivots);
    std::cout << vx1.solve_lu (ma, mLU, nPivots, vb1, dErr);
```

```
    std::cout << dErr << std::endl;
    std::cout << vx2.solve_lu (ma, mLU, nPivots, vb2);
    std::cout << ma * vx1 - vb1 << ma * vx2 - vb2;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
+8.000000000000e+00 -8.000000000000e+00 +3.000000000000e+00
+6.661338147751e-15
-5.000000000000e+00 +4.000000000000e+00 -1.000000000000e+00
+5.329070518201e-15
+0.000000000000e+00 +0.000000000000e+00 +0.000000000000e+00
+0.000000000000e+00 +0.000000000000e+00 +0.000000000000e+00
```

2.3.33 svd

Functions

```
rvector&
rvector::svd (const rmatrix& mArg) throw (cvmexception);
rvector&
rvector::svd (const cmatrix& mArg) throw (cvmexception);

rvector&
rvector::svd (const rmatrix& mArg,
              srmatrix& mU, srmatrix& mVH) throw (cvmexception);
rvector&
rvector::svd (const cmatrix& mArg,
              scmatrix& mU, scmatrix& mVH) throw (cvmexception);
```

set a calling vector to be equal to the singular values

$$\sigma_1 \geqslant \sigma_2 \geqslant \cdots \geqslant \sigma_{\min(m,n)} \geqslant 0$$

of $m \times n$ matrix A (parameter `mArg`). These values are the main diagonal of matrix Σ of the singular value decomposition

$$A = U\Sigma V^H$$

where U and V are orthogonal for real A and unitary for complex A. V^H is transposed V for real one and hermitian conjugated V for complex one. First $\min(m, n)$ columns of the matrices U and V are left and right singular vectors of A respectively. Singular values and singular vectors satisfy

$$Av_i = \sigma_i u_i \quad \text{and} \quad A^H u_i = \sigma_i v_i$$

where u_i and v_i are i-th columns of U and V respectively. Third and fourth versions of the functions set output parameter `mU` to be equal to the matrix mU of size $m \times m$ and `mVH` to be equal to the matrix V^H of size $n \times n$. All the functions return a reference to the object they change and throw exception of type `cvmexception` in case of inappropriate calling object size (it must be equal to $\min(m, n)$), matrix `mU` size (should be $m \times m$), matrix `mVH` size (should be $n \times n$) or in case of convergence error. See also `rvector`, `rmatrix`, `cmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (10);

try {
    double m[] = {1., -1., 1., 2., -2., 1.,
                  3., -2., 1., 0., -2., 1.};
```

```
    rmatrix mA(m,4,3);
    rmatrix mSigma(4,3);
    rvector v(3);
    srmatrix mU(4), mVH(3);

    v.svd(mA, mU, mVH);
    mSigma.diag(0) = v;

    std::cout << mU << std::endl;
    std::cout << mVH << std::endl;
    std::cout << mSigma << std::endl;

    std::cout << (mA * ~mVH - mU * mSigma).norm() << std::endl;
    std::cout << (~mA * mU - ~(mSigma * mVH)).norm() << std::endl;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
-4.8425643615e-01 +1.9516809011e-01 +1.1506232201e-02 -8.5280286542e-01
+2.1685987119e-01 -3.4107922671e-01 -8.8948423927e-01 -2.1320071636e-01
+6.6237057295e-01 +7.1553688692e-01 -6.1787070600e-02 -2.1320071636e-01
-5.2889765022e-01 +5.7756501033e-01 -4.5262319054e-01 +4.2640143271e-01

-2.2124855498e-01 +8.5354150454e-01 -4.7171599183e-01
+9.5937301747e-01 +1.0365951763e-01 -2.6240830353e-01
-1.7507852602e-01 -5.1060905244e-01 -8.4179920723e-01

+4.9561500411e+00 +0.0000000000e+00 +0.0000000000e+00
+0.0000000000e+00 +2.5088408581e+00 +0.0000000000e+00
+0.0000000000e+00 +0.0000000000e+00 +3.7721919242e-01
+0.0000000000e+00 +0.0000000000e+00 +0.0000000000e+00

+1.3710111285e-15
+2.4829995848e-15
```

2.3.34 eig

Functions

```
rvector&
rvector::eig (const srsmatrix& mArg) throw (cvmexception);
rvector&
rvector::eig (const schmatrix& mArg) throw (cvmexception);

rvector&
rvector::eig (const srsmatrix& mArg,
              srmatrix& mEigVect) throw (cvmexception);
rvector&
rvector::eig (const schmatrix& mArg,
              scmatrix& mEigVect) throw (cvmexception);
```

solve a symmetric eigenvalue problem and set a calling vector to be equal to eigenvalues of a square matrix `mArg`. The symmetric eigenvalue problem is defined as follows: given a symmetric or Hermitian matrix A, find the eigenvalues λ and the corresponding eigenvectors z that satisfy the equation

$$Az = \lambda z.$$

All n eigenvalues are real not only for real symmetric but also for complex Hermitian matrices A, and there exists an orthogonal system of n eigenvectors. If A is a symmetric or Hermitian positive-definite matrix, all eigenvalues are positive. See [3] for further details. The third and fourth versions of the functions set an output parameter `mEigVect` to be equal to a square matrix containing eigenvectors as columns. All the functions return a reference to the vector they change and throw an exception of type `cvmexception` in case of inappropriate calling object sizes or in case of convergence error. See also `rvector`, `cvector::eig`, `srsmatrix`, `schmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (3);

try {
    srsmatrix m(3);
    srmatrix me(3);
    rvector v(3);
    m.randomize(1., 3.);

    v.eig (m, me);
    std::cout << v;
```

```
    std::cout << m * me(1) - me(1) * v(1);
    std::cout << m * me(2) - me(2) * v(2);
    std::cout << m * me(3) - me(3) * v(3);
    std::cout << me(1) * me(2) << std::endl; // orthogonality check

    schmatrix mc(3);
    scmatrix mce(3);
    mc.randomize_real(1., 3.);
    mc.randomize_imag(1., 3.);

    v.eig (mc, mce);
    std::cout << v;

    std::cout << mc * mce(1) - mce(1) * v(1);
    std::cout << mc * mce(2) - mce(2) * v(2);
    std::cout << mc * mce(3) - mce(3) * v(3);
    std::cout << mce(1) % mce(2) << std::endl; // orthogonality check
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
-9.360e-01 +3.535e-01 +6.376e+00
-4.441e-16 -5.551e-16 -6.106e-16
+3.331e-16 +1.145e-16 +1.110e-16
-4.441e-16 +0.000e+00 -4.441e-16
+2.060e-17
-3.274e+00 +9.710e-01 +8.209e+00
(-4.441e-16,-1.221e-15) (-1.443e-15,-4.441e-16) (-8.882e-16,+4.683e-16)
(-5.551e-16,-2.776e-16) (+0.000e+00,-4.025e-16) (+6.661e-16,-2.461e-17)
(-5.551e-16,+0.000e+00) (+4.441e-16,-4.441e-16) (+0.000e+00,+3.896e-16)
(+1.608e-16,-2.261e-17)
```

2.3.35 gemv

Function

```
rvector& rvector::gemv (bool bLeft, const rmatrix& m, TR dAlpha,
                        const rvector& v, TR dBeta) throw (cvmexception);
```

calls one of ?GEMV routines of the BLAS library performing a matrix-vector operation defined as

$$c = \alpha M \cdot v + \beta c \quad \text{or} \quad c = \alpha v \cdot M + \beta c,$$

where α and β are real numbers (parameters dAlpha and dBeta), M is a matrix (parameter m) and v and c are vectors (parameter v and calling vector respectively). First operation is performed if bLeft passed is false and second one otherwise. The function returns a reference to the vector changed and throws an exception of type cvmexception in case of inappropriate sizes of the operands. See also rvector, rmatrix. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (7);

try {
    double alpha = 1.3;
    double beta = -0.7;
    rmatrix m(4,3);
    rvector c(4);
    rvector v(3);
    m.randomize(-1., 2.); v.randomize(-1., 3.); c.randomize(0., 2.);

    std::cout << m * v * alpha + c * beta;
    std::cout << c.gemv(false, m, alpha, v, beta);
    std::cout << c * m * alpha + v * beta;
    std::cout << v.gemv(true, m, alpha, c, beta);
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
-3.5397829e-02 +3.1990410e-02 +3.2633344e-01 -5.4669713e-01
-3.5397829e-02 +3.1990410e-02 +3.2633344e-01 -5.4669713e-01
-4.7697026e-01 -2.2544922e-01 -5.5204984e-01
-4.7697026e-01 -2.2544922e-01 -5.5204984e-01
```

2.3.36 gbmv

Function

```
rvector& rvector::gbmv (bool bLeft, const srbmatrix& m, TR dAlpha,
                        const rvector& v, TR dBeta) throw (cvmexception);
```

calls one of ?GBMV routines of the BLAS library performing a matrix-vector operation defined as

$$c = \alpha M \cdot v + \beta c \quad \text{or} \quad c = \alpha v \cdot M + \beta c,$$

where α and β are real numbers (parameters dAlpha and dBeta), M is a band matrix (parameter m) and v and c are vectors (parameter v and calling vector respectively). First operation is performed if bLeft passed is false and second one otherwise. The function returns a reference to the vector changed and throws an exception of type cvmexception in case of inappropriate sizes of the operands. See also rvector, srbmatrix. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (7);

try {
    double alpha = 1.3;
    double beta = -0.7;
    srbmatrix m(3, 1, 0);
    rvector c(3);
    rvector v(3);
    m.randomize(-1., 2.); v.randomize(-1., 3.); c.randomize(0., 2.);

    std::cout << m * v * alpha + c * beta;
    std::cout << c.gbmv(false, m, alpha, v, beta);
    std::cout << c * m * alpha + v * beta;
    std::cout << v.gbmv(true, m, alpha, c, beta);
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
+1.4551599e+00 -5.1882508e-01 -5.2088503e-02
+1.4551599e+00 -5.1882508e-01 -5.2088503e-02
+7.3471591e-01 -2.6952064e-01 -2.0478054e-01
+7.3471591e-01 -2.6952064e-01 -2.0478054e-01
```

2.3.37 randomize

Function

```
rvector& rvector::randomize (TR dFrom, TR dTo);
```

fills a calling vector with pseudo-random numbers distributed between `dFrom` and `dTo`. The function returns a reference to the vector changed. See also `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (7);

rvector v(4);
v.randomize(-2.,3.);
std::cout << v;
```

prints

```
-1.1160314e+000 2.5649586e+000 8.9345988e-001 -1.1631825e+000
```

2.4 cvector

This is end-user class encapsulating a vector in Euclidean space of complex numbers.

```
template <typename TR, typename TC>
class cvector : public Array<TR,TC> {
public:
    cvector ();
    explicit cvector (int nSize);
    cvector (int nSize, TC c);
    cvector (TC* pD, int nSize, int nIncr = 1);
    cvector (const cvector& v);
    cvector (const TR* pRe, const TR* pIm, int nSize, int nIncr = 1);
    cvector (const rvector& vRe, const rvector& vIm);
    cvector (const TR* pA, int nSize,
             bool bRealPart = true, int nIncr = 1);
    explicit cvector (const rvector& v, bool bRealPart = true);
    rvector real ();
    rvector imag ();
    cvector& operator = (const cvector& v) throw (cvmexception);
    cvector& assign (const TC* p, int nIncr = 1);
    cvector& assign (int n, const cvector& v) throw (cvmexception);
    cvector& set (TC x);
    cvector& assign_real (const rvector& vRe) throw (cvmexception);
    cvector& assign_imag (const rvector& vIm) throw (cvmexception);
    cvector& set_real (TR x);
    cvector& set_imag (TR x);
    cvector& resize (int nNewSize) throw (cvmexception);
    bool operator == (const cvector& v) const;
    bool operator != (const cvector& v) const;
    cvector& operator << (const cvector& v) throw (cvmexception);
    cvector operator + (const cvector& v) const throw (cvmexception);
    cvector operator - (const cvector& v) const throw (cvmexception);
    cvector& sum (const cvector& v1,
                  const cvector& v2) const throw (cvmexception);
    cvector& diff (const cvector& v1,
                   const cvector& v2) const throw (cvmexception);
    cvector& operator += (const cvector& v) throw (cvmexception);
    cvector& operator -= (const cvector& v) throw (cvmexception);
    cvector operator - () const throw (cvmexception);
    cvector operator * (TR d) const;
    cvector operator / (TR d) const throw (cvmexception);
    cvector operator * (TC c) const;
```

```
    cvector operator / (TC c) const throw (cvmexception);
    cvector& operator *= (TR d);
    cvector& operator /= (TR d) throw (cvmexception);
    cvector& operator *= (TC c);
    cvector& operator /= (TC c) throw (cvmexception);
    cvector& normalize ();
    cvector operator ~() const throw (cvmexception);
    cvector& conj (const cvector& v) throw (cvmexception);
    cvector& conj ();
    TC operator * (const cvector& v) const throw (cvmexception);
    TC operator % (const cvector& v) const throw (cvmexception);
    cvector operator * (const cvector& v) const throw (cvmexception);
    cvector& mult (const cvector& v, const cmatrix& m)
                  throw (cvmexception);
    cvector& mult (const cmatrix& m, const cvector& v)
                  throw (cvmexception);
    cmatrix rank1update_u (const cvector& v) const;
    cmatrix rank1update_c (const cvector& v) const;
    cvector& solve (const scmatrix& mA,
                   const cvector& vB, TR& dErr) throw (cvmexception);
    cvector& solve (const scmatrix& mA,
                   const cvector& vB) throw (cvmexception);
    cvector& solve_lu (const scmatrix& mA, const scmatrix& mLU,
                      const int* pPivots, const cvector& vB, TR& dErr)
                      throw (cvmexception);
    cvector& solve_lu (const scmatrix& mA, const scmatrix& mLU,
                      const int* pPivots, const cvector& vB)
                      throw (cvmexception);
    cvector& eig (const srmatrix& mArg) throw (cvmexception);
    cvector& eig (const scmatrix& mArg) throw (cvmexception);
    cvector& eig (const srmatrix& mArg,
                 scmatrix& mEigVect) throw (cvmexception);
    cvector& eig (const scmatrix& mArg,
                 scmatrix& mEigVect) throw (cvmexception);
    cvector& gemv (bool bLeft, const cmatrix& m, TC cAlpha,
                  const cvector& v, TC dBeta) throw (cvmexception);
    cvector& gbmv (bool bLeft, const scbmatrix& m, TC dAlpha,
                  const cvector& v, TC dBeta) throw (cvmexception);
    cvector& randomize_real (TR dFrom, TR dTo);
    cvector& randomize_imag (TR dFrom, TR dTo);
};
```

2.4.1 cvector ()

Constructor

`cvector::cvector ();`

creates an empty `cvector` object. See also `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

cvector v;
std::cout << v.size() << std::endl;

v.resize (3);
v(1) = std::complex<double>(1.5, -1.);
std::cout << v;
```

prints

```
0
(1.50e+00,-1.00e+00) (0.00e+00,0.00e+00) (0.00e+00,0.00e+00)
```

2.4.2 cvector (int)

Constructor

```
explicit cvector::cvector (int nSize);
```

creates a `cvector` object of size equal to `nSize`. The constructor throws an exception of type `cvmexception` in case of non-positive size passed or memory allocation failure. See also `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

cvector v(3);
std::cout << v.size() << std::endl;
std::cout << v;
```

prints

```
3
(0.00e+00,0.00e+00) (0.00e+00,0.00e+00) (0.00e+00,0.00e+00)
```

2.4.3 cvector (int, TC)

Constructor

```
cvector::cvector (int nSize, TC c);
```

creates a `cvector` object of size equal to `nSize` and fills it with value of `c`. The constructor throws an exception of type `cvmexception` in case of non-positive size passed or memory allocation failure. See also `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

cvector v (3, std::complex<double>(1.5, -1.));
std::cout << v;
```

prints

```
(1.50e+00,-1.00e+00) (1.50e+00,-1.00e+00) (1.50e+00,-1.00e+00)
```

2.4.4 cvector (TC*,int,int)

Constructor

```
cvector::cvector (TC* pD, int nSize, int nIncr = 1);
```

creates a `cvector` object of size equal to `nSize`. Unlike others, this constructor *does not allocate a memory*. It just shares a memory with an array pointed to by `pD` with the distance between elements equal to `nIncr`. See also `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
cvector v1 ((std::complex<double>*) a, 2, 2);

std::cout << v1;
v1(2) = std::complex<double> (7.77, 8.88);
std::cout << v1 << std::endl;

for (int i = 0; i < 6; i++) {
    std::cout << a[i] << " ";
}
std::cout << std::endl;

cvector v2 ((std::complex<double>*) a, 3);
std::cout << v2;
```

prints

```
(1.00e+00,2.00e+00) (5.00e+00,6.00e+00)
(1.00e+00,2.00e+00) (7.77e+00,8.88e+00)

1.00e+00 2.00e+00 3.00e+00 4.00e+00 7.77e+00 8.88e+00
(1.00e+00,2.00e+00) (3.00e+00,4.00e+00) (7.77e+00,8.88e+00)
```

2.4.5 cvector (const cvector&)

Copy constructor

```
cvector::cvector (const cvector& v);
```

creates a `cvector` object of size equal to size of vector `v` and sets every element of created vector to a value of appropriate element of `v`. The constructor throws an exception of type `cvmexception` in case of memory allocation failure. See also `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9., 10.};
const cvector v ((std::complex<double>*) a, 3, 2);
cvector vc (v);

vc(1) = std::complex<double>(7.77,8.88);
std::cout << vc;
std::cout << v;
```

prints

```
(7.77e+00,8.88e+00) (5.00e+00,6.00e+00) (9.00e+00,1.00e+01)
(1.00e+00,2.00e+00) (5.00e+00,6.00e+00) (9.00e+00,1.00e+01)
```

2.4.6 cvector (const TR*,const TR*,int,int)

Constructor

```
cvector::cvector (const TR* pRe, const TR* pIm, int nSize, int nIncr = 1);
```

creates a `cvector` object of size equal to `nSize` and copies every `nIncr`'s element of arrays pointed to by `pRe` and `pIm` to the real and imaginary part of the object created. The constructor throws an exception of type `cvmexception` in case of non-positive size passed or memory allocation failure. See also `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

double re[] = {1., 2., 3., 4., 5.};
double im[] = {5., 4., 3., 2., 1.};
cvector v (re, im, 3, 2);

std::cout << v;
re[0] = 7.77;
std::cout << v;

const double rec[] = {1., 2., 3.};
const double imc[] = {5., 4., 3.};
const cvector vc (rec, imc, 3);
std::cout << vc;
```

prints

```
(1.00e+00,5.00e+00) (3.00e+00,4.00e+00) (5.00e+00,3.00e+00)
(1.00e+00,5.00e+00) (3.00e+00,4.00e+00) (5.00e+00,3.00e+00)
(1.00e+00,5.00e+00) (2.00e+00,4.00e+00) (3.00e+00,3.00e+00)
```

2.4.7 cvector (const rvector&, const rvector&)

Constructor

```
cvector::cvector (const rvector& vRe, const rvector& vIm);
```

creates a `cvector` object of size equal to `vRe.size()` and `vIm.size()` and copies vectors `vRe` and `vIm` to the real and imaginary part of the object created. The constructor throws an exception of type `cvmexception` in case of non-equal sizes of the parameters passed or memory allocation failure. See also `cvector` and `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

rvector vr(3), vi(3);
vr[1] = 1.;
vr[2] = 2.;
vr[3] = 3.;
vi[1] = 5.;
vi[2] = 4.;
vi[3] = 3.;

const cvector vc(vr, vi);
std::cout << vc;
```

prints

```
(1.00e+00,5.00e+00) (2.00e+00,4.00e+00) (3.00e+00,3.00e+00)
```

2.4.8 cvector (const TR*,int,bool,int)

Constructor

```
cvector::cvector (const TR* pA, int nSize,
                  bool bRealPart = true, int nIncr = 1);
```

creates a `cvector` object of size equal to `nSize` and copies every `nIncr`'s element of array pointed to by `pA` to the real (if `bRealPart` is `true`) or imaginary (if `bRealPart` is `false`) part of the object created. The constructor throws an exception of type `cvmexception` in case of non-positive size passed or memory allocation failure. See also `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

const double a[] = {1., 2., 3., 4., 5.};
cvector v1 (a, 3, false, 2);
cvector v2 (a, 2);

std::cout << v1 << v2;
```

prints

```
(0.00e+00,1.00e+00) (0.00e+00,3.00e+00) (0.00e+00,5.00e+00)
(1.00e+00,0.00e+00) (2.00e+00,0.00e+00)
```

2.4.9 cvector (const rvector&,bool)

Constructor

```
explicit cvector::cvector (const rvector& v, bool bRealPart = true);
```

creates a `cvector` object of size equal to `v.size()` and copies every element of a vector `v` to the real (if `bRealPart` is `true`) or imaginary (if `bRealPart` is `false`) part of the object created. The constructor throws an exception of type `cvmexception` in case of memory allocation failure. See also `cvector` and `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

rvector vr (3);
vr(1) = 1.;
vr(2) = 2.;
vr(3) = 3.;

cvector v1 (vr);
cvector v2 (vr, false);
std::cout << v1 << v2;
```

prints

```
(1.00e+00,0.00e+00) (2.00e+00,0.00e+00) (3.00e+00,0.00e+00)
(0.00e+00,1.00e+00) (0.00e+00,2.00e+00) (0.00e+00,3.00e+00)
```

2.4.10 real

Function

```
rvector cvector::real ();
```

creates a rvector object of size equal to a size of a calling vector sharing a memory with its real part. In other words, the vector returned is an *l-value*. See also cvector. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

cvector vc(3);
vc.set(std::complex<double>(1.,1.));
std::cout << vc << vc.real();
vc.real()(1) = 7.77;
std::cout << vc;
```

prints

```
(1.00e+00,1.00e+00) (1.00e+00,1.00e+00) (1.00e+00,1.00e+00)
1.00e+00 1.00e+00 1.00e+00
(7.77e+00,1.00e+00) (1.00e+00,1.00e+00) (1.00e+00,1.00e+00)
```

2.4.11 imag

function

```
rvector cvector::imag ();
```

creates a `rvector` object of size equal to a size of a calling vector sharing a memory with its imaginary part. In other words, the vector returned is an *l-value*. See also `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

cvector vc(3);
vc.set(std::complex<double>(1.,1.));
std::cout << vc << vc.imag();
vc.imag()(1) = 7.77;
std::cout << vc;
```

prints

```
(1.00e+00,1.00e+00) (1.00e+00,1.00e+00) (1.00e+00,1.00e+00)
1.00e+00 1.00e+00 1.00e+00
(1.00e+00,7.77e+00) (1.00e+00,1.00e+00) (1.00e+00,1.00e+00)
```

2.4.12 operator = (const cvector&)

Operator

```
cvector& cvector::operator = (const cvector& v) throw (cvmexception);
```

sets every element of a calling vector to a value of appropriate element of a vector v and returns a reference to the vector changed. The operator throws an exception of type cvmexception in case of different vector sizes. See also cvector. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    cvector v(3);
    cvector vc(3);
    v(1) = std::complex<double>(1.,2.);

    vc = v;
    std::cout << vc;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(1.00e+00,2.00e+00) (0.00e+00,0.00e+00) (0.00e+00,0.00e+00)
```

2.4.13 assign(const TC*, int)

Function

```
cvector& cvector::assign (const TC* p, int nIncr = 1);
```

sets every element of a calling vector to a value of every nIncr-th element of an array pointed to by parameter p and returns a reference to the vector changed. See also cvector. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

const double a[] = {1., 2., 3., 4., 5., 6., 7.};
cvector v1(3);
cvector v2(2);

v1.assign((const std::complex<double>*) a);
v2.assign((const std::complex<double>*) a, 2);
std::cout << v1;
std::cout << v2;
```

prints

```
(1.00e+00,2.00e+00) (3.00e+00,4.00e+00) (5.00e+00,6.00e+00)
(1.00e+00,2.00e+00) (5.00e+00,6.00e+00)
```

2.4.14 assign (int, const cvector&)

Function

```
cvector& cvector::assign (int n, const cvector& v) throw (cvmexception);
```

sets every element of a calling vector's sub-vector beginning with 1-based index `n` to a vector `v` and returns a reference to the vector changed. The function throws an exception of type `cvmexception` if `n` is not positive or `v.size()+n-1` is greater than a calling vector's size. See also `cvector`. Example:

```
using namespace cvm;

cvector v1(5);
cvector v2(2);
v1.set(std::complex<double>(1.,1.));
v2.set(std::complex<double>(2.,2.));
v1.assign(3, v2);
std::cout << v1;
```

prints

```
(1,1) (1,1) (2,2) (2,2) (1,1)
```

2.4.15 set(TC)

Function

```
cvector& cvector::set (TC x);
```

sets every element of a calling vector to a value of parameter `x` and returns a reference to the vector changed. See also `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

cvector v(3);
v.set(std::complex<double>(3.,1.));
std::cout << v;
```

prints

```
(3.00e+00,1.00e+00) (3.00e+00,1.00e+00) (3.00e+00,1.00e+00)
```

2.4.16 assign_real

Function

```
cvector& cvector::assign_real (const rvector& vRe) throw (cvmexception);
```

sets real part of every element of a calling vector to a value of appropriate element of a vector vRe and returns a reference to the vector changed. The function throws an exception of type cvmexception in case of different sizes of the operands. See also cvector and rvector. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

rvector v(3);
cvector vc(3);
v(1) = 1.; v(2) = 2.; v(3) = 3.;

vc.assign_real(v);
std::cout << vc;
```

prints

```
(1.00e+00,0.00e+00) (2.00e+00,0.00e+00) (3.00e+00,0.00e+00)
```

2.4.17 assign_imag

Function

```
cvector& cvector::assign_imag (const rvector& vIm) throw (cvmexception);
```

sets imaginary part of every element of a calling vector to a value of appropriate element of a vector `vIm` and returns a reference to the vector changed. The function throws an exception of type `cvmexception` in case of different sizes of the operands. See also `cvector` and `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

rvector v(3);
cvector vc(3);
v(1) = 1.; v(2) = 2.; v(3) = 3.;

vc.assign_imag(v);
std::cout << vc;
```

prints

```
(0.00e+00,1.00e+00) (0.00e+00,2.00e+00) (0.00e+00,3.00e+00)
```

2.4.18 set_real

Function

```
cvector& cvector::set_real (TR x);
```

sets real part of every element of a calling vector to a value of parameter `x` and returns a reference to the vector changed. See also `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

cvector v(3);
v.set_real(1.);
std::cout << v;
```

prints

```
(1.00e+00,0.00e+00) (1.00e+00,0.00e+00) (1.00e+00,0.00e+00)
```

2.4.19 set_imag

Function

```
cvector& cvector::set_imag (TR x);
```

sets imaginary part of every element of a calling vector to a value of parameter `x` and returns a reference to the vector changed. See also `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

cvector v(3);
v.set_imag(1.);
std::cout << v;
```

prints

```
(0.00e+00,1.00e+00) (0.00e+00,1.00e+00) (0.00e+00,1.00e+00)
```

2.4.20 resize

Function

```
cvector& cvector::resize (int nNewSize) throw (cvmexception);
```

changes a size of a calling vector to be equal to `nNewSize` and returns a reference to the vector changed. In case of increasing of its size, the vector is filled up with zeroes. The function throws an exception of type `cvmexception` in case of negative size passed or memory allocation failure. See also `basic_array::resize` and `cvector`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4.};
    rvector v (a, 3);
    std::cout << v;
    v.resize(2);
    std::cout << v;
    v.resize(4);
    std::cout << v;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(1,2) (3,4) (5,6)
(1,2) (3,4)
(1,2) (3,4) (0,0) (0,0)
```

2.4.21 operator ==

Operator

```
bool cvector::operator == (const cvector& v) const;
```

compares a calling vector with a vector `v` and returns `true` if they have the same sizes and their appropriate elements differ by not more than the smallest normalized positive number. Returns `false` otherwise. See also `cvector`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4.};
cvector v1 ((std::complex<double>*)a, 2);
cvector v2 (2);

v2(1) = std::complex<double>(1.,2.);
v2(2) = std::complex<double>(3.,4.);

std::cout << (v1 == v2) << std::endl;
```

prints

```
1
```

2.4.22 operator !=

Operator

```
bool cvector::operator != (const cvector& v) const;
```

compares a calling vector with a vector `v` and returns `true` if they have different sizes or some of their appropriate elements differ by more than the smallest normalized positive number. Returns `false` otherwise. See also `cvector`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4.};
cvector v1 ((std::complex<double>*)a, 2);
cvector v2 (2);

std::cout << (v1 != v2) << std::endl;
```

prints

```
1
```

2.4.23 operator <<

Operator

```
cvector& cvector::operator << (const cvector& v) throw (cvmexception);
```

destroys a calling vector, creates a new one as a copy of v and returns a reference to the vector changed. See also cvector. The operator throws an exception of type cvmexception in case of memory allocation failure. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    cvector v(2);
    cvector vc(3);
    v(1) = std::complex<double> (1.,2.);
    v(2) = std::complex<double> (3.,4.);
    std::cout << v << vc << std::endl;

    vc << v;
    std::cout << vc;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(1.00e+000,2.00e+000) (3.00e+000,4.00e+000)
(0.00e+000,0.00e+000) (0.00e+000,0.00e+000) (0.00e+000,0.00e+000)

(1.00e+000,2.00e+000) (3.00e+000,4.00e+000)
```

2.4.24 operator +

Operator

```
cvector cvector::operator + (const cvector& v) const
throw (cvmexception);
```

creates an object of type `cvector` as a sum of a calling vector and vector `v`. It throws an exception of type `cvmexception` in case of different sizes of the operands or memory allocation failure. See also `cvector::sum`, `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    cvector va(3);
    cvector vb(3);
    va.set(std::complex<double>(1.,1.));
    vb.set(std::complex<double>(2.,2.));

    std::cout << va + vb;
    std::cout << va + va;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(3.00e+000,3.00e+000) (3.00e+000,3.00e+000) (3.00e+000,3.00e+000)
(2.00e+000,2.00e+000) (2.00e+000,2.00e+000) (2.00e+000,2.00e+000)
```

2.4.25 operator -

Operator

```
cvector cvector::operator - (const cvector& v) const
throw (cvmexception);
```

creates an object of type `cvector` as a difference of a calling vector and vector v. It throws an exception of type `cvmexception` in case of different sizes of the operands or memory allocation failure. See also `cvector::diff`, `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    cvector va(3);
    cvector vb(3);
    va.set(std::complex<double> (1.,1.));
    vb.set(std::complex<double> (2.,2.));

    std::cout << va - vb;
    std::cout << va - va;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(-1.00e+000,-1.00e+000) (-1.00e+000,-1.00e+000) (-1.00e+000,-1.00e+000)
(0.00e+000,0.00e+000) (0.00e+000,0.00e+000) (0.00e+000,0.00e+000)
```

2.4.26 sum

Function

```
cvector& cvector::sum (const cvector& v1, const cvector& v2)
throw (cvmexception);
```

assigns a result of addition of vectors v1 and v2 to a calling vector and returns a reference to the vector changed. It throws an exception of type cvmexception in case of different sizes of the operands. See also cvector::operator + , cvector. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    cvector va(3);
    cvector vb(3);
    cvector v(3);
    va.set(std::complex<double> (1.,1.));
    vb.set(std::complex<double> (2.,2.));

    std::cout << v.sum(va, vb);
    std::cout << v.sum(v, va);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(3.00e+000,3.00e+000) (3.00e+000,3.00e+000) (3.00e+000,3.00e+000)
(4.00e+000,4.00e+000) (4.00e+000,4.00e+000) (4.00e+000,4.00e+000)
```

2.4.27 diff

Function

```
cvector& cvector::diff (const cvector& v1, const cvector& v2)
throw (cvmexception);
```

assigns a result of subtraction of vectors `v1` and `v2` to a calling vector and returns a reference to the vector changed. It throws an exception of type `cvmexception` in case of different sizes of the operands. See also `cvector::operator -` , `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    cvector va(3);
    cvector vb(3);
    cvector v(3);
    va.set(std::complex<double> (1.,1.));
    vb.set(std::complex<double> (2.,2.));

    std::cout << v.diff(va, vb);
    std::cout << v.diff(v, va);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(-1.00e+000,-1.00e+000) (-1.00e+000,-1.00e+000) (-1.00e+000,-1.00e+000)
(-2.00e+000,-2.00e+000) (-2.00e+000,-2.00e+000) (-2.00e+000,-2.00e+000)
```

2.4.28 operator +=

Operator

```
cvector& cvector::operator += (const cvector& v) throw (cvmexception);
```

adds to a calling vector a vector v and returns a reference to the vector changed. It throws an exception of type cvmexception in case of different sizes of the operands. See also cvector::operator + , cvector::sum, cvector. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    cvector v1(3);
    cvector v2(3);
    v1.set(std::complex<double> (1.,1.));
    v2.set(std::complex<double> (2.,2.));

    v1 += v2;
    std::cout << v1;

    // well, you can do this too, but temporary object would be created
    v2 += v2;
    std::cout << v2;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(3.00e+000,3.00e+000) (3.00e+000,3.00e+000) (3.00e+000,3.00e+000)
(4.00e+000,4.00e+000) (4.00e+000,4.00e+000) (4.00e+000,4.00e+000)
```

2.4.29 operator -=

Operator

```
cvector& cvector::operator -= (const cvector& v) throw (cvmexception);
```

subtracts from a calling vector a vector v and returns a reference to the vector changed. It throws an exception of type cvmexception in case of different sizes of the operands. See also cvector::operator - , cvector::diff, cvector. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    cvector v1(3);
    cvector v2(3);
    v1.set(std::complex<double> (1.,1.));
    v2.set(std::complex<double> (2.,2.));

    v1 -= v2;
    std::cout << v1;

    // well, you can do this too, but temporary object would be created
    v2 -= v2;
    std::cout << v2;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(-1.00e+000,-1.00e+000) (-1.00e+000,-1.00e+000) (-1.00e+000,-1.00e+000)
(0.00e+000,0.00e+000) (0.00e+000,0.00e+000) (0.00e+000,0.00e+000)
```

2.4.30 operator - ()

Operator

```
cvector cvector::operator - () const throw (cvmexception);
```

creates an object of type `cvector` as a calling vector multiplied by -1. It throws an exception of type `cvmexception` in case of memory allocation failure. See also `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

double a[] = {1., 2., 3., 4.};
const cvector v ((std::complex<double>*) a, 2);

std::cout << - v;
```

prints

```
(-1.00e+000,-2.00e+000) (-3.00e+000,-4.00e+000)
```

2.4.31 operator * (TR)

Operator

```
cvector cvector::operator * (TR d) const;
```

creates an object of type cvector as a product of a calling vector and a number d. See also cvector. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

double a[] = {1., 2., 3., 4.};
const cvector v ((std::complex<double>*) a, 2);

std::cout << v * 5.;
```

prints

```
(5.00e+000,1.00e+001) (1.50e+001,2.00e+001)
```

2.4.32 operator / (TR)

Operator

```
cvector cvector::operator / (TR d) const throw (cvmexception);
```

creates an object of type `cvector` as a quotient of a calling vector and a number `d`. The operator throws an exception of type `cvmexception` if `d` has an absolute value equal or less than the smallest normalized positive number. See also `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

double a[] = {1., 2., 3., 4.};
const cvector v ((std::complex<double>*) a, 2);

std::cout << v / 4.;
```

prints

```
(2.50e-001,5.00e-001) (7.50e-001,1.00e+000)
```

2.4.33 operator * (TC)

Operator

```
cvector cvector::operator * (TC c) const;
```

creates an object of type `cvector` as a product of a calling vector and a complex number `c`. See also `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

double a[] = {1., 2., 3., 4.};
const cvector v ((std::complex<double>*) a, 2);

std::cout << v * std::complex<double>(1.,1.);
```

prints

```
(-1.00e+000,3.00e+000) (-1.00e+000,7.00e+000)
```

2.4.34 operator / (TC)

Operator

```
cvector cvector::operator / (TC c) const
throw (cvmexception);
```

creates an object of type `cvector` as a quotient of a calling vector and a complex number `c`. The operator throws an exception of type `cvmexception` if `c` has an absolute value equal or less than the smallest normalized positive number. See also `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

double a[] = {1., 2., 3., 4.};
const cvector v ((std::complex<double>*) a, 2);

std::cout << v / std::complex<double>(1.,1.);
```

prints

```
(1.50e+000,5.00e-001) (3.50e+000,5.00e-001)
```

2.4.35 operator *= (TR)

Operator

```
cvector& cvector::operator *= (TR d);
```

multiplies a calling vector by real number d and returns a reference to the vector changed. See also cvector::operator * , cvector. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

double a[] = {1., 2., 3., 4.};
cvector v ((std::complex<double>*) a, 2);

std::cout << (v *= 2.);
```

prints

```
(2.00e+000,4.00e+000) (6.00e+000,8.00e+000)
```

2.4.36 operator /= (TR)

Operator

```
cvector& cvector::operator /= (TR d) throw (cvmexception);
```

divides a calling vector by real number d and returns a reference to the vector changed. It throws an exception of type cvmexception if d has an absolute value equal or less than the smallest normalized positive number. See also cvector::operator / , cvector. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

double a[] = {1., 2., 3., 4.};
cvector v ((std::complex<double>*) a, 2);

std::cout << (v /= 2.);
```

prints

```
(5.00e-001,1.00e+000) (1.50e+000,2.00e+000)
```

2.4.37 operator *= (TC)

Operator

```
cvector& cvector::operator *= (TC c);
```

multiplies a calling vector by complex number c and returns a reference to the vector changed. See also cvector::operator * , cvector. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

double a[] = {1., 2., 3., 4.};
cvector v ((std::complex<double>*) a, 2);

v *= std::complex<double>(1.,1.);
std::cout << v;
```

prints

```
(-1.00e+000,3.00e+000) (-1.00e+000,7.00e+000)
```

2.4.38 operator /= (TC)

Operator

```
cvector& cvector::operator /= (TC c) throw (cvmexception);
```

divides a calling vector by complex number c and returns a reference to the vector changed. It throws an exception of type cvmexception if c has an absolute value equal or less than the smallest normalized positive number. See also cvector::operator / , cvector. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

double a[] = {1., 2., 3., 4.};
cvector v ((std::complex<double>*) a, 2);

v /= std::complex<double>(1.,1.);
std::cout << v;
```

prints

```
(1.50e+000,5.00e-001) (3.50e+000,5.00e-001)
```

2.4.39 normalize

Function

```
cvector& cvector::normalize ();
```

normalizes a calling vector so its Euclidean norm becomes equal to 1 if it was greater than the smallest normalized positive number before the call (otherwise the function nothing). See also cvector. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

double a[] = {1., 2., 3., 4.};
cvector v ((std::complex<double>*) a, 2);

std::cout << v.normalize();
std::cout << v.norm() << std::endl;
```

prints

```
(1.83e-01,3.65e-01) (5.48e-01,7.30e-01)
1.00e+00
```

2.4.40 conjugation

Operator and functions

```
cvector  cvector::operator ~ () const throw (cvmexception);
cvector& cvector::conj (const cvector& v) throw (cvmexception);
cvector& cvector::conj ();
```

encapsulate complex conjugation. First operator creates an object of type `cvector` as a complex conjugated calling vector (it throws an exception of type `cvmexception` in case of memory allocation failure). Second function sets calling vector to be equal to vector `v` conjugated (it throws an exception of type `cvmexception` in case of different sizes of the operands), third one makes it to be equal to conjugated itself. See also `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
double a[] = {1., 2., 3., 4.};
const cvector v ((std::complex<double>*) a, 2);
cvector vc(2);

std::cout << ~v;
std::cout << vc.conj(v);
std::cout << vc.conj();
```

prints

```
(1.00e+00,-2.00e+00) (3.00e+00,-4.00e+00)
(1.00e+00,-2.00e+00) (3.00e+00,-4.00e+00)
(1.00e+00,2.00e+00) (3.00e+00,4.00e+00)
```

2.4.41 operator * (const cvector&)

Operator

```
TC cvector::operator * (const cvector& v) const throw (cvmexception);
```

returns a scalar product of a calling vector and v. The operator throws an exception of type cvmexception if the operands have different sizes. See also cvector. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

double a[] = {1., 2., 3., 4.};
double b[] = {1., -1., 1., 2.};
const cvector v1((std::complex<double>*) a, 2);
const cvector v2((std::complex<double>*) b, 2);

std::cout << v1 * v2 << std::endl;
```

prints

```
(-2.00e+00,1.10e+01)
```

2.4.42 operator %

Operator

```
TC cvector::operator % (const cvector& v) const throw (cvmexception);
```

returns a scalar product of a complex conjugated calling vector and v. The operator throws an exception of type cvmexception if the operands have different sizes. See also cvector. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

double a[] = {1., 2., 3., 4.};
double b[] = {1., -1., 1., 2.};
const cvector v1((std::complex<double>*) a, 2);
const cvector v2((std::complex<double>*) b, 2);

std::cout << v1 % v2 << std::endl;
std::cout << ~v1 * v2 << std::endl;
```

prints

```
(1.00e+01,-1.00e+00)
(1.00e+01,-1.00e+00)
```

2.4.43 operator * (const cmatrix&)

Operator

```
cvector cvector::operator * (const cmatrix& m) const
throw (cvmexception);
```

creates an object of type `cvector` as a product of a calling vector and a matrix `m`. The operator throws an exception of type `cvmexception` if the calling vector's size is differ from the number of rows of matrix `m`'. See also `cvector::mult (const cvector&, const cmatrix&)`, `cvector`, `cmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

double a[] = {1., 2., 3., 3., 2., 1.};
double b[] = {1., -1., 1., 2., -2., 1.,
              3., -2., 1., 2., -1., 3.};
const cvector v((std::complex<double>*) a, 3);
const cmatrix m((std::complex<double>*) b, 3, 2);

std::cout << v << m << std::endl << v * m;
```

prints

```
(1.00e+00,2.00e+00) (3.00e+00,3.00e+00) (2.00e+00,1.00e+00)
(1.00e+00,-1.00e+00) (3.00e+00,-2.00e+00)
(1.00e+00,2.00e+00) (1.00e+00,2.00e+00)
(-2.00e+00,1.00e+00) (-1.00e+00,3.00e+00)

(-5.00e+00,1.00e+01) (-1.00e+00,1.80e+01)
```

2.4.44 mult (const cvector&, const cmatrix&)

Function

```
cvector& cvector::mult (const cvector& v, const cmatrix& m)
throw (cvmexception);
```

sets a calling vector to be equal to a product of a vector `v` by a matrix `m` and returns the reference to the object changed. The function throws an exception of type `cvmexception` if case of inappropriate sizes of the operands. See also `cvector::mult (const cmatrix&, const cvector&)`, `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    double a[] = {1., 2., 3., 1., 2., 3.};
    double b[] = {1., -1., 1., -1., 1., -1.,
                  2., -1., 2., -1., 2., -1.};
    const cvector v ((std::complex<double>*) a, 3);
    const cmatrix m ((std::complex<double>*) b, 3, 2);
    const scmatrix sm ((std::complex<double>*) b, 2);
    cvector vm (2);

    std::cout << vm.mult(v, m) << std::endl;
    std::cout << sm << std::endl;
    std::cout << vm.mult(vm, sm);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(1.20e+01,0.00e+00) (1.80e+01,6.00e+00)

(1.00e+00,-1.00e+00) (1.00e+00,-1.00e+00)
(1.00e+00,-1.00e+00) (2.00e+00,-1.00e+00)

(3.60e+01,-2.40e+01) (5.40e+01,-1.80e+01)
```

2.4.45 mult (const cmatrix&, const cvector&)

Function

```
cvector& cvector::mult (const cmatrix& m, const cvector& v)
throw (cvmexception);
```

sets a calling vector to be equal to a product of a matrix `m` by a vector `v` and returns a reference to the object changed. The function throws an exception of type `cvmexception` if case of inappropriate sizes of the operands. See also `cvector::mult (const cvector&, const cmatrix&)`, `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    double a[] = {1., 2., 3., 1., 2., 3.};
    double b[] = {1., -1., 1., -1., 1., -1.,
                  2., -1., 2., -1., 2., -1.};
    const cvector v ((std::complex<double>*) a, 3);
    const cmatrix m ((std::complex<double>*) b, 2, 3);
    const scmatrix sm ((std::complex<double>*) b, 2);
    cvector vm (2);

    std::cout << vm.mult(m, v) << std::endl;
    std::cout << sm << std::endl;;
    std::cout << vm.mult(vm, sm);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(1.40e+01,3.00e+00) (1.70e+01,4.00e+00)

(1.00e+00,-1.00e+00) (1.00e+00,-1.00e+00)
(1.00e+00,-1.00e+00) (2.00e+00,-1.00e+00)

(3.80e+01,-2.40e+01) (5.50e+01,-2.00e+01)
```

2.4.46 rank1update_u

Function

```
cmatrix cvector::rank1update_u (const cvector& v) const;
```

creates an object of type cmatrix as a rank-1 update (unconjugated) of a calling vector and a vector v. The rank-1 update (unconjugated) operation of a vector-column x of a size m and a vector-row y of a size n is defined as $m \times n$ matrix

$$\begin{pmatrix} x_1 y_1 & x_1 y_2 & \cdots & x_1 y_n \\ x_2 y_1 & x_2 y_2 & \cdots & x_2 y_n \\ \dots & \dots & \dots & \dots \\ x_m y_1 & x_m y_2 & \cdots & x_m y_n \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix} \begin{pmatrix} y_1 & y_2 & \cdots & y_n \end{pmatrix}$$

See also cvector, cmatrix. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
double a[] = {1., 2., 3., -2., -1., 1.};
double b[] = {4., 5., 3., 2.};
cvector v1((std::complex<double>*) a, 3);
cvector v2((std::complex<double>*) b, 2);

std::cout << v1.rank1update_u (v2);
```

prints

```
(-6.00e+00,1.30e+01) (-1.00e+00,8.00e+00)
(2.20e+01,7.00e+00) (1.30e+01,0.00e+00)
(-9.00e+00,-1.00e+00) (-5.00e+00,1.00e+00)
```

2.4.47 rank1update_c

Function

```
cmatrix cvector::rank1update_c (const cvector& v) const;
```

creates an object of type cmatrix as a rank-1 update (conjugated) of a calling vector and a complex conjugated vector v. The rank-1 update (conjugated) operation of a vector-column x of a size m and a complex conjugated vector-column y of a size n is defined as $m \times n$ matrix

$$\begin{pmatrix} x_1 y_1^* & x_1 y_2^* & \cdots & x_1 y_n^* \\ x_2 y_1^* & x_2 y_2^* & \cdots & x_2 y_n^* \\ \hdotsfor{4} \\ x_m y_1^* & x_m y_2^* & \cdots & x_m y_n^* \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix} \mathrm{conj} \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix} \begin{pmatrix} y_1^* & y_2^* & \dots & y_n^* \end{pmatrix},$$

where y_i^* is i-th complex conjugated element of y. See also cvector, cmatrix. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
double a[] = {1., 2., 3., -2., -1., 1.};
double b[] = {4., 5., 3., 2.};
cvector v1((std::complex<double>*) a, 3);
cvector v2((std::complex<double>*) b, 2);

std::cout << v1.rank1update_c (v2) << std::endl;
std::cout << v1.rank1update_u (~v2);
```

prints

```
(1.40e+01,3.00e+00) (7.00e+00,4.00e+00)
(2.00e+00,-2.30e+01) (5.00e+00,-1.20e+01)
(1.00e+00,9.00e+00) (-1.00e+00,5.00e+00)

(1.40e+01,3.00e+00) (7.00e+00,4.00e+00)
(2.00e+00,-2.30e+01) (5.00e+00,-1.20e+01)
(1.00e+00,9.00e+00) (-1.00e+00,5.00e+00)
```

2.4.48 solve

Functions

```
cvector&
cvector::solve (const scmatrix& mA,
                const cvector& vB, TR& dErr) throw (cvmexception);
cvector&
cvector::solve (const scmatrix& mA,
                const cvector& vB) throw (cvmexception);
```

set a calling vector to be equal to a solution x of a linear equation $Ax = b$ where parameter `mA` is the square matrix A and parameter `vB` is the vector b. Every function returns a reference to the vector changed. The first version also sets output parameter `dErr` to be equal to a norm of computation error. These functions throw exception of type `cvmexception` in case of inappropriate sizes of the objects or when the matrix A is close to cingular. See also `cvector`, `scmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (7);

try {
    double m[] = {1., -1., 1., 2., -2., 1., 3., -3.};
    double b[] = {1., 2., 5., -3.};
    scmatrix ma((std::complex<double>*) m, 2);
    cvector vb((std::complex<double>*) b, 2);
    cvector vx(2);
    double dErr = 0.;

    std::cout << vx.solve (ma, vb, dErr);
    std::cout << dErr << std::endl;
    std::cout << ma * vx - vb;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(3.5200000e+00,6.4000000e-01) (2.2400000e+00,-1.3200000e+00)
3.2788531e-15
(-7.7715612e-16,4.4408921e-16) (0.0000000e+00,0.0000000e+00)
```

2.4.49 solve_lu

Functions

```
cvector&
cvector::solve_lu (const scmatrix& mA, const scmatrix& mLU,
                   const int* pPivots, const cvector& vB, TR& dErr)
                   throw (cvmexception);
cvector&
cvector::solve_lu (const scmatrix& mA, const scmatrix& mLU,
                   const int* pPivots, const cvector& vB)
                   throw (cvmexception);
```

set a calling vector to be equal to a solution x of a linear equation $Ax = b$ where parameter `mA` is the square matrix A, parameter `mLU` is LU factorization of the matrix A, parameter `pPivots` is an array of pivot numbers created while factorizing the matrix A and parameter `vB` is the vector b. Every function returns a reference to the vector changed. The first version also sets output parameter `dErr` to be equal to a norm of computation error. These functions are useful when you need to solve few linear equations of kind $Ax = b$ with the same matrix A and different vectors b. In such case you save on matrix A factorization since it's needed to be performed just one time. These functions throw exception of type `cvmexception` in case of inappropriate sizes of the objects or when the matrix A is close to cingular. See also `cvector`, `scmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (7);

try {
    double m[] = {1., -1., 1., 2., -2., 1., 3., -3.};
    double b1[] = {1., 2., 5., -3.};
    double b2[] = {3., -1., 1., 7.};
    scmatrix ma((std::complex<double>*) m, 2);
    scmatrix mLU(2);
    cvector vb1((std::complex<double>*) b1, 2);
    cvector vb2((std::complex<double>*) b2, 2);
    cvector vx1(2);
    cvector vx2(2);
    iarray nPivots(2);
    double dErr = 0.;

    mLU.low_up(ma, nPivots);
    std::cout << vx1.solve_lu (ma, mLU, nPivots, vb1, dErr);
    std::cout << dErr << std::endl;
```

```
    std::cout << vx2.solve_lu (ma, mLU, nPivots, vb2);
    std::cout << ma * vx1 - vb1 << ma * vx2 - vb2;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(3.5200000e+00,6.4000000e-01) (2.2400000e+00,-1.3200000e+00)
3.2788531e-15
(2.2800000e+00,1.9600000e+00) (3.6000000e-01,5.2000000e-01)
(-7.7715612e-16,4.4408921e-16) (0.0000000e+00,0.0000000e+00)
(-8.8817842e-16,0.0000000e+00) (-2.2204460e-16,0.0000000e+00)
```

2.4.50 eig

Functions

```
cvector&
cvector::eig (const srmatrix& mArg) throw (cvmexception);
cvector&
cvector::eig (const scmatrix& mArg) throw (cvmexception);

cvector&
cvector::eig (const srmatrix& mArg,
              scmatrix& mEigVect) throw (cvmexception);
cvector&
cvector::eig (const scmatrix& mArg,
              scmatrix& mEigVect) throw (cvmexception);
```

solve a nonsymmetric eigenvalue problem and set a calling vector to be equal to eigenvalues of a square matrix `mArg`. The nonsymmetric eigenvalue problem is defined as follows: given a nonsymmetric (or non-Hermitian) matrix A, find the eigenvalues λ and the corresponding eigenvectors z that satisfy the equation

$$Az = \lambda z.$$

Some eigenvalues may be complex even for real matrix A. Moreover, if a real nonsymmetric matrix has a complex eigenvalue $a + bi$ corresponding to an eigenvector z, then $a - bi$ is also an eigenvalue. The eigenvalue $a - bi$ corresponds to the eigenvector whose elements are complex conjugate to the elements of z. The third and fourth versions of the functions set an output parameter `mEigVect` to be equal to a square matrix containing eigenvectors as columns. All the functions return a reference to the vector they change and throw an exception of type `cvmexception` in case of inappropriate calling object sizes or in case of convergence error. See also `cvector`, `rvector::eig`, `scmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    srmatrix m(3);
    scmatrix me(3);
    cvector vl(3);

    m(1,1) = 0.1;  m(1,2) = 0.2;  m(1,3) = 0.1;
    m(2,1) = 0.11; m(2,2) = 2.9; m(2,3) = -8.4;
    m(3,1) = 0.;   m(3,2) = 2.91; m(3,3) = 8.2;
```

```
    std::cout << vl.eig (m, me);
    std::cout << scmatrix(m) * me(1) - me(1) * vl(1);
    std::cout << scmatrix(m) * me(2) - me(2) * vl(2);
    std::cout << scmatrix(m) * me(3) - me(3) * vl(3);

    scmatrix mc(m);
    mc.randomize_imag(-1., 1.);

    std::cout << std::endl << vl.eig (mc, me);
    std::cout << mc * me(1) - me(1) * vl(1);
    std::cout << mc * me(2) - me(2) * vl(2);
    std::cout << mc * me(3) - me(3) * vl(3);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(9.69e-02,0.00e+00) (5.55e+00,4.17e+00) (5.55e+00,-4.17e+00)
(0.00e+00,0.00e+00) (8.46e-18,0.00e+00) (7.59e-18,0.00e+00)
(5.55e-17,-2.78e-17) (8.88e-16,1.78e-15) (-1.33e-15,-1.33e-15)
(5.55e-17,2.78e-17) (8.88e-16,-1.78e-15) (-1.33e-15,1.33e-15)

(3.57e-02,-8.47e-01) (6.22e+00,4.76e+00) (4.95e+00,-3.90e+00)
(5.55e-17,0.00e+00) (1.18e-16,1.35e-16) (2.36e-16,6.76e-16)
(3.33e-16,0.00e+00) (-1.55e-15,-3.55e-15) (-4.44e-16,0.00e+00)
(0.00e+00,-3.33e-16) (-1.78e-15,-2.22e-15) (2.22e-16,1.55e-15)
```

2.4.51 gemv

Function

```
cvector& cvector::gemv (bool bLeft, const cmatrix& m, TC dAlpha,
                        const cvector& v, TC dBeta) throw (cvmexception);
```

calls one of ?GEMV routines of the BLAS library performing a matrix-vector operation defined as

$$c = \alpha M \cdot v + \beta c \quad \text{or} \quad c = \alpha v \cdot M + \beta c,$$

where α and β are complex numbers (parameters dAlpha and dBeta), M is a complex matrix (parameter m) and v and c are complex vectors (parameter v and calling vector respectively). First operation is performed if bLeft passed is false and second one otherwise. The function returns a reference to the vector changed and throws an exception of type cvmexception in case of inappropriate calling object sizes. See also cvector, cmatrix. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    std::complex<double> alpha = std::complex<double>(1.3,-0.7);
    std::complex<double> beta  = std::complex<double>(0.15,-1.09);
    cmatrix m(3,2);
    cvector c(3);
    cvector v(2);
    m.randomize_real(-1., 2.); m.randomize_imag(0., 1.);
    v.randomize_real(-1., 3.); v.randomize_imag(2., 4.);
    c.randomize_real(0., 2.); c.randomize_imag(3., 7.);
    std::cout << m * v * alpha + c * beta;
    std::cout << c.gemv(false, m, alpha, v, beta);
    std::cout << c * m * alpha + v * beta;
    std::cout << v.gemv(true, m, alpha, c, beta);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(2.71e-01,2.44e+00) (2.20e+01,7.16e+00) (-7.89e-01,2.45e+00)
(2.71e-01,2.44e+00) (2.20e+01,7.16e+00) (-7.89e-01,2.45e+00)
(5.92e+01,-1.47e+01) (3.54e+01,-3.14e+00)
(5.92e+01,-1.47e+01) (3.54e+01,-3.14e+00)
```

2.4.52 gbmv

Function

```
cvector& cvector::gbmv (bool bLeft, const scbmatrix& m, TC dAlpha,
                        const cvector& v, TC dBeta) throw (cvmexception);
```

calls one of ?GBMV routines of the BLAS library performing a matrix-vector operation defined as

$$c = \alpha M \cdot v + \beta c \quad \text{or} \quad c = \alpha v \cdot M + \beta c,$$

where α and β are complex numbers (parameters `dAlpha` and `dBeta`), M is a complex band matrix (parameter `m`) and v and c are complex vectors (parameter `v` and calling vector respectively). First operation is performed if `bLeft` passed is `false` and second one otherwise. The function returns a reference to the vector changed and throws an exception of type `cvmexception` in case of inappropriate calling object sizes. See also `cvector`, `scbmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    std::complex<double> alpha = std::complex<double>(1.3,-0.7);
    std::complex<double> beta  = std::complex<double>(0.15,-1.09);
    scbmatrix m(3,1,0);
    cvector c(3);
    cvector v(3);
    m.randomize_real(-1., 2.); m.randomize_imag(0., 1.);
    v.randomize_real(-1., 3.); v.randomize_imag(2., 4.);
    c.randomize_real(0., 2.); c.randomize_imag(3., 7.);
    std::cout << m * v * alpha + c * beta;
    std::cout << c.gbmv(false, m, alpha, v, beta);
    std::cout << c * m * alpha + v * beta;
    std::cout << v.gbmv(true, m, alpha, c, beta);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(3.73e+00,7.96e-01) (6.89e+00,1.07e+01) (2.16e+00,3.28e+00)
(3.73e+00,7.96e-01) (6.89e+00,1.07e+01) (2.16e+00,3.28e+00)
(3.11e+01,2.51e+01) (-4.93e+00,1.34e+01) (1.70e+00,3.93e+00)
(3.11e+01,2.51e+01) (-4.93e+00,1.34e+01) (1.70e+00,3.93e+00)
```

2.4.53 randomize_real

Function

```
cvector& cvector::randomize_real (TR dFrom, TR dTo);
```

fills a real part of a calling vector with pseudo-random numbers distributed between dFrom and dTo. The function returns a reference to the vector changed. See also cvector. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

cvector v(3);
v.randomize_real(-2.,3.);
std::cout << v;
```

prints

```
(-4.93e-01,0.00e+00) (1.37e+00,0.00e+00) (-1.49e-01,0.00e+00)
```

2.4.54 randomize_imag

Function

```
cvector& cvector::randomize_imag (TR dFrom, TR dTo);
```

fills an imaginary part of a calling vector with pseudo-random numbers distributed between dFrom and dTo. The function returns a reference to the vector changed. See also cvector. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

cvector v(3);
v.randomize_imag(-2.,3.);
std::cout << v;
```

prints

```
(0.00e+00,-4.37e-01) (0.00e+00,-1.59e+00) (0.00e+00,2.42e+00)
```

2.5 Matrix

This base class contains member functions common for all matrices. This class is not designed to be instantiated.

```
template <typename TR, typename TC>
class Matrix : public Array<TR,TC> {
public:
    int msize () const;
    int nsize () const;
    int ld () const;
    int rowofmax () const;
    int rowofmin () const;
    int colofmax () const;
    int colofmin () const;
    virtual TR norm1 () const;
    <typename TR, typename TC>
    friend std::ostream& operator << <> (std::ostream& os,
                                         const Array<TR,TC>& aOut);
};
```

2.5.1 msize

Function

```
int Matrix<TR,TC>::msize () const;
```

returns a number of rows of a calling matrix. The function is *inherited* in all matrix classes of the library: `rmatrix`, `cmatrix`, `srmatrix`, `scmatrix`, `srbmatrix`, `scbmatrix`, `srsmatrix` and `schmatrix`. See also `Matrix`. Example:

```
using namespace cvm;

rmatrix m (100, 200);
std::cout << m.msize() << std::endl;
```

prints

```
100
```

2.5.2 nsize

Function

```
int Matrix<TR,TC>::nsize () const;
```

returns a number of columns of a calling matrix. The function is *inherited* in all matrix classes of the library: `rmatrix`, `cmatrix`, `srmatrix`, `scmatrix`, `srbmatrix`, `scbmatrix`, `srsmatrix` and `schmatrix`. See also `Matrix`. Example:

```
using namespace cvm;

rmatrix m (100, 200);

std::cout << m.nsize() << std::endl;
```

prints

```
200
```

2.5.3 ld

Function

```
int Matrix<TR,TC>::ld () const;
```

returns a leading dimension of a calling matrix. Leading dimension is equal to a number of rows for every matrix except submatrices. For submatrices it's equal to a number of rows of parent matrix. The function is *inherited* in all matrix classes of the library: `rmatrix`, `cmatrix`, `srmatrix`, `scmatrix`, `srbmatrix`, `scbmatrix`, `srsmatrix` and `schmatrix`. See also `Matrix`. Example:

```
using namespace cvm;

rmatrix m (100, 200);
srmatrix ms (m, 30, 40, 5); // 5x5 submatrix
std::cout << ms.ld() << std::endl;
```

prints

```
100
```

2.5.4 rowofmax

Function

```
int Matrix<TR,TC>::rowofmax () const;
```

returns a 1-based number of a row of a calling matrix where the element with the maximum absolute value is located. The function is *inherited*[7] in all matrix classes of the library: rmatrix, cmatrix, srmatrix, scmatrix, srbmatrix, scbmatrix, srsmatrix and schmatrix. See also Matrix. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., -3., 1., -1.};
rmatrix m (a, 2, 3);

std::cout << m << std::endl << m.rowofmax() << std::endl;
```

prints

```
1 2 1
0 -3 -1

2
```

[7] Calls virtual function inside

2.5.5 rowofmin

Function

```
int Matrix<TR,TC>::rowofmin () const;
```

returns a 1-based number of a row of a calling matrix where the element with the minimum absolute value is located. The function is *inherited*[8] in all matrix classes of the library: rmatrix, cmatrix, srmatrix, scmatrix, srbmatrix, scbmatrix, srsmatrix and schmatrix. See also Matrix. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., -3., 1., -1.};
rmatrix m (a, 2, 3);

std::cout << m << std::endl << m.rowofmin() << std::endl;
```

prints

```
1 2 1
0 -3 -1

2
```

[8]Calls virtual function inside

2.5.6 colofmax

Function

```
int Matrix<TR,TC>::colofmax () const;
```

returns a 1-based number of a column of a calling matrix where the element with the maximum absolute value is located. The function is *inherited*[9] in all matrix classes of the library: rmatrix, cmatrix, srmatrix, scmatrix, srbmatrix, scbmatrix, srsmatrix and schmatrix. See also Matrix. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., -3., 1., -1.};
rmatrix m (a, 2, 3);

std::cout << m << std::endl << m.colofmax() << std::endl;
```

prints

```
1 2 1
0 -3 -1

2
```

[9]Calls virtual function inside

2.5.7 colofmin

Function

```
int Matrix<TR,TC>::colofmin () const;
```

returns a 1-based number of a column of a calling matrix where the element with the minimum absolute value is located. The function is *inherited*[10] in all matrix classes of the library: rmatrix, cmatrix, srmatrix, scmatrix, srbmatrix, scbmatrix, srsmatrix and schmatrix. See also Matrix. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., -3., 1., -1.};
rmatrix m (a, 2, 3);

std::cout << m << std::endl << m.colofmin() << std::endl;
```

prints

```
1 2 1
0 -3 -1

1
```

[10] Calls virtual function inside

2.5.8 norm1

Virtual function

```
virtual TR Matrix<TR,TC>::norm1 () const;
```

returns a 1-norm of a calling matrix that is defined as

$$\|A\|_1 = \max_{j=1,\dots,n} \sum_{i=1}^{m} |a_{ij}|,$$

where A is $m \times n$ matrix. The function is *inherited* in the following classes of the library: `rmatrix`, `cmatrix`, `srmatrix`, `scmatrix`, `srsmatrix` and `schmatrix`. It's *redefined* in `srbmatrix` and `scbmatrix`. See also `Array::norminf` and `Matrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., -3., 1., 0.};
rmatrix m (a, 2, 3);

std::cout << m << std::endl << m.norm1()
          << std::endl << m.norminf() << std::endl;
```

prints

```
1 2 1
0 -3 0

5
4
```

2.5.9 operator << <> (std::ostream& os, const Matrix<TR,TC>& aOut)

Friend template operator

```
template <typename TR, typename TC>
friend std::ostream& operator << <> (std::ostream& os,
                                     const Matrix<TR,TC>& aOut);
```

outputs a matrix referenced by parameter aOut into os stream. See also Array::operator << , Matrix. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

srmatrix m(3);
m(1,1) = 1.;
m(2,3) = 3.;

std::cout << m;
```

prints

```
1.00e+00 0.00e+00 0.00e+00
0.00e+00 0.00e+00 3.00e+00
0.00e+00 0.00e+00 0.00e+00
```

2.6 rmatrix

This is end-user class encapsulating a matrix in Euclidean space of real numbers.

```
template <typename TR>
class rmatrix : public Matrix <TR,TR> {
public:
    rmatrix ();
    rmatrix (int nM, int nN);
    rmatrix (TR* pD, int nM, int nN);
    rmatrix (const rmatrix& m);
    explicit rmatrix (const rvector& v, bool bBeColumn = true);
    rmatrix (rmatrix& m, int nRow, int nCol, int nHeight, int nWidth);
    TR& operator () (int im, int in) throw (cvmexception);
    TR operator () (int im, int in) const throw (cvmexception);
    rvector operator () (int i) throw (cvmexception);
    const rvector operator () (int i) const throw (cvmexception);
    rvector operator [] (int i) throw (cvmexception);
    const rvector operator [] (int i) const throw (cvmexception);
    rvector diag (int i) throw (cvmexception);
    const rvector diag (int i) const throw (cvmexception);
    rmatrix& operator = (const rmatrix& m) throw (cvmexception);
    rmatrix& assign (const rvector& v);
    rmatrix& assign (const TR* pD);
    rmatrix& assign (int nRow, int nCol, const rmatrix& m)
                     throw (cvmexception);
    rmatrix& set (TR x);
    rmatrix& resize (int nNewM, int nNewN) throw (cvmexception);
    bool operator == (const rmatrix& m) const;
    bool operator != (const rmatrix& m) const;
    rmatrix& operator << (const rmatrix& m) throw (cvmexception);
    rmatrix operator + (const rmatrix& m) const
                        throw (cvmexception);
    rmatrix operator - (const rmatrix& m) const
                        throw (cvmexception);
    rmatrix& sum (const rmatrix& m1,
                  const rmatrix& m2) throw (cvmexception);
    rmatrix& diff (const rmatrix& m1,
                   const rmatrix& m2) throw (cvmexception);
    rmatrix& operator += (const rmatrix& m) throw (cvmexception);
    rmatrix& operator -= (const rmatrix& m) throw (cvmexception);
    rmatrix operator - () const;
    rmatrix operator * (TR d) const;
```

```
rmatrix operator / (TR d) const
                    throw (cvmexception);
rmatrix& operator *= (TR d);
rmatrix& operator /= (TR d) throw (cvmexception);
rmatrix& normalize ();
rmatrix operator ~ () const throw (cvmexception);
rmatrix& transpose (const rmatrix& m) throw (cvmexception);
rmatrix& transpose () throw (cvmexception);
rvector operator * (const rvector& v) const
                    throw (cvmexception);
rmatrix operator * (const rmatrix& m) const
                    throw (cvmexception);
rmatrix& mult (const rmatrix& m1, const rmatrix& m2)
               throw (cvmexception);
rmatrix& rank1update (const rvector& vCol,
                      const rvector& vRow)
                      throw (cvmexception);
rmatrix& swap_rows (int n1, int n2) throw (cvmexception);
rmatrix& swap_cols (int n1, int n2) throw (cvmexception);
rmatrix& solve (const srmatrix& mA,
                const rmatrix& mB, TR& dErr)
                throw (cvmexception);
rmatrix& solve (const srmatrix& mA,
                const rmatrix& mB) throw (cvmexception);
rmatrix& solve_lu (const srmatrix& mA, const srmatrix& mLU,
                   const int* pPivots, const rmatrix& mB, TR& dErr)
                   throw (cvmexception);
rmatrix& solve_lu (const srmatrix& mA, const srmatrix& mLU,
                   const int* pPivots, const rmatrix& mB)
                   throw (cvmexception);
rvector svd () const throw (cvmexception);
rvector svd (srmatrix& mU, srmatrix& mVH) const
             throw (cvmexception);
rmatrix pinv (TR threshold = cvmMachSp()) const
              throw (cvmexception);
rmatrix& pinv (const rmatrix& mA,
               TR threshold = cvmMachSp())
               throw (cvmexception);
int rank (TR eps = cvmMachSp ()) const throw (cvmexception);
rmatrix& ger (TR dAlpha, const rvector& vCol,
              const rvector& vRow) throw (cvmexception);
rmatrix& gemm (const rmatrix& m1, bool bTrans1,
               const rmatrix& m2, bool bTrans2,
```

```
               TR dAlpha, TR dBeta) throw (cvmexception);
    rmatrix& symm (bool bLeft, const srsmatrix& ms,
               const rmatrix& m, TR dAlpha, TR dBeta)
               throw (cvmexception);
    void qr(rmatrix& mQ, srmatrix& mR) const throw (cvmexception);
    void qr(srmatrix& mQ, rmatrix& mR) const throw (cvmexception);
    rmatrix& vanish ();
    rmatrix& randomize (TR dFrom, TR dTo);
};
```

2.6.1 rmatrix ()

Default constructor

```
rmatrix::rmatrix ();
```

creates an empty rmatrix object. See also rmatrix. Example:

```
using namespace cvm;

rmatrix m;
std::cout << m.msize() << std::endl << m.nsize() << std::endl;
std::cout << m.size() << std::endl;

m.resize (2, 3);
std::cout << m;
```

prints

```
0
0
0
0 0 0
0 0 0
```

2.6.2 rmatrix (int,int)

Constructor

```
rmatrix::rmatrix (int nM, int nN);
```

creates an $m \times n$ `rmatrix` object where m is passed in `nM` parameter (number of rows) and n is passed in `nN` (number of columns). The constructor throws an exception of type `cvmexception` in case of non-positive sizes passed or memory allocation failure. See also `rmatrix`. Example:

```
using namespace cvm;

rmatrix m (3, 4);
std::cout << m.msize() << std::endl << m.nsize()
          << std::endl << m.size() << std::endl << m;
```

prints

```
3
4
12
0 0 0 0
0 0 0 0
0 0 0 0
```

2.6.3 rmatrix (TR*,int,int)

Constructor

```
rmatrix::rmatrix (TR* pD, int nM, int nN);
```

creates an $m \times n$ `rmatrix` object where m is passed in `nM` parameter (number of rows) and n is passed in `nN` (number of columns). Unlike others, this constructor *does not allocate a memory*. It just shares a memory with an array pointed to by `pD`. See also `rmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
double a[] = {1., 2., 3., 4., 5., 6.};
rmatrix m (a, 2, 3);

std::cout << m << std::endl;

a[1] = 7.77;
std::cout << m;
```

prints

```
1.00e+000 3.00e+000 5.00e+000
2.00e+000 4.00e+000 6.00e+000

1.00e+000 3.00e+000 5.00e+000
7.77e+000 4.00e+000 6.00e+000
```

2.6.4 rmatrix (const rmatrix&)

Copy constructor

```
rmatrix::rmatrix (const rmatrix& m);
```

creates a `rmatrix` object as a copy of `m`. The constructor throws an exception of type `cvmexception` in case of memory allocation failure. See also `rmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
double a[] = {1., 2., 3., 4., 5., 6.};
rmatrix m (a, 2, 3);
rmatrix mc(m);

m(1,1) = 7.77;
std::cout << m << std::endl << mc;
```

prints

```
7.77e+000 3.00e+000 5.00e+000
2.00e+000 4.00e+000 6.00e+000

1.00e+000 3.00e+000 5.00e+000
2.00e+000 4.00e+000 6.00e+000
```

2.6.5 rmatrix (const rvector&,bool)

Constructor

```
explicit rmatrix::rmatrix (const rvector& v, bool bBeColumn = true);
```

creates a `rmatrix` object containing `v.size()` rows and 1 column if `bBeColumn` is `true` or 1 row and `v.size()` columns otherwise. After that it copies the vector `v`'s elements to the matrix created. The constructor throws an exception of type `cvmexception` in case of memory allocation failure. See also `rmatrix`, `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
rvector v(3);
v(1) = 1.;
v(2) = 2.;
v(3) = 3.;

rmatrix mc (v);
rmatrix mr (v, false);

std::cout << mc << std::endl << mr;
```

prints

```
1.00e+000
2.00e+000
3.00e+000

1.00e+000 2.00e+000 3.00e+000
```

2.6.6 submatrix

Submatrix constructor

```
rmatrix::rmatrix (rmatrix& m, int nRow, int nCol,
                              int nHeight, int nWidth);
```

creates a `rmatrix` object as a *submatrix* of `m`. It means that the matrix object created shares a memory with some part of `m`. This part is defined by its upper left corner (parameters `nRow` and `nCol`, both are 1-based) and its height and width (parameters `nHeight` and `nWidth`). See also `rmatrix`. Example:

```
using namespace cvm;
rmatrix m(4,5);
rmatrix subm(m, 2, 2, 2, 2);
subm.set(1.);

std::cout << m;
```

prints

```
0 0 0 0 0
0 1 1 0 0
0 1 1 0 0
0 0 0 0 0
```

2.6.7 operator (,)

Indexing operators

```
TR& rmatrix::operator () (int im, int in) throw (cvmexception);
TR rmatrix::operator () (int im, int in) const throw (cvmexception);
```

provide access to an element of a matrix. The first version of the operator is applicable to a non-constant object. This version returns an *l-value* in order to make possible write access to an element. Both operators are 1-based. The operators throw an exception of type `cvmexception` if `im` is outside of `[1,msize()]` range or `in` is outside of `[1,nsize()]` range. The operators are *inherited* in the the classes `srmatrix` and `srbmatrix`. The operators are *redefined* in the the class `srsmatrix`. See also `rmatrix`, `Matrix::msize()`, `Matrix::nsize()`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    double a[] = {1., 2., 3., 4., 5., 6.};
    const rmatrix m (a, 2, 3);
    rmatrix ms(m);

    std::cout << m(1,1) << " " << m(2,3) << std::endl << std::endl;

    ms(2,2) = 7.77;
    std::cout << ms;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1.00e+00 6.00e+00

1.00e+00 3.00e+00 5.00e+00
2.00e+00 7.77e+00 6.00e+00
```

2.6.8 operator ()

Indexing operators

```
rvector rmatrix::operator () (int i) throw (cvmexception);
const rvector rmatrix::operator () (int i) const throw (cvmexception);
```

provide access to an i-th column of a matrix. The first version of the operator is applicable to a non-constant object and *returns an l-value*, i.e. the vector returned shares a memory with the i-th column of the matrix in order to make possible write access to it. The second version creates a *copy* of a column and therefore is *not an l-value*. Both operators are 1-based. The operators throw an exception of type `cvmexception` if the parameter `i` is outside of `[1,nsize()]` range. The operators are *inherited* in the the class `srmatrix`. The operators are *redefined* in the the classes `srbmatrix` and `srsmatrix`. See also `rmatrix`, `Matrix::nsize()`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    double a[] = {1., 2., 3., 4., 5., 6.};
    const rmatrix m (a, 2, 3);
    srmatrix ms(2);

    std::cout << m(2) << std::endl;

    ms(2) = m(3);
    std::cout << ms;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
3.00e+00 4.00e+00

0.00e+00 5.00e+00
0.00e+00 6.00e+00
```

2.6.9 operator []

Indexing operators

```
rvector rmatrix::operator [] (int i) throw (cvmexception);
const rvector rmatrix::operator [] (int i) const throw (cvmexception);
```

provide access to an i-th row of a matrix. The first version of the operator is applicable to a non-constant object and *returns an l-value*, i.e. the vector returned shares a memory with the i-th row of the matrix in order to make possible write access to it. The second version creates a *copy* of a row and therefore is *not an l-value*. Both operators are 1-based. The operators throw an exception of type `cvmexception` if `i` is outside of `[1,msize()]` range. The operators are *inherited* in the the class `srmatrix`. The operators are *redefined* in the the classes `srbmatrix` and `srsmatrix`. See also `rmatrix`, `Matrix::msize()`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    double a[] = {1., 2., 3., 4., 5., 6.};
    const rmatrix m (a, 2, 3);
    srmatrix ms(3);

    std::cout << m[1] << std::endl;

    ms[1] = m[2];
    std::cout << ms;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1.00e+00 3.00e+00 5.00e+00

2.00e+00 4.00e+00 6.00e+00
0.00e+00 0.00e+00 0.00e+00
0.00e+00 0.00e+00 0.00e+00
```

2.6.10 diag

Functions

```
rvector rmatrix::diag (int i) throw (cvmexception);
const rvector rmatrix::diag (int i) const throw (cvmexception);
```

provide access to an i-th diagonal of a matrix, where $i = 0$ for main diagonal, $i < 0$ for lower diagonals and $i > 0$ for upper ones. The first version of the function is applicable to a non-constant object and *returns an l-value*, i.e. the vector returned shares a memory with the i-th diagonal of the matrix in order to make possible write access to it. The second version creates a *copy* of the diagonal and therefore is *not an l-value*. The functions throw an exception of type `cvmexception` if the parameter `i` is outside of `[-msize()+1,nsize()-1]` range. The functions are *inherited* in the the classes `srmatrix` and `srbmatrix`. The functions are *redefined* in the the class `srsmatrix`. See also `rmatrix`. Example:

```
using namespace cvm;
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    rmatrix  m(2,3);
    const srmatrix ms(a,3);

    m.diag(-1).set(1.);
    m.diag(0).set(2.);
    m.diag(1).set(3.);
    m.diag(2).set(4.);
    std::cout << m << std::endl;

    std::cout << ms << std::endl;
    std::cout << ms.diag(0) << ms.diag(1);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
2 3 4
1 2 3

1 4 7
2 5 8
3 6 9

1 5 9
4 8
```

2.6.11 operator = (const rmatrix&)

Operator

`rmatrix& rmatrix::operator = (const rmatrix& m) throw (cvmexception);`

sets an every element of a calling matrix to a value of appropriate element of a matrix `m` and returns a reference to the matrix changed. The operator throws an exception of type `cvmexception` in case of different matrix sizes. The operator is *redefined* in the the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    double a[] = {1., 2., 3., 4., 5., 6.};
    const rmatrix m1(a, 3, 2);
    rmatrix m2(3, 2);

    m2 = m1;
    std::cout << m2;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1.00e+000 4.00e+000
2.00e+000 5.00e+000
3.00e+000 6.00e+000
```

2.6.12 assign (const TR*)

Function

```
rmatrix& rmatrix::assign (const rvector& v);
rmatrix& rmatrix::assign (const TR* pD);
```

sets every element of a calling matrix to a value of appropriate element of a vector `v` or an array pointed to by `pD` and returns a reference to the matrix changed. The function is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
const double a[] = {1., 2., 3., 4., 5., 6.};
rmatrix m(2, 3);

m.assign(a);
std::cout << m;
```

prints

```
1.00e+000 3.00e+000 5.00e+000
2.00e+000 4.00e+000 6.00e+000
```

2.6.13 assign (int, int, const rmatrix&)

Function

```
rmatrix& rmatrix::assign (int nRow, int nCol, const rmatrix& m)
throw (cvmexception);
```

sets sub-matrix of a calling matrix beginning with 1-based row `nRow` and column `nCol` to a matrix `m` and returns a reference to the matrix changed. The function throws an exception of type `cvmexception` if `nRow` or `nCol` are not positive or matrix `m` doesn't fit. The function is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix`. Example:

```
using namespace cvm;

rmatrix m1(4,5);
rmatrix m2(2,2);
m1.set(1.);
m2.set(2.);
m1.assign(2,3,m2);
std::cout << m1;
```

prints

```
1 1 1 1 1
1 1 2 2 1
1 1 2 2 1
1 1 1 1 1
```

2.6.14 set (TR)

Function

```
rmatrix& rmatrix::set (TR x);
```

sets every element of a calling matrix to a value of parameter `x` and returns a reference to the matrix changed. Use `vanish` to set every element of a calling matrix to be equal to zero. The function is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
rmatrix m(2, 3);

m.set(3.);
std::cout << m;
```

prints

```
3.00e+000 3.00e+000 3.00e+000
3.00e+000 3.00e+000 3.00e+000
```

2.6.15 resize

Function

```
rmatrix& rmatrix::resize (int nNewM, int nNewN)
throw (cvmexception);
```

changes a size of a calling matrix to `nNewM` by `nNewN` and returns a reference to the matrix changed. In case of increasing of its size, the matrix is filled up with zeroes. The function throws an exception of type `cvmexception` in case of negative size passed or memory allocation failure. The function is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    double a[] = {1., 2., 3., 4., 5., 6.};
    rmatrix m(a, 2, 3);
    std::cout << m << std::endl;
    m.resize (2, 2);
    std::cout << m << std::endl;
    m.resize (3, 3);
    std::cout << m;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1.00e+000 3.00e+000 5.00e+000
2.00e+000 4.00e+000 6.00e+000

1.00e+000 3.00e+000
2.00e+000 4.00e+000

1.00e+000 3.00e+000 0.00e+000
2.00e+000 4.00e+000 0.00e+000
0.00e+000 0.00e+000 0.00e+000
```

2.6.16 `operator ==`

Operator

```
bool rmatrix::operator == (const rmatrix& m) const;
```

compares a calling matrix with a matrix `m` and returns `true` if they have the same sizes and their appropriate elements differ by not more than the smallest normalized positive number. Returns `false` otherwise. The operator is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix`. Example:

```
using namespace cvm;
double a[] = {1., 2., 3., 4.};
rmatrix m1(a, 2, 2);
rmatrix m2(2, 2);

m2(1,1) = 1.; m2(1,2) = 3.;
m2(2,1) = 2.; m2(2,2) = 4.;

std::cout << (m1 == m2) << std::endl;
```

prints

```
1
```

2.6.17 operator !=

Operator

```
bool rmatrix::operator != (const rmatrix& m) const;
```

compares a calling matrix with a matrix `m` and returns `true` if they have different sizes or at least one of their appropriate elements differs by more than the smallest normalized positive number. Returns `false` otherwise. The operator is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix`. Example:

```
using namespace cvm;
double a[] = {1., 2., 3., 4.};
rmatrix m1(a, 2, 2);
rmatrix m2(2, 2);

m2(1,1) = 1.; m2(1,2) = 3.;
m2(2,1) = 2.; m2(2,2) = 4.;

std::cout << (m1 != m2) << std::endl;
```

prints

```
0
```

2.6.18 operator <<

Operator

```
rmatrix& rmatrix::operator << (const rmatrix& m)
throw (cvmexception);
```

destroys a calling matrix, creates a new one as a copy of m and returns a reference to the matrix changed. The operator throws an exception of type cvmexception in case of memory allocation failure. The operator is *redefined* in the classes srmatrix, srbmatrix and srsmatrix. See also rmatrix. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    rmatrix m(3,4);
    rmatrix mc(1,1);
    m(1,2) = 1.;
    m(3,4) = 2.;
    std::cout << m << mc << std::endl;

    mc << m;
    std::cout << mc;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
0.00e+000 1.00e+000 0.00e+000 0.00e+000
0.00e+000 0.00e+000 0.00e+000 0.00e+000
0.00e+000 0.00e+000 0.00e+000 2.00e+000
0.00e+000

0.00e+000 1.00e+000 0.00e+000 0.00e+000
0.00e+000 0.00e+000 0.00e+000 0.00e+000
0.00e+000 0.00e+000 0.00e+000 2.00e+000
```

2.6.19 operator +

Operator

```
rmatrix rmatrix::operator + (const rmatrix& m) const
throw (cvmexception);
```

creates an object of type `rmatrix` as a sum of a calling matrix and a matrix `m`. It throws an exception of type `cvmexception` in case of different sizes of the operands. The operator is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix::sum`, `rmatrix`. Example:

```
using namespace cvm;
try {
    double a[] = {1., 2., 3., 4., 5., 6.};
    const rmatrix ma(a,2,3);
    rmatrix mb(2,3);
    mb.set(1.);

    std::cout << ma + mb << std::endl;
    std::cout << ma + ma;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
2 4 6
3 5 7

2 6 10
4 8 12
```

2.6.20 operator -

Operator

```
rmatrix rmatrix::operator - (const rmatrix& m) const
throw (cvmexception);
```

creates an object of type `rmatrix` as a difference of a calling matrix and a matrix `m`. It throws an exception of type `cvmexception` in case of different sizes of the operands. The operator is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix::diff`, `rmatrix`. Example:

```
using namespace cvm;
try {
    double a[] = {1., 2., 3., 4., 5., 6.};
    const rmatrix ma(a,2,3);
    rmatrix mb(2,3);
    mb.set(1.);

    std::cout << ma - mb << std::endl;
    std::cout << ma - ma;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
0 2 4
1 3 5

0 0 0
0 0 0
```

2.6.21 sum

Function

```
rmatrix& rmatrix::sum (const rmatrix& m1, const rmatrix& m2)
throw (cvmexception);
```

assigns a result of addition of matrices `m1` and `m2` to a calling matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes of the operands. The function is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix::operator +` , `rmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6.};
    const rmatrix m1(a, 2, 3);
    rmatrix m2(2, 3);
    rmatrix m(2, 3);
    m2.set(1.);

    std::cout << m.sum(m1, m2) << std::endl;
    std::cout << m.sum(m, m2);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
2 4 6
3 5 7

3 5 7
4 6 8
```

2.6.22 diff

Function

```
rmatrix& rmatrix::diff (const rmatrix& m1, const rmatrix& m2)
throw (cvmexception);
```

assigns a result of subtraction of matrices `m1` and `m2` to a calling matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes of the operands. The function is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix::operator -` , `rmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6.};
    const rmatrix m1(a, 2, 3);
    rmatrix m2(2, 3);
    rmatrix m(2, 3);
    m2.set(1.);

    std::cout << m.diff(m1, m2) << std::endl;
    std::cout << m.diff(m, m2);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
0 2 4
1 3 5

-1 1 3
0 2 4
```

2.6.23 operator +=

Operator

```
rmatrix& rmatrix::operator += (const rmatrix& m) throw (cvmexception);
```

adds a matrix `m` to a calling matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes of the operands. The operator is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix::operator +` , `rmatrix::sum`, `rmatrix`. Example:

```
using namespace cvm;
try {
    rmatrix m1(2, 3);
    rmatrix m2(2, 3);
    m1.set(1.);
    m2.set(2.);

    m1 += m2;
    std::cout << m1 << std::endl;

    // well, you can do this too, but temporary object would be created
    m2 += m2;
    std::cout << m2;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
3 3 3
3 3 3

4 4 4
4 4 4
```

2.6.24 operator -=

Operator

`rmatrix& rmatrix::operator -= (const rmatrix& m) throw (cvmexception);`

subtracts a matrix `m` from a calling matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes of the operands. The operator is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix::operator -` , `rmatrix::diff`, `rmatrix`. Example:

```
using namespace cvm;
try {
    rmatrix m1(2, 3);
    rmatrix m2(2, 3);
    m1.set(1.);
    m2.set(2.);

    m1 -= m2;
    std::cout << m1 << std::endl;

    // well, you can do this too, but temporary object would be created
    m2 -= m2;
    std::cout << m2;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
-1 -1 -1
-1 -1 -1

0 0 0
0 0 0
```

2.6.25 operator - ()

Operator

```
rmatrix rmatrix::operator - () const throw (cvmexception);
```

creates an object of type `rmatrix` as a calling matrix multiplied by -1. The operator is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6.};
const rmatrix ma(a, 2, 3);

std::cout << - ma;
```

prints

```
-1 -3 -5
-2 -4 -6
```

2.6.26 operator * (TR)

Operator

```
rmatrix rmatrix::operator * (TR d) const;
```

creates an object of type `rmatrix` as a product of a calling matrix and a number `d`. The operator is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix::operator *=` , `rmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6.};
rmatrix m(a, 2, 3);

std::cout << m * 2.;
```

prints

```
2 6 10
4 8 12
```

2.6.27 operator / (TR)

Operator

```
rmatrix rmatrix::operator / (TR d) const throw (cvmexception);
```

creates an object of type `rmatrix` as a quotient of a calling matrix and a number `d`. It throws an exception of type `cvmexception` if `d` has an absolute value equal or less than the smallest normalized positive number. The operator is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix::operator /=` , `rmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    double a[] = {1., 2., 3., 4., 5., 6.};
    rmatrix m(a, 2, 3);

    std::cout << m / 2.;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
5.00e-01 1.50e+00 2.50e+00
1.00e+00 2.00e+00 3.00e+00
```

2.6.28 operator *= (TR)

Operator

```
rmatrix& rmatrix::operator *= (TR d);
```

multiplies a calling matrix by a number `d` and returns a reference to the matrix changed. The operator is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix::operator *` , `rmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6.};
rmatrix m(a, 2, 3);

m *= 2.;
std::cout << m;
```

prints

```
2 6 10
4 8 12
```

2.6.29 operator /= (TR)

Operator

```
rmatrix& rmatrix::operator /= (TR d) throw (cvmexception);
```

divides a calling matrix by a number d and returns a reference to the matrix changed. It throws an exception of type cvmexception if d has an absolute value equal or less than the smallest normalized positive number. The operator is *redefined* in the classes srmatrix, srbmatrix and srsmatrix. See also rmatrix::operator / , rmatrix. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    double a[] = {1., 2., 3., 4., 5., 6.};
    rmatrix m(a, 2, 3);

    m /= 2.;
    std::cout << m;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
5.00e-01 1.50e+00 2.50e+00
1.00e+00 2.00e+00 3.00e+00
```

2.6.30 normalize

Function

```
rmatrix& rmatrix::normalize ();
```

normalizes a calling matrix so its Euclidean norm becomes equal to 1 if it was greater than the smallest normalized positive number before the call (otherwise the function does nothing). The function is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

double a[] = {1., 2., 3., 4., 5., 6.};
rmatrix m(a, 2, 3);

m.normalize();
std::cout << m;
```

prints

```
1.05e-01 3.14e-01 5.24e-01
2.10e-01 4.19e-01 6.29e-01
```

2.6.31 transposition

Operator and functions

```
rmatrix rmatrix::operator ~ () const throw (cvmexception);
rmatrix& rmatrix::transpose (const rmatrix& m) throw (cvmexception);
rmatrix& rmatrix::transpose () throw (cvmexception);
```

encapsulate matrix transposition. First operator creates an object of type `rmatrix` as a transposed calling matrix (it throws an exception of type `cvmexception` in case of memory allocation failure). Second function sets a calling matrix to be equal to a matrix `m` transposed (it throws an exception of type `cvmexception` in case of not appropriate sizes of the operands), third one makes it to be equal to transposed itself (it also throws an exception of type `cvmexception` in case of memory allocation failure). The functions are *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6.};
    rmatrix m(a,2,3);
    rmatrix mt(3,2);

    std::cout << m << std::endl << ~m << std::endl ;
    mt.transpose(m);
    std::cout << mt << std::endl;
    mt.transpose();
    std::cout << mt;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1 3 5
2 4 6

1 2
3 4
5 6

1 2
3 4
5 6
```

```
1 3 5
2 4 6
```

2.6.32 operator * (const rvector&)

Operator

```
rvector rmatrix::operator * (const rvector& v) const
throw (cvmexception);
```

creates an object of type `rvector` as a product of a calling matrix and a vector `v`. It throws an exception of type `cvmexception` if the number of columns of the calling matrix differs from the size of the vector `v`. Use `rvector::mult` in order to get rid of a new object creation. The function is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix`, `rvector`. Example:

```
using namespace cvm;

try {
    rmatrix m(2, 3);
    rvector v(3);
    m.set(1.);
    v.set(1.);

    std::cout << m * v;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
3 3
```

2.6.33 operator * (const rmatrix&)

Operator

```
rmatrix rmatrix::operator * (const rmatrix& m) const
throw (cvmexception);
```

creates an object of type `rmatrix` as a product of a calling matrix and a matrix `m`. It throws an exception of type `cvmexception` if the number of columns of the calling matrix differs from the number of rows of the matrix `m`. Use `rmatrix::mult` in order to get rid of a new object creation. The operator is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix`. Example:

```
using namespace cvm;

try {
    rmatrix m1(2, 3);
    rmatrix m2(3, 2);
    m1.set(1.);
    m2.set(1.);

    std::cout << m1 * m2;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
3 3
3 3
```

2.6.34 mult

Function

```
rmatrix& rmatrix::mult (const rmatrix& m1, const rmatrix& m2)
throw (cvmexception);
```

sets a calling matrix to be equal to a product of a matrix `m1` by a matrix `m2` and returns a reference to the matrix changed. The function throws an exception of type `cvmexception` in case of inappropriate sizes of the operands. The function is *inherited* in the class `srmatrix` and *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `rmatrix`. Example:

```
using namespace cvm;

try {
    rmatrix m1(2, 3);
    rmatrix m2(3, 2);
    rmatrix m(2, 2);
    m1.set(1.);
    m2.set(1.);

    std::cout << m.mult(m1, m2) << std::endl;
    std::cout << m1.mult(m, m1);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
3 3
3 3

6 6 6
6 6 6
```

2.6.35 rank1update

Function

```
rmatrix&
rmatrix::rank1update (const rvector& vCol, const rvector& vRow)
throw (cvmexception);
```

sets a calling matrix to be equal to the rank-1 update of vectors `vCol` and `vRow` and returns a reference to the matrix changed. The function throws an exception of type `cvmexception` if the number of rows of the calling matrix is not equal to `vCol.size()` or the number of columns of the calling matrix is not equal to `vRow.size()`. The function is *inherited* in the the class `srmatrix` and *not applicable* to objects of the classes `srbmatrix` and `srsmatrix` (i.e. an exception of type `cvmexception` would be thrown in case of using it for objects of those classes). See also rvector::rank1update, `rmatrix`. Example:

```
using namespace cvm;

try {
    rvector vc(3), vr(2);
    rmatrix m(3, 2);
    vc(1) = 1.;
    vc(2) = 2.;
    vc(3) = 3.;
    vr(1) = 4.;
    vr(2) = 5.;

    std::cout << m.rank1update (vc, vr);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
4 5
8 10
12 15
```

2.6.36 swap_rows

Function

```
rmatrix& rmatrix::swap_rows (int n1, int n2) throw (cvmexception);
```

swaps two rows of a calling matrix and returns a reference to the matrix changed. `n1` and `n2` are the numbers of rows to be swapped, both are 1-based). The function throws an exception of type `cvmexception` if one of the parameters is outside of the range `[1,msize()]`. The function is *redefined* in the the class `srmatrix` and *not applicable* to objects of the classes `srbmatrix` and `srsmatrix` (i.e. an exception of type `cvmexception` would be thrown in case of using it for objects of those classes). See also `rmatrix`. Example:

```
using namespace cvm;
try {
    double a[] = {1., 2., 3., 4., 5., 6.};
    rmatrix m (a, 3, 2);

    std::cout << m << std::endl;
    std::cout << m.swap_rows(2,3);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1 4
2 5
3 6

1 4
3 6
2 5
```

2.6.37 swap_cols

Function

```
rmatrix& rmatrix::swap_cols (int n1, int n2) throw (cvmexception);
```

swaps two columns of a calling matrix and returns a reference to the matrix changed. n1 and n2 are the numbers of columns to be swapped, both are 1-based). The function throws an exception of type cvmexception if one of the parameters is outside of the range [1,nsize()]. The function is *redefined* in the the class srmatrix and *not applicable* to objects of the classes srbmatrix and srsmatrix (i.e. an exception of type cvmexception would be thrown in case of using it for objects of those classes). See also rmatrix. Example:

```
using namespace cvm;
try {
    double a[] = {1., 2., 3., 4., 5., 6.};
    rmatrix m (a, 2, 3);

    std::cout << m << std::endl;
    std::cout << m.swap_cols(2,3);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1 3 5
2 4 6

1 5 3
2 6 4
```

2.6.38 solve

Functions

```
rmatrix&
rmatrix::solve (const srmatrix& mA,
                const rmatrix& mB, TR& dErr) throw (cvmexception);
rmatrix&
rmatrix::solve (const srmatrix& mA,
                const rmatrix& vB) throw (cvmexception);
```

set a calling matrix to be equal to a solution X of the matrix linear equation $AX = B$ where the parameter `mA` is the square matrix A and the parameter `vB` is the matrix B. Every function returns a reference to the matrix changed. The first version also sets the output parameter `dErr` to be equal to the norm of computation error. The functions throw exception of type `cvmexception` in case of inappropriate sizes of the operands or when the matrix A is close to cingular. The functions are *redefined* in the the class `srmatrix` and *inherited* thereafter in the classes `srbmatrix` and `srsmatrix`. See also `rvector::solve`, `rmatrix`, `srmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 10.};
    srmatrix ma(a, 3);
    rmatrix  mb(3,4);
    rmatrix  mx(3,4);
    double dErr;
    mb.randomize(1., 10.);

    mx.solve (ma, mb, dErr);
    std::cout << mx << std::endl << ma * mx - mb;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
-4.47e-001 -4.17e+000 -4.21e+000 -8.49e-002
-8.41e+000 2.30e+001 1.57e+001 -7.21e+000
5.70e+000 -1.21e+001 -7.57e+000 5.10e+000

-8.88e-015 5.33e-015 7.99e-015 1.78e-015
```

```
-4.44e-015 1.78e-015 3.55e-015 3.55e-015
1.78e-015 0.00e+000 -4.44e-015 -8.88e-016
```

2.6.39 solve_lu

Functions

```
rmatrix&
rmatrix::solve_lu (const srmatrix& mA, const srmatrix& mLU,
                   const int* pPivots, const rmatrix& mB, TR& dErr)
                   throw (cvmexception);
rmatrix&
rmatrix::solve_lu (const srmatrix& mA, const srmatrix& mLU,
                   const int* pPivots, const rmatrix& mB)
                   throw (cvmexception);
```

set a calling matrix to be equal to a solution X of the matrix linear equation $AX = B$ where the parameter `mA` is the square matrix A, parameter `mLU` is LU factorization of the matrix A, parameter `pPivots` is an array of pivot numbers created while factorizing the matrix A and the parameter `mB` is the matrix B. Every function returns a reference to the matrix changed. The first version also sets output parameter `dErr` to be equal to a norm of computation error. These functions are useful when you need to solve few linear equations of kind $AX = B$ with the same matrix A and different matrices B. In such case you save on matrix A factorization since it's needed to be performed just one time. The functions throw exception of type `cvmexception` in case of inappropriate sizes of the operands or when the matrix A is close to cingular. See also `rvector::solve`, `rmatrix`, `srmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    double a[] = {1., -1., 1., 2., -2., 1., 3., -2., 1.};
    srmatrix ma(a,3);
    srmatrix mLU(3);
    rmatrix  mb1(3,2);
    rmatrix  mb2(3,2);
    rmatrix  mx1(3,2);
    rmatrix  mx2(3,2);
    iarray   nPivots(3);
    double   dErr = 0.;
    mb1.randomize(-1.,3.);
    mb2.randomize(2.,5.);

    mLU.low_up(ma, nPivots);
    std::cout << mx1.solve_lu (ma, mLU, nPivots, mb1, dErr);
    std::cout << dErr << std::endl;
```

```
    std::cout << mx2.solve_lu (ma, mLU, nPivots, mb2) << std::endl;
    std::cout << ma * mx1 - mb1 << std::endl << ma * mx2 - mb2;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
3.85e+00 5.90e-01
-4.23e+00 -3.67e+00
2.10e+00 2.55e+00
7.04e-15
9.49e+00 8.93e+00
-1.00e+01 -1.42e+01
4.21e+00 7.55e+00

0.00e+00 0.00e+00
0.00e+00 0.00e+00
4.44e-16 -1.11e-16

4.44e-16 0.00e+00
-4.44e-16 0.00e+00
8.88e-16 0.00e+00
```

2.6.40 svd

Functions

```
rvector
rmatrix::svd () const throw (cvmexception);
rvector
rmatrix::svd (srmatrix& mU, srmatrix& mVH) const throw (cvmexception);
```

create an object of type `rvector` as a vector of singular values of a calling matrix The second version of the function sets output parameter `mU` to be equal to the matrix U of size $m \times m$ and `mVH` to be equal to the matrix V^H of size $n \times n$. All the functions throw an exception of type `cvmexception` in case of inappropriate sizes of the operands or in case of convergence error. Use `rvector::svd` in order to get rid of a new vector creation. The function is *redefined* in the the classes `srmatrix`, `srbmatrix`, `srsmatrix`. See also `rvector`, `rmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    double m[] = {1., -1., 1., 2., -2., 1.,
                  3., -2., 1., 0., -2., 1.};
    rmatrix mA(m,4,3);
    rmatrix mSigma(4,3);
    rvector v;
    srmatrix mU(4), mVH(3);

    v << mA.svd(mU, mVH);
    mSigma.diag(0) = v;

    std::cout << mU << std::endl;
    std::cout << mVH << std::endl;
    std::cout << mSigma << std::endl;

    std::cout << (mA * ~mVH - mU * mSigma).norm() << std::endl;
    std::cout << (~mA * mU - ~(mSigma * mVH)).norm() << std::endl;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
-4.84e-01 1.95e-01 1.15e-02 -8.53e-01
2.17e-01 -3.41e-01 -8.89e-01 -2.13e-01
```

```
6.62e-01 7.16e-01 -6.18e-02 -2.13e-01
-5.29e-01 5.78e-01 -4.53e-01 4.26e-01

-2.21e-01 8.54e-01 -4.72e-01
9.59e-01 1.04e-01 -2.62e-01
-1.75e-01 -5.11e-01 -8.42e-01

4.96e+00 0.00e+00 0.00e+00
0.00e+00 2.51e+00 0.00e+00
0.00e+00 0.00e+00 3.77e-01
0.00e+00 0.00e+00 0.00e+00

1.37e-15
2.48e-15
```

2.6.41 pinv

Functions

```
rmatrix  rmatrix::pinv (TR threshold = cvmMachSp()) const
                        throw (cvmexception);
rmatrix& rmatrix::pinv (const basic_rmatrix& mA,
                        TR threshold = cvmMachSp())
                        throw (cvmexception);
```

encapsulate matrix pseudo inversion [6], p. 33 (or Moore-Penrose generalized inversion [5], p. 421). Strictly defined, an $n \times m$ matrix A^+ is a *pseudo inversion* of $m \times n$ matrix A if the following two equations are satisfied:

$$AA^+A = A,$$
$$A^+ = QA^H = A^H P$$

where Q and P are some matrices. To compute the pseudo inversion, we use Singular Value Decomposition (SVD)

$$A = U\Sigma V^H$$

of matrix A, thus

$$A^+ = V\Sigma^{-1}U^H,$$

where Σ^{-1} is a diagonal $n \times m$ matrix having inverted diagonal values of the matrix Σ if they are greater than some threshold, and zeros otherwise.

First version creates an object of type `rmatrix` as a pseudo inverted calling matrix (it throws an exception of type `cvmexception` in case of memory allocation failure). Second function sets a calling matrix to be equal to a matrix `mA` pseudo inverted (it throws an exception of type `cvmexception` in case of not appropriate sizes of the operands). The threshold parameter sets a minimum distinguishable from zero singular value to be used to compute the pseudo inversion. All values equal or less than the threshold are treated as zeros. The functions are *inherited* in the classes `srmatrix` and `srsmatrix` and *redefined* in `srbmatrix`. See also `rmatrix`. Example:

```
using namespace cvm;

try {
    rmatrix mA(3,4);
    mA(1,1) =  1.; mA(1,2) = -1.; mA(1,3) =  2.; mA(1,4) =  0.;
    mA(2,1) = -1.; mA(2,2) =  2.; mA(2,3) = -3.; mA(2,4) =  1.;
    mA(3,1) =  0.; mA(3,2) =  1.; mA(3,3) = -1.; mA(3,4) =  1.;
    rmatrix mX = mA.pinv(1.e-13);
    std::cout << mX << (mA * mX * mA - mA).norm2() << std::endl;
}
catch (exception& e) {
```

```
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
+3.3333333e-001 -2.1510571e-016 +3.3333333e-001
+1.1111111e-001 +1.1111111e-001 +2.2222222e-001
+2.2222222e-001 -1.1111111e-001 +1.1111111e-001
+4.4444444e-001 +1.1111111e-001 +5.5555556e-001
+2.5460202e-015
```

Band matrix example:

```
using namespace cvm;

try {
    srbmatrix mA (40, 1, 0);
    mA.diag(0).randomize(-1.,1.);
    mA.diag(-1).randomize(5.,10.);

    srmatrix mX (40);
    mX.pinv(mA);
    std::cout << (mA * mX * mA - mA).norm2() << std::endl;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
+8.1956952e-14
```

2.6.42 rank

Function

```
int rmatrix::rank (TR eps = cvmMachSp ()) const throw (cvmexception);
```

returns a rank of a calling matrix as a number of singular values with normalized absolute value greater than or equal to a parameter `eps` (this is the largest relative spacing by default). The function throws an exception of type `cvmexception` in case of convergence error. The function is *inherited* in the the classes `srmatrix`, `srbmatrix`, `srsmatrix`. See also `rmatrix`. Example:

```
using namespace cvm;
try {
    double a[] = {1., 2., 3., 4., 5., 6.,
                  7., 8., 9., 10., 11., 12.};
    rmatrix m(a,3,4);

    std::cout << m << m.rank() << std::endl;
    m(3,4) = 13.;
    std::cout << m.rank() << std::endl;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1 4 7 10
2 5 8 11
3 6 9 12
2
3
```

2.6.43 ger

Function

```
rmatrix&
rmatrix::ger (TR dAlpha, const rvector& vCol, const rvector& vRow)
throw (cvmexception);
```

calls one of ?GER routines of the BLAS library performing a rank-1 update matrix-vector operation defined as $M = \alpha\, x \cdot y' + M$, where α is a real number (parameter dAlpha), M is a calling matrix and x and y are real vectors (parameters vCol and vRow respectively). The function returns a reference to the matrix changed and throws an exception of type cvmexception in case of inappropriate sizes of the operands. The function is *inherited* in the the class srmatrix and *not applicable* to objects of the classes srbmatrix and srsmatrix (i.e. an exception of type cvmexception would be thrown in case of using it for objects of those classes). See also rvector, rmatrix. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (4);
try {
    double alpha = 1.3;
    rmatrix m(3,4);
    rvector vc(3);
    rvector vr(4);
    m.randomize(-1., 2.); vc.randomize(-1., 3.); vr.randomize(0., 2.);

    std::cout << m + vc.rank1update (vr) * alpha << std::endl;
    std::cout << m.ger(alpha, vc, vr);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
-1.7127e-01 2.9410e+00 1.3449e+00 3.6055e+00
1.9057e+00 2.6726e+00 1.7134e+00 2.2154e+00
1.7217e-01 1.3508e+00 8.8949e-01 2.2551e+00

-1.7127e-01 2.9410e+00 1.3449e+00 3.6055e+00
1.9057e+00 2.6726e+00 1.7134e+00 2.2154e+00
1.7217e-01 1.3508e+00 8.8949e-01 2.2551e+00
```

2.6.44 gemm

Function

```
rmatrix& rmatrix::gemm (const rmatrix& m1, bool bTrans1,
                        const rmatrix& m2, bool bTrans2,
                        TR dAlpha, TR dBeta) throw (cvmexception);
```

calls one of ?GEMM routines of the BLAS library performing a matrix-matrix operation defined as

$$M = \alpha \mathcal{T}(M_1) \cdot \mathcal{T}(M_2) + \beta M,$$

where α and β are real numbers (parameters dAlpha and dBeta), M is a calling matrix and M_1 and M_2 are matrices (parameters m1 and m2 respectively). Function $\mathcal{T}(M_i)$ transposes matrix M_i if appropriate boolean parameter bTrans* is equal to true and does nothing otherwise. The function returns a reference to the matrix changed and throws an exception of type cvmexception in case of inappropriate sizes of the operands. The function is *inherited* in the the class srmatrix and *not applicable* to objects of the classes srbmatrix and srsmatrix (i.e. an exception of type cvmexception would be thrown in case of using it for objects of those classes). See also rmatrix. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (4);
try {
    double alpha = 1.3;
    double beta = -0.7;
    rmatrix m1(4,3); rmatrix m2(4,3);
    rmatrix m(3,3);
    m.randomize(-1., 2.); m1.randomize(-1., 3.); m2.randomize(0., 2.);
    std::cout << ~m1 * m2 * alpha + m * beta << std::endl;
    std::cout << m.gemm(m1, true, m2, false, alpha, beta);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
5.0504e+00 6.8736e+00 3.1171e+00
2.3915e+00 2.2544e+00 3.9205e+00
3.4607e+00 3.5351e+00 4.8622e+00

5.0504e+00 6.8736e+00 3.1171e+00
2.3915e+00 2.2544e+00 3.9205e+00
3.4607e+00 3.5351e+00 4.8622e+00
```

2.6.45 symm

Function

```
rmatrix& rmatrix::symm (bool bLeft, const srsmatrix& ms,
                        const rmatrix& m, TR dAlpha, TR dBeta)
                        throw (cvmexception);
```

calls one of ?SYMM routines of the BLAS library performing one of matrix-matrix operations defined as

$$M = \alpha M_s \cdot M_1 + \beta M \quad \text{or} \quad M = \alpha M_1 \cdot M_s + \beta M,$$

where α and β are real numbers (parameters dAlpha and dBeta), M is a calling matrix, M_s is a symmetric matrix and M_1 is a real matrix (parameters ms and m respectively). First operation is performed if bLeft passed is true and second one otherwise. The function returns a reference to the matrix changed and throws an exception of type cvmexception in case of inappropriate sizes of the operands. The function is *inherited* in the the classes srmatrix and srsmatrix and *not applicable* to objects of the class srbmatrix (i.e. an exception of type cvmexception would be thrown in case of using it for objects of that class). See also srsmatrix, rmatrix. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (4);
try {
    double alpha = 1.3;
    double beta = -0.7;
    rmatrix m1(3,4);
    rmatrix m2(4,3);
    srsmatrix ms(3);
    rmatrix m(3,4);
    m.randomize(-1., 2.); m1.randomize(-1., 3.); m2.randomize(0., 2.);
    ms.randomize(-3., 1.);

    std::cout << ms * m1 * alpha + m * beta << std::endl;
    std::cout << m.symm (true, ms, m1, alpha, beta) << std::endl;

    m.resize(4,3);
    std::cout << m2 * ms * alpha + m * beta << std::endl;
    std::cout << m.symm (false, ms, m2, alpha, beta);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
-3.3733e+00 -5.0566e+00 -6.3018e+00 -5.4907e+00
-1.8629e+00 -1.5133e+00 -1.1372e+00 -2.5557e+00
-3.5695e+00 -1.0012e+01 -1.4239e+00 -6.1786e-01

-3.3733e+00 -5.0566e+00 -6.3018e+00 -5.4907e+00
-1.8629e+00 -1.5133e+00 -1.1372e+00 -2.5557e+00
-3.5695e+00 -1.0012e+01 -1.4239e+00 -6.1786e-01

-6.4072e+00 7.0534e-01 1.5349e+00
-4.8219e+00 -6.9891e+00 -5.1766e+00
6.8503e-01 3.5828e+00 -3.2174e+00
2.3469e-01 -9.3921e-01 -2.1961e+00

-6.4072e+00 7.0534e-01 1.5349e+00
-4.8219e+00 -6.9891e+00 -5.1766e+00
6.8503e-01 3.5828e+00 -3.2174e+00
2.3469e-01 -9.3921e-01 -2.1961e+00
```

2.6.46 qr

Functions

```
void rmatrix::qr (rmatrix& mQ, srmatrix& mR) const throw (cvmexception);
void rmatrix::qr (srmatrix& mQ, rmatrix& mR) const throw (cvmexception);
```

compute QR factorization as

$$M = QR$$

where M is a calling matrix, orthogonal matrix Q and upper triangular matrix R are `mQ` and `mR` respectively. First version encapsulates so-called "economy" algorithm which for a given $m \times n$ matrix M computes $m \times n$ matrix Q and $n \times n$ matrix R. Second version is a "full" mode one computing $m \times m$ matrix Q and $m \times n$ matrix R. The functions throw an exception of type `cvmexception` in case of inappropriate sizes of the operands. The functions are *redefined* in the the class `srmatrix`. See also `rmatrix`, `srmatrix`. Example:

```
using namespace cvm;

treal a[] = {1., 2., 3., 4., 5., 6.};
const cvm::rmatrix mh(a, 2, 3);
const cvm::rmatrix mv(a, 3, 2);
cvm::srmatrix s2(2), s3(3);
cvm::rmatrix  h(2,3), v(3,2);

mh.qr(h,s3);
std::cout << (eye_real(2) - ~rmatrix(h,1,1,2,2)*rmatrix(h,1,1,2,2)).norm()
          << " " << (mh - h * s3).norm() << std::endl;
mh.qr(s2,h);
std::cout << (eye_real(2) - ~s2 * s2).norm()
          << " " << (mh - s2 * h).norm() << std::endl;
mv.qr(v,s2);
std::cout << (eye_real(2) - ~v * v).norm()
          << " " << (mv - v * s2).norm() << std::endl;
mv.qr(s3,v);
std::cout << (eye_real(3) - ~s3 * s3).norm()
          << " " << (mv - s3 * v).norm() << std::endl;
```

prints

```
+4.6933177e-16 +2.2342807e-15
+4.6933177e-16 +2.2342807e-15
+5.1302953e-16 +1.4432899e-15
+5.2889959e-16 +1.4432899e-15
```

2.6.47 vanish

Function

```
rmatrix& rmatrix::vanish();
```

sets every element of a calling matrix to be equal to zero and returns a reference to the matrix changed. This function is faster than, for example, `rmatrix::set(TR)` with zero operand passed. The function is *redefined* in the classes `srmatrix`, `srsmatrix`, `srbmatrix`. See also `rmatrix`. Example:

```
using namespace cvm;

rmatrix m(3, 4);
m.randomize(0.,1.);

std::cout << m << std::endl;
std::cout << m.vanish ();
```

prints

```
0.856532 0.938261 0.275704 0.186834
0.651173 0.812159 0.100467 0.536912
0.0726646 0.695914 0.661824 0.554613

0 0 0 0
0 0 0 0
0 0 0 0
```

2.6.48 randomize

Function

```
rmatrix& rmatrix::randomize (TR dFrom, TR dTo);
```

fills a calling matrix with pseudo-random numbers distributed between `dFrom` and `dTo`. The function returns a reference to the matrix changed. See also `rmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (7);

rmatrix m(3,4);
m.randomize(-2.,3.);
std::cout << m;
```

prints

```
9.6853542e-01 2.7761467e+00 2.3791009e+00 -3.4452345e-01
2.9029511e+00 -9.5519883e-01 -4.9131748e-01 -1.2561113e+00
1.5219886e+00 -1.4494461e+00 2.8193304e+00 4.8817408e-01
```

2.7 cmatrix

This is end-user class encapsulating a matrix in Euclidean space of complex numbers.

```
template <typename TR, typename TC>
class cmatrix : public Matrix <TR,TC> {
public:
    cmatrix ();
    cmatrix (int nM, int nN);
    cmatrix (TC* pD, int nM, int nN);
    cmatrix (const cmatrix& m);
    explicit cmatrix (const cvector& v, bool bBeColumn = true);
    explicit cmatrix (const rmatrix& m, bool bRealPart = true);
    cmatrix (const TR* pRe, const TR* pIm, int nM, int nN);
    cmatrix (const rmatrix& mRe, const rmatrix& mIm);
    cmatrix (cmatrix& m, int nRow, int nCol, int nHeight, int nWidth);
    TC& operator () (int im, int in) throw (cvmexception);
    TC operator () (int im, int in) const throw (cvmexception);
    cvector operator () (int i) throw (cvmexception);
    const cvector operator () (int i) const throw (cvmexception);
    cvector operator [] (int i) throw (cvmexception);
    const cvector operator [] (int i) const throw (cvmexception);
    cvector diag (int i) throw (cvmexception);
    const cvector diag (int i) const throw (cvmexception);
    const rmatrix real () const;
    const rmatrix imag () const;
    cmatrix& operator = (const cmatrix& m) throw (cvmexception);
    cmatrix& assign (const cvector& v);
    cmatrix& assign (const TC* pD);
    rmatrix& assign (int nRow, int nCol, const cmatrix& m)
                     throw (cvmexception);
    cmatrix& set (TC z);
    cmatrix& assign_real (const rmatrix& mRe) throw (cvmexception);
    cmatrix& assign_imag (const rmatrix& mIm) throw (cvmexception);
    cmatrix& set_real (TR d);
    cmatrix& set_imag (TR d);
    cmatrix& resize (int nNewM, int nNewN) throw (cvmexception);
    bool operator == (const cmatrix& v) const;
    bool operator != (const cmatrix& v) const;
    cmatrix& operator << (const cmatrix& m) throw (cvmexception);
    cmatrix operator + (const cmatrix& m) const throw (cvmexception);
    cmatrix operator - (const cmatrix& m) const throw (cvmexception);
    cmatrix& sum (const cmatrix& m1,
```

```
              const cmatrix& m2) throw (cvmexception);
cmatrix& diff (const cmatrix& m1,
               const cmatrix& m2) throw (cvmexception);
cmatrix& operator += (const cmatrix& m) throw (cvmexception);
cmatrix& operator -= (const cmatrix& m) throw (cvmexception);
cmatrix operator - () const;
cmatrix operator * (TR d) const;
cmatrix operator / (TR d) const throw (cvmexception);
cmatrix operator * (TC z) const;
cmatrix operator / (TC z) const throw (cvmexception);
cmatrix& operator *= (TR d);
cmatrix& operator /= (TR d) throw (cvmexception);
cmatrix& operator *= (TC z);
cmatrix& operator /= (TC z) throw (cvmexception);
cmatrix& normalize ();
cmatrix operator ~() const throw (cvmexception);
cmatrix& conj (const cmatrix& m) throw (cvmexception);
cmatrix& conj () throw (cvmexception);
cvector operator * (const cvector& v) const
                    throw (cvmexception);
cmatrix operator * (const cmatrix& m) const
                    throw (cvmexception);
cmatrix& mult (const cmatrix& m1, const cmatrix& m2)
               throw (cvmexception);
cmatrix& rank1update_u (const rvector& vCol,
                        const rvector& vRow) throw (cvmexception);
cmatrix& rank1update_c (const rvector& vCol,
                        const rvector& vRow) throw (cvmexception);
cmatrix& swap_rows (int n1, int n2) throw (cvmexception);
cmatrix& swap_cols (int n1, int n2) throw (cvmexception);
cmatrix& solve (const scmatrix& mA,
                const cmatrix& mB, TR& dErr)
                throw (cvmexception);
cmatrix& solve (const scmatrix& mA,
                const cmatrix& mB) throw (cvmexception);
cmatrix& solve_lu (const scmatrix& mA, const scmatrix& mLU,
                   const int* pPivots, const cmatrix& mB, TR& dErr)
                   throw (cvmexception);
cmatrix& solve_lu (const scmatrix& mA, const scmatrix& mLU,
                   const int* pPivots, const cmatrix& mB)
                   throw (cvmexception);
rvector svd () throw (cvmexception);
rvector svd (cmatrix& mLSingVect, cmatrix& mRSingVect)
```

```
                 throw (cvmexception);
    cmatrix pinv (TR threshold = basic_cvmMachSp<TR>()) const
                  throw (cvmexception);
    cmatrix& pinv (const cmatrix& mA,
                   TR threshold = cvmMachSp<TR>())
                   throw (cvmexception);
    void qr(cmatrix& mQ, scmatrix& mR) const throw (cvmexception);
    void qr(scmatrix& mQ, cmatrix& mR) const throw (cvmexception);
    int rank (TR eps = cvmMachSp ()) const throw (cvmexception);
    cmatrix& vanish ();
    cmatrix& geru (TC alpha, const cvector& vCol,
                   const cvector& vRow) throw (cvmexception);
    cmatrix& gerc (TC alpha, const cvector& vCol,
                   const cvector& vRow) throw (cvmexception);
    cmatrix& gemm (const cmatrix& m1, bool bTrans1,
                   const cmatrix& m2, bool bTrans2,
                   TC dAlpha, TC dBeta) throw (cvmexception);
    cmatrix& hemm (bool bLeft, const schmatrix& ms, const cmatrix& m,
                   TC dAlpha, TC dBeta) throw (cvmexception);
    cmatrix& randomize_real (TR dFrom, TR dTo);
    cmatrix& randomize_imag (TR dFrom, TR dTo);
};
```

2.7.1 cmatrix ()

Constructor

```
cmatrix::cmatrix ();
```

creates an empty `cmatrix` object. See also `cmatrix`. Example:

```
using namespace cvm;

cmatrix m;
std::cout << m.msize() << std::endl
          << m.nsize() << std::endl;

m << eye_complex(3);
std::cout << m;
```

prints

```
0
0
(1,0) (0,0) (0,0)
(0,0) (1,0) (0,0)
(0,0) (0,0) (1,0)
```

2.7.2 cmatrix (int,int)

Constructor

```
cmatrix::cmatrix (int nM, int nN);
```

creates an $m \times n$ `cmatrix` object where m is passed in `nM` parameter (number of rows) and n is passed in `nN` (number of columns). The constructor throws an exception of type `cvmexception` in case of non-positive sizes passed or memory allocation failure. See also `cmatrix`. Example:

```
using namespace cvm;

cmatrix m (3, 4);
std::cout << m.msize() << std::endl
          << m.nsize() << std::endl
          << m.size()  << std::endl << m;
```

prints

```
3
4
12
(0,0) (0,0) (0,0) (0,0)
(0,0) (0,0) (0,0) (0,0)
(0,0) (0,0) (0,0) (0,0)
```

2.7.3 cmatrix (TC*,int,int)

Constructor

```
cmatrix::cmatrix (TC* pD, int nM, int nN);
```

creates an $m \times n$ `cmatrix` object where m is passed in `nM` parameter (number of rows) and n is passed in `nN` (number of columns). Unlike others, this constructor *does not allocate memory*. It just shares a memory with an array pointed to by `pD`. See also `cmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6.,
              7., 8., 9., 10., 11., 12.};
cmatrix m ((std::complex<double>*) a, 2, 3);

std::cout << m << std::endl;

a[1] = 7.77;
std::cout << m;
```

prints

```
(1,2) (5,6) (9,10)
(3,4) (7,8) (11,12)

(1,7.77) (5,6) (9,10)
(3,4) (7,8) (11,12)
```

2.7.4 cmatrix (const cmatrix&)

Copy constructor

```
cmatrix::cmatrix (const cmatrix& m);
```

creates a `cmatrix` object as a copy of `m`. The constructor throws an exception of type `cvmexception` in case of memory allocation failure. See also `cmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6.,
              7., 8., 9., 10., 11., 12.};
cmatrix m ((std::complex<double>*) a, 2, 3);
cmatrix mc(m);

m(1,1) = std::complex<double>(7.77,7.77);
std::cout << m << std::endl << mc;
```

prints

```
(7.77,7.77) (5,6) (9,10)
(3,4) (7,8) (11,12)

(1,2) (5,6) (9,10)
(3,4) (7,8) (11,12)
```

2.7.5 cmatrix (const cvector&,bool)

Constructor

```
explicit cmatrix::cmatrix (const cvector& v, bool bBeColumn = true);
```

creates a `cmatrix` object containing `v.size()` rows and 1 column if `bBeColumn` is `true` or 1 row and `v.size()` columns otherwise. After that it copies the vector `v`'s elements to the matrix created. The constructor throws an exception of type `cvmexception` in case of memory allocation failure. See also `cmatrix`, `cvector`. Example:

```
using namespace cvm;

cvector v(3);
v(1) = std::complex<double>(1.,2.);
v(2) = std::complex<double>(2.,3.);
v(3) = std::complex<double>(3.,4.);

cmatrix mc (v);
cmatrix mr (v, false);

std::cout << mc << std::endl << mr;
```

prints

```
(1,2)
(2,3)
(3,4)

(1,2) (2,3) (3,4)
```

2.7.6 cmatrix (const rmatrix&,bool)

Constructor

```
explicit cmatrix::cmatrix (const rmatrix& m, bool bRealPart = true);
```

creates a `cmatrix` object containing `m.msize()` rows and `m.nsize()` columns and copies the matrix `m` to its real part if `bRealPart` is `true` or to its imaginary part otherwise. The constructor throws an exception of type `cvmexception` in case of memory allocation failure. See also `cmatrix`, `rmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6.};
const rmatrix m (a, 2, 3);
cmatrix mr(m), mi(m, false);

std::cout << mr << std::endl << mi;
```

prints

```
(1,0) (3,0) (5,0)
(2,0) (4,0) (6,0)

(0,1) (0,3) (0,5)
(0,2) (0,4) (0,6)
```

2.7.7 cmatrix (const TR*,const TR*,int,int)

Constructor

```
cmatrix::cmatrix (const TR* pRe, const TR* pIm,
                  const int nM, const int nN);
```

creates a `cmatrix` object of size `nM` by `nN` and copies every element of arrays pointed to by `pRe` and `pIm` to a real and imaginary part of the matrix created respectively. Use `NULL` pointer to fill up appropriate part with zero values. The constructor throws an exception of type `cvmexception` in case of memory allocation failure. See also `cmatrix`. Example:

```
using namespace cvm;

double re[] = {1., 2., 3., 4., 5., 6.};
double im[] = {6., 5., 4., 3., 2., 1.};
cmatrix m (re, im, 3, 2);
std::cout << m << std::endl;
re[1] = 7.77;
std::cout << m << std::endl;

const double re2[] = {1., 2., 3., 4.};
const cmatrix m2 (re2, NULL, 2, 2);
std::cout << m2;
```

prints

```
(1,6) (4,3)
(2,5) (5,2)
(3,4) (6,1)

(1,6) (4,3)
(2,5) (5,2)
(3,4) (6,1)

(1,0) (3,0)
(2,0) (4,0)
```

2.7.8 cmatrix (const rmatrix&, const rmatrix&)

Constructor

```
cmatrix::cmatrix (const rmatrix& mRe, const rmatrix& mIm);
```

creates a `cmatrix` object of size `mRe.msize()` by `mRe.nsize()` (if one of these sizes differs from appropriate size of matrix `mIm` then the constructor throws an exception of type `cvmexception`) and copies matrices `mRe` and `mIm` to a real and imaginary part of the matrix created respectively. The constructor throws an exception of type `cvmexception` in case of memory allocation failure. See also `cmatrix`, `rmatrix`. Example:

```
using namespace cvm;

rmatrix mr(3,3), mi(3,3);
mr(1,1) = 1.;
mr(2,2) = 2.;
mr(3,3) = 3.;
mi(1,3) = 6.;
mi(2,2) = 5.;
mi(3,1) = 4.;

const cmatrix mc(mr, mi);
std::cout << mc;
```

prints

```
(1,0) (0,0) (0,6)
(0,0) (2,5) (0,0)
(0,4) (0,0) (3,0)
```

2.7.9 submatrix

Submatrix constructor

```
cmatrix::cmatrix (cmatrix& m, int nRow, int nCol,
                              int nHeight, int nWidth);
```

creates a `cmatrix` object as a *submatrix* of `m`. It means that the matrix object created shares a memory with some part of `m`. This part is defined by its upper left corner (parameters `nRow` and `nCol`, both are 1-based) and its height and width (parameters `nHeight` and `nWidth`). See also `cmatrix`. Example:

```
using namespace cvm;

cmatrix m(4,5);
cmatrix subm(m, 2, 2, 2, 2);
subm.set(std::complex<double>(1.,1.));

std::cout << m;
```

prints

```
(0,0) (0,0) (0,0) (0,0) (0,0)
(0,0) (1,1) (1,1) (0,0) (0,0)
(0,0) (1,1) (1,1) (0,0) (0,0)
(0,0) (0,0) (0,0) (0,0) (0,0)
```

2.7.10 operator (,)

Indexing operators

```
TC& cmatrix::operator () (int im, int in) throw (cvmexception);
TC cmatrix::operator () (int im, int in) const throw (cvmexception);
```

provide access to an element of a matrix. The first version of the operator is applicable to a non-constant object. This version returns an *l-value* in order to make possible write access to an element. Both operators are 1-based. The operators throw an exception of type `cvmexception` if `im` is outside of `[1,msize()]` range or `in` is outside of `[1,nsize()]` range. The operators are *inherited* in the the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6.,
                  7., 8., 9., 10., 11., 12.};
    const cmatrix m ((std::complex<double>*) a, 2, 3);
    scmatrix ms(2);
    std::cout << m(1,1) << " "
              << m(2,3) << std::endl << std::endl;
    ms(2,2) = std::complex<double>(7.77,7.77);
    std::cout << ms;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(1,2) (11,12)

(0,0) (0,0)
(0,0) (7.77,7.77)
```

2.7.11 operator ()

Indexing operators

```
cvector cmatrix::operator () (int i) throw (cvmexception);
const cvector cmatrix::operator () (int i) const throw (cvmexception);
```

provide access to an i-th column of a matrix. The first version of the operator is applicable to a non-constant object and *returns an l-value*, i.e. the vector returned shares a memory with the i-th column of the matrix in order to make possible write access to it. The second version creates a *copy* of the column and therefore is *not an l-value*. Both operators are 1-based. The operators throw an exception of type `cvmexception` if the parameter `i` is outside of `[1,nsize()]` range. The operators are *inherited* in the the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix`, `Matrix::msize()`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6.,
                  7., 8., 9., 10., 11., 12.};
    const cmatrix m ((std::complex<double>*) a, 2, 3);
    scmatrix ms(2);

    std::cout << m(2) << std::endl;

    ms(2) = m(3);
    std::cout << ms;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(5,6) (7,8)

(0,0) (9,10)
(0,0) (11,12)
```

2.7.12 operator []

Indexing operators

```
cvector cmatrix::operator [] (int i) throw (cvmexception);
const cvector cmatrix::operator [] (int i) const throw (cvmexception);
```

provide access to an i-th row of a matrix. The first version of the operator is applicable to a non-constant object and *returns an l-value*, i.e. the vector returned shares a memory with the i-th row of the matrix in order to make possible write access to it. The second version creates a *copy* of the row and therefore is *not an l-value*. Both operators are 1-based. The operators throw an exception of type `cvmexception` if `i` is outside of `[1,msize()]` range. The operators are *inherited* in the the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix`, `Matrix::msize()`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6.,
                  7., 8., 9., 10., 11., 12.};
    const cmatrix m ((std::complex<double>*) a, 2, 3);
    scmatrix ms(3);
    std::cout << m[1] << std::endl;
    ms[1] = m[2];
    std::cout << ms;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(1,2) (5,6) (9,10)

(3,4) (7,8) (11,12)
(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
```

2.7.13 diag

Functions

```
cvector cmatrix::diag (int i) throw (cvmexception);
const cvector cmatrix::diag (int i) const throw (cvmexception);
```

provide access to an i-th diagonal of a matrix, where $i = 0$ for main diagonal, $i < 0$ for lower diagonals and $i > 0$ for upper ones. The first version of the function is applicable to a non-constant object and *returns an l-value*, i.e. the vector returned shares a memory with the i-th diagonal of the matrix in order to make possible write access to it. The second version creates a *copy* of the diagonal and therefore is *not an l-value*. The functions throw an exception of type `cvmexception` if the parameter `i` is outside of `[-msize()+1,nsize()-1]` range. The functions are *inherited* in the the classes `scmatrix` and `scbmatrix`. The functions are *redefined* in the class `schmatrix`. See also `cmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
                  10., 11., 12., 13., 14., 15., 16., 17., 18.};
    cmatrix m (2, 3);
    const scmatrix ms((std::complex<double>*)a, 3);
    m.diag(-1).set(std::complex<double>(1.,1.));
    m.diag(0).set(std::complex<double>(2.,2.));
    m.diag(1).set(std::complex<double>(3.,3.));
    m.diag(2).set(std::complex<double>(4.,4.));
    std::cout << m << std::endl;
    std::cout << ms << std::endl;
    std::cout << ms.diag(0) << ms.diag(1);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(2,2) (3,3) (4,4)
(1,1) (2,2) (3,3)

(1,2) (7,8) (13,14)
(3,4) (9,10) (15,16)
(5,6) (11,12) (17,18)

(1,2) (9,10) (17,18)
(7,8) (15,16)
```

2.7.14 real

Function

```
const rmatrix cmatrix::real () const;
```

creates an object of type `const rmatrix` as a real part of a calling matrix. Please note that, unlike cvector::real, this function creates new object *not sharing* a memory with a real part of the calling matrix, i.e. the matrix returned is *not an l-value*. The function is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `rmatrix`, `cmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6.,
              7., 8., 9., 10., 11., 12.};
cmatrix m((std::complex<double>*) a, 2, 3);

std::cout << m << std::endl << m.real();
```

prints

```
(1,2) (5,6) (9,10)
(3,4) (7,8) (11,12)

1 5 9
3 7 11
```

2.7.15 imag

Function

```
const rmatrix cmatrix::imag () const;
```

creates an object of type `const rmatrix` as an imaginary part of a calling matrix. Please note that, unlike cvector::imag, this function creates new object *not sharing* a memory with an imaginary part of the calling matrix, i.e. the matrix returned is *not an l-value*. The function is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `rmatrix`, `cmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6.,
              7., 8., 9., 10., 11., 12.};
cmatrix m((std::complex<double>*) a, 2, 3);

std::cout << m << std::endl << m.imag();
```

prints

```
(1,2) (5,6) (9,10)
(3,4) (7,8) (11,12)

2 6 10
4 8 12
```

2.7.16 operator = (const cmatrix&)

Operator

```
cmatrix& cmatrix::operator = (const cmatrix& m) throw (cvmexception);
```

sets an every element of a calling rmatrix to a value of appropriate element of a matrix m and returns a reference to the matrix changed. The operator throws an exception of type cvmexception in case of different matrix sizes. The operator is *redefined* in the the classes scmatrix, scbmatrix and schmatrix. See also cmatrix. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6.,
                  7., 8., 9., 10., 11., 12.};
    const cmatrix m1((std::complex<double>*) a, 2, 3);
    cmatrix m2(2, 3);

    m2 = m1;
    std::cout << m2;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(1,2) (5,6) (9,10)
(3,4) (7,8) (11,12)
```

2.7.17 assign (const TC*)

Function

```
cmatrix& cmatrix::assign (const cvector& v);
cmatrix& cmatrix::assign (const TC* pD);
```

sets every element of a calling matrix to a value of appropriate element of a vector `v` or an array pointed to by `pD` and returns a reference to the matrix changed. The function is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix`. Example:

```
using namespace cvm;

const double a[] = {1., 2., 3., 4., 5., 6.,
                    7., 8., 9., 10., 11., 12.};
cmatrix m(2, 3);

m.assign ((const std::complex<double>*) a);
std::cout << m;
```

prints

```
(1,2) (5,6) (9,10)
(3,4) (7,8) (11,12)
```

2.7.18 assign (int, int, const cmatrix&)

Function

```
cmatrix& cmatrix::assign (int nRow, int nCol, const cmatrix& m)
throw (cvmexception);
```

sets sub-matrix of a calling matrix beginning with 1-based row `nRow` and column `nCol` to a matrix `m` and returns a reference to the matrix changed. The function throws an exception of type `cvmexception` if `nRow` or `nCol` are not positive or matrix `m` doesn't fit. The function is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix`. Example:

```
using namespace cvm;

rmatrix m1(4,5);
rmatrix m2(2,2);
m1.set(1.);
m2.set(2.);
m1.assign(2,3,m2);
std::cout << m1;
```

prints

```
(1,1) (1,1) (1,1) (1,1) (1,1)
(1,1) (1,1) (2,2) (2,2) (1,1)
(1,1) (1,1) (2,2) (2,2) (1,1)
(1,1) (1,1) (1,1) (1,1) (1,1)
```

2.7.19 set (TC)

Function

```
cmatrix& cmatrix::set (TC z);
```

sets every element of a calling matrix to a value of parameter z and returns a reference to the matrix changed. Use vanish to set every element of a calling matrix to be equal to zero. The function is *redefined* in the classes scmatrix, scbmatrix and schmatrix. See also cmatrix. Example:

```
using namespace cvm;

cmatrix m(3, 4);
m.set(std::complex<double>(3.,4.));
std::cout << m;
```

prints

```
(3,4) (3,4) (3,4) (3,4)
(3,4) (3,4) (3,4) (3,4)
(3,4) (3,4) (3,4) (3,4)
```

2.7.20 assign_real

Function

```
cmatrix& cmatrix::assign_real (const rmatrix& mRe) throw (cvmexception);
```

sets real part of every element of a calling matrix to a value of appropriate element of a matrix mRe and returns a reference to the matrix changed. The function throws an exception of type cvmexception in case of different sizes of the operands. See also cmatrix and rmatrix. The function is *redefined* in the classes scmatrix, scbmatrix and schmatrix. Example:

```
using namespace cvm;

rmatrix m (2,3);
cmatrix mc(2,3);
m.randomize (0., 1.);

mc.assign_real(m);
std::cout << mc;
```

prints

```
(0.126835,0) (0.57271,0) (0.28312,0)
(0.784417,0) (0.541673,0) (0.663869,0)
```

2.7.21 assign_imag

Function

```
cmatrix& cmatrix::assign_imag (const rmatrix& mIm) throw (cvmexception);
```

sets imaginary part of every element of a calling matrix to a value of appropriate element of a matrix mIm and returns a reference to the matrix changed. The function throws an exception of type cvmexception in case of different sizes of the operands. The function is *redefined* in the classes scmatrix, scbmatrix and schmatrix. See also cmatrix and rmatrix. Example:

```
using namespace cvm;

rmatrix m (2,3);
cmatrix mc(2,3);
m.randomize (0., 1.);

mc.assign_imag(m);
std::cout << mc;
```

prints

```
(0,0.13831) (0,0.267373) (0,0.482345)
(0,0.50618) (0,0.992401) (0,0.444777)
```

2.7.22 set_real

Function

```
cmatrix& cmatrix::set_real (TR d);
```

sets real part of every element of a calling matrix to a value of parameter d and returns a reference to the matrix changed. See also cmatrix. The function is *redefined* in the classes scmatrix, scbmatrix and schmatrix. Example:

```
using namespace cvm;

cmatrix m(2,3);
m.set_real(1.);
std::cout << m;
```

prints

```
(1,0) (1,0) (1,0)
(1,0) (1,0) (1,0)
```

2.7.23 set_imag

Function

```
cmatrix& cmatrix::set_imag (TR d);
```

sets imaginary part of every element of a calling matrix to a value of parameter `d` and returns a reference to the matrix changed. See also `cmatrix`. The function is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. Example:

```
using namespace cvm;

cmatrix m(2,3);
m.set_imag(1.);
std::cout << m;
```

prints

```
(0,1) (0,1) (0,1)
(0,1) (0,1) (0,1)
```

2.7.24 resize

Function

```
cmatrix& cmatrix::resize (int nNewM, int nNewN) throw (cvmexception);
```

changes a size of a calling matrix to nNewM by nNewN and returns a reference to the matrix changed. In case of increasing of its size, the matrix is filled up with zeroes. The function throws an exception of type cvmexception in case of negative size passed or memory allocation failure. The function is *redefined* in the classes scmatrix, scbmatrix and schmatrix. See also cmatrix. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6.,
                  7., 8., 9., 10., 11., 12.};
    cmatrix m((std::complex<double>*) a, 2, 3);
    std::cout << m << std::endl;
    m.resize (2, 2);
    std::cout << m << std::endl;
    m.resize (3, 3);
    std::cout << m;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(1,2) (5,6) (9,10)
(3,4) (7,8) (11,12)

(1,2) (5,6)
(3,4) (7,8)

(1,2) (5,6) (0,0)
(3,4) (7,8) (0,0)
(0,0) (0,0) (0,0)
```

2.7.25 operator ==

Operator

```
bool cmatrix::operator == (const cmatrix& m) const;
```

compares a calling matrix with a matrix `m` and returns `true` if they have the same sizes and their appropriate elements differ by not more than the smallest normalized positive number. Returns `false` otherwise. The operator is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix`. Example:

```
using namespace cvm;

cmatrix m1(2, 3);
cmatrix m2(2, 3);
m1.set_real(1.);
m2.set_real(1.);
std::cout << (m1 == m2) << std::endl;
```

prints

```
1
```

2.7.26 operator !=

Operator

```
bool cmatrix::operator != (const cmatrix& m) const;
```

compares a calling matrix with a matrix `m` and returns `true` if they have different sizes or at least of their appropriate elements differs by more than the smallest normalized positive number. Returns `false` otherwise. The operator is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix`. Example:

```
using namespace cvm;

cmatrix m1(2, 3);
cmatrix m2(2, 3);
m1.set_real(1.);
m2.set_real(1.);
std::cout << (m1 != m2) << std::endl;
```

prints

```
0
```

2.7.27 operator <<

Operator

```
cmatrix& cmatrix::operator << (const cmatrix& m) throw (cvmexception);
```

destroys a calling matrix, creates a new one as a copy of m and returns a reference to the matrix changed. The operator throws an exception of type cvmexception in case of memory allocation failure. The operator is *redefined* in the classes scmatrix, scbmatrix and schmatrix. See also cmatrix. Example:

```
using namespace cvm;

try {
    cmatrix m(2,3);
    cmatrix mc(1,1);
    m(1,2) = std::complex<double>(1.,2.);
    m(2,3) = std::complex<double>(2.,4.);
    std::cout << m << mc << std::endl;

    mc << m;
    std::cout << mc;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(0,0) (1,2) (0,0)
(0,0) (0,0) (2,4)
(0,0)

(0,0) (1,2) (0,0)
(0,0) (0,0) (2,4)
```

2.7.28 operator +

Operator

```
cmatrix cmatrix::operator + (const cmatrix& m) const
throw (cvmexception);
```

creates an object of type `cmatrix` as the sum of a calling matrix and a matrix `m`. It throws an exception of type `cvmexception` in case of different sizes of the operands. The operator is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cvector::sum`, `cmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6.,
                  7., 8., 9., 10., 11., 12.};
    const cmatrix ma ((std::complex<double>*) a, 2, 3);
    cmatrix mb (2, 3);
    mb.set (std::complex<double>(1.,1.));

    std::cout << ma + mb << std::endl;
    std::cout << ma + ma;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(2,3) (6,7) (10,11)
(4,5) (8,9) (12,13)

(2,4) (10,12) (18,20)
(6,8) (14,16) (22,24)
```

2.7.29 operator -

Operator

```
cmatrix cmatrix::operator - (const cmatrix& m) const
throw (cvmexception);
```

creates an object of type `cmatrix` as the difference of a calling matrix and a matrix `m`. It throws an exception of type `cvmexception` in case of different sizes of the operands. The operator is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cvector::diff`, `cmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6.,
                  7., 8., 9., 10., 11., 12.};
    const cmatrix ma ((std::complex<double>*) a, 2, 3);
    cmatrix mb (2, 3);
    mb.set (std::complex<double>(1.,1.));

    std::cout << ma - mb << std::endl;
    std::cout << ma - ma;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(0,1) (4,5) (8,9)
(2,3) (6,7) (10,11)

(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
```

2.7.30 sum

Function

```
cmatrix& cmatrix::sum (const cmatrix& m1, const cmatrix& m2)
throw (cvmexception);
```

assigns a result of addition of matrices `m1` and `m2` to a calling matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes of the operands. The function is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix::operator +` , `cmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6.,
                  7., 8., 9., 10., 11., 12.};
    const cmatrix ma ((std::complex<double>*) a, 2, 3);
    cmatrix mb (2, 3);
    cmatrix m (2, 3);
    mb.set (std::complex<double>(1.,1.));

    std::cout << m.sum(ma, mb) << std::endl;
    std::cout << m.sum(m, mb);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(2,3) (6,7) (10,11)
(4,5) (8,9) (12,13)

(3,4) (7,8) (11,12)
(5,6) (9,10) (13,14)
```

2.7.31 diff

Function

```
cmatrix& cmatrix::diff (const cmatrix& m1, const cmatrix& m2)
throw (cvmexception);
```

assigns a result of subtraction of matrices m1 and m2 to a calling matrix and returns a reference to the matrix changed. It throws an exception of type cvmexception in case of different sizes of the operands. The function is *redefined* in the classes scmatrix, scbmatrix and schmatrix. See also cmatrix::operator - , cmatrix. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6.,
                  7., 8., 9., 10., 11., 12.};
    const cmatrix ma ((std::complex<double>*) a, 2, 3);
    cmatrix mb (2, 3);
    cmatrix m (2, 3);
    mb.set (std::complex<double>(1.,1.));

    std::cout << m.diff(ma, mb) << std::endl;
    std::cout << m.diff(m, mb);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(0,1) (4,5) (8,9)
(2,3) (6,7) (10,11)

(-1,0) (3,4) (7,8)
(1,2) (5,6) (9,10)
```

2.7.32 operator +=

Operator

```
cmatrix& cmatrix::operator += (const cmatrix& m) throw (cvmexception);
```

adds a matrix `m` to a calling matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes of the operands. The operator is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix::operator +` , `cmatrix::sum`, `cmatrix`. Example:

```
using namespace cvm;

try {
    cmatrix m1(2, 3);
    cmatrix m2(2, 3);
    m1.set(std::complex<double>(1.,1.));
    m2.set(std::complex<double>(2.,2.));

    m1 += m2;
    std::cout << m1 << std::endl;

    // well, you can do this too, but temporary object would be created
    m2 += m2;
    std::cout << m2;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(3,3) (3,3) (3,3)
(3,3) (3,3) (3,3)

(4,4) (4,4) (4,4)
(4,4) (4,4) (4,4)
```

2.7.33 operator -=

Operator

```
cmatrix& cmatrix::operator -= (const cmatrix& m) throw (cvmexception);
```

subtracts a matrix `m` from a calling matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes of the operands. The operator is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix::operator -` , `cmatrix::diff`, `cmatrix`. Example:

```
using namespace cvm;

try {
    cmatrix m1(2, 3);
    cmatrix m2(2, 3);
    m1.set(std::complex<double>(1.,1.));
    m2.set(std::complex<double>(2.,2.));

    m1 -= m2;
    std::cout << m1 << std::endl;

    // well, you can do this too, but temporary object would be created
    m2 -= m2;
    std::cout << m2;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(-1,-1) (-1,-1) (-1,-1)
(-1,-1) (-1,-1) (-1,-1)

(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
```

2.7.34 operator - ()

Operator

```
cmatrix cmatrix::operator - () const throw (cvmexception);
```

creates an object of type `cmatrix` as a calling matrix multiplied by -1. The operator is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6.,
              7., 8., 9., 10., 11., 12.};
const cmatrix ma ((std::complex<double>*) a, 2, 3);

std::cout << - ma;
```

prints

```
(-1,-2) (-5,-6) (-9,-10)
(-3,-4) (-7,-8) (-11,-12)
```

2.7.35 operator * (TR)

Operator

```
cmatrix cmatrix::operator * (TR d) const;
```

creates an object of type cmatrix as a product of a calling matrix and a real number d. The operator is *redefined* in the classes scmatrix, scbmatrix and schmatrix. See also cmatrix::operator *= , cmatrix. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6.,
              7., 8., 9., 10., 11., 12.};
const cmatrix ma ((std::complex<double>*) a, 2, 3);

std::cout << ma * 5.;
```

prints

```
(5,10) (25,30) (45,50)
(15,20) (35,40) (55,60)
```

2.7.36 operator / (TR)

Operator

```
cmatrix cmatrix::operator / (TR d) const throw (cvmexception);
```

creates an object of type `cmatrix` as a quotient of a calling matrix and a real number `d`. It throws an exception of type `cvmexception` if `d` has an absolute value equal or less than the smallest normalized positive number. The operator is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix::operator /=` , `cmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6.,
                  7., 8., 9., 10., 11., 12.};
    const cmatrix ma ((std::complex<double>*) a, 2, 3);

    std::cout << ma / 4.;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(0.25,0.5) (1.25,1.5) (2.25,2.5)
(0.75,1) (1.75,2) (2.75,3)
```

2.7.37 operator * (TC)

Operator

```
cmatrix cmatrix::operator * (TC z) const;
```

creates an object of type `cmatrix` as a product of a calling matrix and a complex number `z`. The operator is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix::operator *=` , `cmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6.,
              7., 8., 9., 10., 11., 12.};
const cmatrix ma ((std::complex<double>*) a, 2, 3);

std::cout << ma * std::complex<double>(5.,2.);
```

prints

```
(1,12) (13,40) (25,68)
(7,26) (19,54) (31,82)
```

2.7.38 operator / (TC)

Operator

```
cmatrix cmatrix::operator / (TC z) const throw (cvmexception);
```

creates an object of type `cmatrix` as a quotient of a calling matrix and a complex number `z`. It throws an exception of type `cvmexception` if `z` has an absolute value equal or less than the smallest normalized positive number. The operator is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix::operator /=` , `cmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6.,
                  7., 8., 9., 10., 11., 12.};
    const cmatrix ma ((std::complex<double>*) a, 2, 3);

    std::cout << ma / std::complex<double>(4.,2.);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(0.4,0.3) (1.6,0.7) (2.8,1.1)
(1,0.5) (2.2,0.9) (3.4,1.3)
```

2.7.39 operator *= (TR)

Operator

```
cmatrix& cmatrix::operator *= (TR d);
```

multiplies a calling matrix by a real number d and returns a reference to the matrix changed. The operator is *redefined* in the classes scmatrix, scbmatrix and schmatrix. See also cmatrix::operator * , cmatrix. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6.,
              7., 8., 9., 10., 11., 12.};
cmatrix ma ((std::complex<double>*) a, 2, 3);

ma *= 2.;
std::cout << ma;
```

prints

```
(2,4) (10,12) (18,20)
(6,8) (14,16) (22,24)
```

2.7.40 operator /= (TR)

Operator

```
cmatrix& cmatrix::operator /= (TR d) throw (cvmexception);
```

divides a calling matrix by a real number `d` and returns a reference to the matrix changed. It throws an exception of type `cvmexception` if `d` has an absolute value equal or less than the smallest normalized positive number. The operator is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix::operator /` , `cmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6.,
                  7., 8., 9., 10., 11., 12.};
    cmatrix ma ((std::complex<double>*) a, 2, 3);

    ma /= 2.;
    std::cout << ma;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(0.5,1) (2.5,3) (4.5,5)
(1.5,2) (3.5,4) (5.5,6)
```

2.7.41 operator *= (TC)

Operator

```
cmatrix& cmatrix::operator *= (TC z);
```

multiplies a calling matrix by a complex number `z` and returns a reference to the matrix changed. The operator is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix::operator *` , `cmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6.,
              7., 8., 9., 10., 11., 12.};
cmatrix ma ((std::complex<double>*) a, 2, 3);

ma *= std::complex<double>(2.,1.);
std::cout << ma;
```

prints

```
(0,5) (4,17) (8,29)
(2,11) (6,23) (10,35)
```

2.7.42 operator /= (TC)

Operator

`cmatrix& cmatrix::operator /= (TC z) throw (cvmexception);`

divides a calling matrix by a complex number z and returns a reference to the matrix changed. It throws an exception of type cvmexception if z has an absolute value equal or less than the smallest normalized positive number. The operator is *redefined* in the classes scmatrix, scbmatrix and schmatrix. See also cmatrix::operator / , cmatrix. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6.,
                  7., 8., 9., 10., 11., 12.};
    cmatrix ma ((std::complex<double>*) a, 2, 3);

    ma /= std::complex<double>(2.,1.);
    std::cout << ma;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(0.8,0.6) (3.2,1.4) (5.6,2.2)
(2,1) (4.4,1.8) (6.8,2.6)
```

2.7.43 normalize

Function

```
cmatrix& cmatrix::normalize ();
```

normalizes a calling matrix so its Euclidean norm becomes equal to 1 if it was greater than the smallest normalized positive number before the call (otherwise the function does nothing). The function is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6.,
              7., 8., 9., 10., 11., 12.};
cmatrix ma ((std::complex<double>*) a, 2, 3);

ma.normalize();
std::cout << ma << ma.norm() << std::endl;
```

prints

```
(0.0392232,0.0784465) (0.196116,0.235339) (0.353009,0.392232)
(0.11767,0.156893) (0.274563,0.313786) (0.431455,0.470679)
1
```

2.7.44 conjugation

Operator and functions

```
cmatrix cmatrix::operator ~ () const throw (cvmexception);
cmatrix& cmatrix::conj (const cmatrix& m) throw (cvmexception);
cmatrix& cmatrix::conj () throw (cvmexception);
```

encapsulate complex matrix conjugation. First operator creates an object of type `cmatrix` as a conjugated calling matrix (it throws an exception of type `cvmexception` in case of memory allocation failure). Second function sets a calling matrix to be equal to a matrix `m` conjugated (it throws an exception of type `cvmexception` in case of not appropriate sizes of the operands), third one makes it to be equal to conjugated itself (it also throws an exception of type `cvmexception` in case of memory allocation failure). The functions are *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6.,
              7., 8., 9., 10., 11., 12.};
cmatrix m((std::complex<double>*) a, 2, 3);
cmatrix mc(3,2);
std::cout << m << std::endl << ~m << std::endl ;
mc.conj(m);
std::cout << mc << std::endl;
mc.conj();
std::cout << mc;
```

prints

```
(1,2) (5,6) (9,10)
(3,4) (7,8) (11,12)

(1,-2) (3,-4)
(5,-6) (7,-8)
(9,-10) (11,-12)

(1,-2) (3,-4)
(5,-6) (7,-8)
(9,-10) (11,-12)

(1,2) (5,6) (9,10)
(3,4) (7,8) (11,12)
```

2.7.45 operator * (const cvector&)

Operator

```
cvector cmatrix::operator * (const cvector& v) const
throw (cvmexception);
```

creates an object of type cvector as a product of a calling matrix and a vector v. It throws an exception of type cvmexception if the number of columns of the calling matrix differs from the size of the vector v. Use cvector::mult in order to get rid of a new object creation. The function is *redefined* in the classes scmatrix, scbmatrix and schmatrix. See also cmatrix, cvector. Example:

```
using namespace cvm;

cmatrix m(2, 3);
cvector v(3);
m.set(std::complex<double>(1.,1.));
v.set(std::complex<double>(1.,1.));

std::cout << m * v;
```

prints

```
(0,6) (0,6)
```

2.7.46 operator * (const cmatrix&)

Operator

```
cmatrix cmatrix::operator * (const cmatrix& m) const
throw (cvmexception);
```

creates an object of type `cmatrix` as a product of a calling matrix and a matrix `m`. It throws an exception of type `cvmexception` if the number of columns of the calling matrix differs from the number of rows of the matrix `m`. Use `cmatrix::mult` in order to get rid of a new object creation. The operator is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix`. Example:

```
using namespace cvm;

cmatrix m1(2, 3);
cmatrix m2(3, 2);
m1.set(std::complex<double>(1.,1.));
m2.set(std::complex<double>(1.,1.));

std::cout << m1 * m2;
```

prints

```
(0,6) (0,6)
(0,6) (0,6)
```

2.7.47 mult

Function

```
cmatrix& cmatrix::mult (const cmatrix& m1, const cmatrix& m2)
throw (cvmexception);
```

sets a calling matrix to be equal to a product of a matrix `m1` by a matrix `m2` and returns a reference to the matrix changed. The function throws an exception of type `cvmexception` in case of inappropriate sizes of the operands. The function is is *inherited* in the class `scmatrix` and *redefined* in the classes `scbmatrix` and `schmatrix`. See also `cmatrix`. Example:

```
using namespace cvm;

cmatrix m1(2, 3);
cmatrix m2(3, 2);
scmatrix m(2);
m1.set(std::complex<double>(1.,1.));
m2.set(std::complex<double>(1.,1.));

m.mult(m1, m2);
std::cout << m;
```

prints

```
(0,6) (0,6)
(0,6) (0,6)
```

2.7.48 rank1update_u

Function

```
cmatrix&
cmatrix::rank1update_u (const cvector& vCol, const cvector& vRow)
throw (cvmexception);
```

sets a calling matrix to be equal to the rank-1 update (uncojugated) of vectors `vCol` and `vRow` and returns a reference to the matrix changed. The function throws an exception of type `cvmexception` if the number of rows of the calling matrix is not equal to `vCol.size()` or the number of columns of the calling matrix is not equal to `vRow.size()`. The function is *inherited* in the the class `scmatrix` and *not applicable* to objects of the classes `scbmatrix` and `schmatrix` (i.e. an exception of type `cvmexception` would be thrown in case of using it for objects of those classes). See also cvector::rank1update_u, `cmatrix`. Example:

```
using namespace cvm;

cvector vc(3), vr(2);
cmatrix m(3, 2);
vc.set(std::complex<double>(1.,1.));
vr.set(std::complex<double>(1.,1.));

std::cout << m.rank1update_u (vc, vr);
```

prints

```
(0,2) (0,2)
(0,2) (0,2)
(0,2) (0,2)
```

2.7.49 rank1update_c

Function

```
cmatrix&
cmatrix::rank1update_c (const cvector& vCol, const cvector& vRow)
throw (cvmexception);
```

sets a calling matrix to be equal to the rank-1 update (conjugated) of vectors `vCol` and `vRow` and returns a reference to the matrix changed. The function throws an exception of type `cvmexception` if the number of rows of the calling matrix is not equal to `vCol.size()` or the number of columns of the calling matrix is not equal to `vRow.size()`. The function is *inherited* in the the class `scmatrix` and *not applicable* to objects of the classes `scbmatrix` and `schmatrix` (i.e. an exception of type `cvmexception` would be thrown in case of using it for objects of those classes). See also cvector::rank1update_c, `cmatrix`. Example:

```
using namespace cvm;

cvector vc(3), vr(2);
cmatrix m(3, 2);
vc.set(std::complex<double>(1.,1.));
vr.set(std::complex<double>(1.,1.));

std::cout << m.rank1update_c (vc, vr);
```

prints

```
(2,0) (2,0)
(2,0) (2,0)
(2,0) (2,0)
```

2.7.50 swap_rows

Function

`cmatrix& cmatrix::swap_rows (int n1, int n2) throw (cvmexception);`

swaps two rows of a calling matrix and returns a reference to the matrix changed. `n1` and `n2` are the numbers of rows to be swapped, both are 1-based). The function throws an exception of type `cvmexception` if one of the parameters is outside of the range `[1,msize()]`. The function is *redefined* in the the class `scmatrix` and *not applicable* to objects of the classes `scbmatrix` and `schmatrix` (i.e. an exception of type `cvmexception` would be thrown in case of using it for objects of those classes). See also `cmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6.,
              7., 8., 9., 10., 11., 12.};
cmatrix m ((std::complex<double>*)a, 3, 2);

std::cout << m << std::endl;
std::cout << m.swap_rows(2,3);
```

prints

```
(1,2) (7,8)
(3,4) (9,10)
(5,6) (11,12)

(1,2) (7,8)
(5,6) (11,12)
(3,4) (9,10)
```

2.7.51 swap_cols

Function

```
cmatrix& cmatrix::swap_cols (int n1, int n2) throw (cvmexception);
```

swaps two columns of a calling matrix and returns a reference to the matrix changed. `n1` and `n2` are the numbers of columns to be swapped, both are 1-based). The function throws an exception of type `cvmexception` if one of the parameters is outside of the range `[1,nsize()]`. The function is *redefined* in the the class `scmatrix` and *not applicable* to objects of the classes `scbmatrix` and `schmatrix` (i.e. an exception of type `cvmexception` would be thrown in case of using it for objects of those classes). See also `cmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6.,
              7., 8., 9., 10., 11., 12.};
cmatrix m ((std::complex<double>*)a, 2, 3);

std::cout << m << std::endl;
std::cout << m.swap_cols(2,3);
```

prints

```
(1,2) (5,6) (9,10)
(3,4) (7,8) (11,12)

(1,2) (9,10) (5,6)
(3,4) (11,12) (7,8)
```

2.7.52 solve

Functions

```
cmatrix&
cmatrix::solve (const scmatrix& mA,
                const cmatrix& mB, TR& dErr) throw (cvmexception);
cmatrix&
cmatrix::solve (const scmatrix& mA,
                const cmatrix& vB) throw (cvmexception);
```

set a calling matrix to be equal to a solution X of the matrix linear equation $AX = B$ where the parameter mA is the square matrix A and the parameter vB is the matrix B. Every function returns a reference to the matrix changed. The first version also sets the output parameter dErr to be equal to the norm of computation error. The functions throw exception of type cvmexception in case of inappropriate sizes of the operands or when the matrix A is close to cingular. The functions are *redefined* in the the class scmatrix and *inherited* thereafter in the classes scbmatrix and schmatrix. See also cvector::solve, cmatrix, scmatrix. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (4);

scmatrix ma(3);
cmatrix  mb(3,2);
cmatrix  mx(3,2);
double dErr;
ma.randomize_real(0.,10.); ma.randomize_imag(0.,10.);
mb.randomize_real(0.,10.); mb.randomize_imag(0.,10.);

mx.solve (ma, mb, dErr);
std::cout << mx << std::endl << ma * mx - mb;
```

prints

```
(-4.7267e-01,-1.2929e+00) (1.5912e+00,-2.1678e-01)
(1.3961e+00,6.3385e-01) (-9.2872e-01,1.6793e-01)
(7.0048e-01,8.1763e-01) (-1.7369e-01,1.1588e-01)

(0.0000e+00,-8.8818e-16) (1.7764e-15,8.8818e-16)
(-4.4409e-16,-8.8818e-16) (0.0000e+00,-8.8818e-16)
(0.0000e+00,-1.7764e-15) (8.8818e-16,4.4409e-16)
```

2.7.53 solve_lu

Functions

```
cmatrix&
cmatrix::solve_lu (const scmatrix& mA, const scmatrix& mLU,
                   const int* pPivots, const cmatrix& mB, TR& dErr)
                   throw (cvmexception);
cmatrix&
cmatrix::solve_lu (const scmatrix& mA, const scmatrix& mLU,
                   const int* pPivots, const cmatrix& vB)
                   throw (cvmexception);
```

set a calling matrix to be equal to a solution X of the matrix linear equation $AX = B$ where the parameter `mA` is the square complex matrix A, parameter `mLU` is LU factorization of the matrix A, parameter `pPivots` is an array of pivot numbers created while factorizing the matrix A and the parameter `vB` is the matrix B. Every function returns a reference to the matrix changed. The first version also sets the output parameter `dErr` to be equal to a norm of computation error. These functions are useful when you need to solve few linear equations of kind $AX = B$ with the same matrix A and different matrices B. In such case you save on matrix A factorization since it's needed to be performed just one time. The functions throw exception of type `cvmexception` in case of inappropriate sizes of the operands or when the matrix A is close to cingular. See also `cvector::solve`, `cmatrix`, `scmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (4);
try {
    scmatrix ma(3);
    scmatrix mLU(3);
    cmatrix  mb1(3,2);
    cmatrix  mb2(3,2);
    cmatrix  mx1(3,2);
    cmatrix  mx2(3,2);
    iarray   nPivots(3);
    double   dErr = 0.;
    ma.randomize_real(0.,10.); ma.randomize_imag(0.,10.);
    mb1.randomize_real(0.,10.); mb1.randomize_imag(0.,10.);
    mb2.randomize_real(0.,10.); mb2.randomize_imag(0.,10.);

    mLU.low_up(ma, nPivots);
    std::cout << mx1.solve_lu (ma, mLU, nPivots, mb1, dErr);
```

```
    std::cout << dErr << std::endl;
    std::cout << mx2.solve_lu (ma, mLU, nPivots, mb2) << std::endl;
    std::cout << ma * mx1 - mb1 << std::endl << ma * mx2 - mb2;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(4.2888e-01,8.2409e-02) (-1.1261e-01,-5.7778e-01)
(5.8052e-01,3.2179e-01) (2.5811e-01,-3.8609e-02)
(-3.1499e-02,-7.0014e-01) (1.2652e+00,4.5309e-01)
5.2931e-15
(3.0153e-01,-5.6606e-01) (-1.6308e-01,1.8217e-01)
(7.4971e-01,-1.1305e-01) (5.2187e-01,2.3441e-01)
(-1.9916e-01,1.4493e+00) (9.1046e-02,3.5242e-01)

(0.0000e+00,-8.8818e-16) (0.0000e+00,-8.8818e-16)
(-4.4409e-16,0.0000e+00) (0.0000e+00,-8.8818e-16)
(0.0000e+00,0.0000e+00) (0.0000e+00,0.0000e+00)

(-8.8818e-16,8.8818e-16) (-8.8818e-16,-1.7764e-15)
(0.0000e+00,0.0000e+00) (2.2204e-16,-8.8818e-16)
(4.4409e-16,-8.8818e-16) (1.3878e-17,-4.4409e-16)
```

2.7.54 svd

Functions

```
rvector
cmatrix::svd () throw (cvmexception);
rvector
cmatrix::svd (scmatrix& mU, scmatrix& mVH) throw (cvmexception);
```

create an object of type `rvector` as a vector of singular values of a calling matrix. The second version of the function set the output parameter `mU` to be equal to the matrix U of size $m \times m$ (and change the size of the object if it's needed) and `mVH` to be equal to the matrix V^H of size $n \times n$. All the functions throw an exception of type `cvmexception` in case of inappropriate sizes of the operands or in case of convergence error. Use `rvector::svd` in order to get rid of a new vector creation. The function is *redefined* in the the classes `scmatrix`, `scbmatrix`, `schmatrix`. See also `rvector`, `cmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (3);
try {
    double m[] = {1., -1., 1., 2., -2., 1.,
                  3., -2., 1., 0., -2., 1.};
    cmatrix mA((std::complex<double>*) m, 2, 3);
    cmatrix mSigma(2,3);
    rvector v(2);
    scmatrix mU(2), mVH(3);

    v = mA.svd(mU, mVH);
    mSigma.diag(0) = cvector(v);

    std::cout << mU << std::endl;
    std::cout << mVH << std::endl;
    std::cout << mSigma << std::endl;

    std::cout << (mA * ~mVH - mU * mSigma).norm() << std::endl;
    std::cout << (~mA * mU - ~(mSigma * mVH)).norm() << std::endl;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(-4.861e-01,0.000e+00) (8.739e-01,0.000e+00)
(7.956e-01,-3.616e-01) (4.425e-01,-2.012e-01)

(-7.590e-02,4.474e-01) (7.488e-01,-1.820e-01) (-4.474e-01,1.327e-02)
(8.084e-01,1.878e-01) (-1.576e-02,5.238e-01) (-1.878e-01,3.558e-02)
(1.065e-01,3.065e-01) (3.597e-01,4.669e-02) (8.727e-01,4.012e-02)

(5.452e+00,0.000e+00) (0.000e+00,0.000e+00) (0.000e+00,0.000e+00)
(0.000e+00,0.000e+00) (1.131e+00,0.000e+00) (0.000e+00,0.000e+00)

1.357e-15
1.267e-15
```

2.7.55 pinv

Functions

```
cmatrix  cmatrix::pinv (TR threshold = cvmMachSp()) const
                        throw (cvmexception);
cmatrix& cmatrix::pinv (const basic_rmatrix& mA,
                        TR threshold = cvmMachSp())
                        throw (cvmexception);
```

encapsulate complex matrix pseudo inversion [6], p. 33 (or Moore-Penrose generalized inversion [5], p. 421). Strictly defined, an $n \times m$ matrix A^+ is a *pseudo inversion* of $m \times n$ matrix A if the following two equations are satisfied:

$$AA^+A = A,$$
$$A^+ = QA^H = A^H P$$

where Q and P are some matrices. To compute the pseudo inversion, we use Singular Value Decomposition (SVD)

$$A = U\Sigma V^H$$

of matrix A, thus

$$A^+ = V\Sigma^{-1}U^H,$$

where Σ^{-1} is a diagonal $n \times m$ matrix having inverted diagonal values of the matrix Σ if they are greater than some threshold, and zeros otherwise.

First version creates an object of type `cmatrix` as a pseudo inverted calling matrix (it throws an exception of type `cvmexception` in case of memory allocation failure). Second function sets a calling matrix to be equal to a matrix `mA` pseudo inverted (it throws an exception of type `cvmexception` in case of not appropriate sizes of the operands). The threshold parameter sets a minimum distinguishable from zero singular value to be used to compute the pseudo inversion. All values equal or less than the threshold are treated as zeros. The functions are *inherited* in the classes `scmatrix` and `schmatrix` and *redefined* in `scbmatrix`. See also `cmatrix`. Example:

```
using namespace cvm;

try {
    cmatrix mA(2,3);
    mA(1,1) =  tcomplex(1.,2.);  mA(1,2) = tcomplex(-1.,3.);
    mA(1,3) = tcomplex(0.,-1.);
    mA(2,1) =  tcomplex(1.,-2.); mA(2,2) = tcomplex(1.,-4.);
    mA(2,3) = tcomplex(1.,0.);

    cmatrix mX = mA.pinv();
    std::cout << mX << (mA * mX * mA - mA).norm2() << std::endl;
```

```
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(+3.2407407e-01,-1.1111111e-01) (+2.5925926e-01,-1.8518519e-02)
(-1.3888889e-01,-2.7777778e-02) (-5.5555556e-02,+1.6666667e-01)
(+1.4814815e-01,+2.3148148e-01) (+1.6666667e-01,+1.4814815e-01)
+3.3217718e-15
```

Band matrix example:

```
using namespace cvm;

try {
    scbmatrix mA (40, 1, 0);
    mA.diag(0).randomize_real(-1.,1.);
    mA.diag(0).randomize_imag(-3.,2.);
    mA.diag(-1).randomize_real(5.,10.);
    mA.diag(-1).randomize_imag(-3.,7.);

    scmatrix mX (40);
    mX.pinv(mA);
    std::cout << (mA * mX * mA - mA).norm2() << std::endl;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
+6.3113999e-14
```

2.7.56 rank

Function

```
int cmatrix::rank (TR eps = cvmMachSp ()) const throw (cvmexception);
```

returns a rank of a calling matrix as a number of singular values with normalized absolute value greater than or equal to a parameter `eps` (this is the largest relative spacing by default). The function throws an exception of type `cvmexception` in case of convergence error. The function is *inherited* in the the classes `scmatrix`, `scbmatrix`, `schmatrix`. See also `cmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6.,
              7., 8., 9., 10., 11., 12.};
cmatrix m(a, NULL, 3, 4);

std::cout << m << m.rank() << std::endl;
m(3,4) = std::complex<double>(0.,1.);
std::cout << m.rank() << std::endl;
```

prints

```
(1,0) (4,0) (7,0) (10,0)
(2,0) (5,0) (8,0) (11,0)
(3,0) (6,0) (9,0) (12,0)
2
3
```

2.7.57 qr

Functions

```
void cmatrix::qr (cmatrix& mQ, scmatrix& mR) const throw (cvmexception);
void cmatrix::qr (scmatrix& mQ, cmatrix& mR) const throw (cvmexception);
```

compute QR factorization as

$$M = QR$$

where M is a calling matrix, unitary matrix Q and upper triangular matrix R are `mQ` and `mR` respectively. First version encapsulates so-called "economy" algorithm which for a given $m \times n$ matrix M computes $m \times n$ matrix Q and $n \times n$ matrix R. Second version is a "full" mode one computing $m \times m$ matrix Q and $m \times n$ matrix R. The functions throw an exception of type `cvmexception` in case of inappropriate sizes of the operands. The functions are *redefined* in the the class `scmatrix`. See also `cmatrix`, `scmatrix`. Example:

```
using namespace cvm;

treal ar[] = {1., 2., 3., 4., 5., 6.};
treal ai[] = {1., -1., 2., -2., 3., -3.};
const cvm::cmatrix mh(ar, ai, 2, 3);
const cvm::cmatrix mv(ar, ai, 3, 2);
cvm::scmatrix s2(2), s3(3);
cvm::cmatrix  h(2,3), v(3,2);

mh.qr(h,s3);
std::cout << (eye_complex(2)-~cmatrix(h,1,1,2,2)*cmatrix(h,1,1,2,2)).norm()
          << " " << (mh - h * s3).norm() << std::endl;
mh.qr(s2,h);
std::cout << (eye_complex(2) - ~s2 * s2).norm()
          << " " << (mh - s2 * h).norm() << std::endl;
mv.qr(v,s2);
std::cout << (eye_complex(2) - ~v * v).norm()
          << " " << (mv - v * s2).norm() << std::endl;
mv.qr(s3,v);
std::cout << (eye_complex(3) - ~s3 * s3).norm()
          << " " << (mv - s3 * v).norm() << std::endl;
```

prints

```
+2.5145832e-16 +1.3506446e-15
+2.5145832e-16 +1.3506446e-15
+2.5367068e-16 +1.4432899e-15
+3.8435519e-16 +1.4432899e-15
```

2.7.58 vanish

Function

```
cmatrix& cmatrix::vanish();
```

sets every element of a calling matrix to be equal to zero and returns a reference to the matrix changed. This function is faster than, for example, `cmatrix::set(TC)` with zero operand passed. The function is *redefined* in the classes `scmatrix`, `scbmatrix`, `schmatrix`. See also `cmatrix`. Example:

```
using namespace cvm;

cmatrix m(4, 3);
m.randomize_real(0.,1.);
m.randomize_imag(1.,2.);

std::cout << m << std::endl;
std::cout << m.vanish ();
```

prints

```
(0.851527,1.16376) (0.557512,1.90188) (0.0343638,1.52068)
(0.478042,1.29106) (0.561724,1.19764) (0.320994,1.35804)
(0.264534,1.40986) (0.113468,1.75137) (0.37727,1.54994)
(0.521409,1.83035) (0.559465,1.35072) (0.809198,1.12537)

(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
```

2.7.59 geru

Function

```
cmatrix&
cmatrix::geru (TC dAlpha, const cvector& vCol, const cvector& vRow)
throw (cvmexception);
```

calls one of ?GERU routines of the BLAS library performing a rank-1 update (unconjugated) matrix-vector operation defined as $M = \alpha x \cdot y + M$, where α is a complex number (parameter dAlpha), M is a calling matrix and x and y are complex vectors (parameters vCol and vRow respectively). The function returns a reference to the matrix changed and throws an exception of type cvmexception in case of inappropriate sizes of the operands. The function is *inherited* in the the class scmatrix and *not applicable* to objects of the classes scbmatrix and schmatrix (i.e. an exception of type cvmexception would be thrown in case of using it for objects of those classes). See also cvector, cmatrix. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (5);

std::complex<double> alpha = std::complex<double>(1.2,4.11);
cmatrix m(3,2);
cvector vc(3);
cvector vr(2);
m.randomize_real(-1., 2.); m.randomize_imag(-3., 2.);
vc.randomize_real(-1., 3.); vc.randomize_imag(1., 3.);
vr.randomize_real(0., 2.); vr.randomize_imag(-1., 2.);

std::cout << m + vc.rank1update_u (vr) * alpha << std::endl;
std::cout << m.geru(alpha, vc, vr);
```

prints

```
(2.88144e+00,3.54299e+00) (-8.14760e+00,-1.03789e+00)
(6.33361e-01,3.35209e+00) (-4.81787e+00,-8.53964e+00)
(5.44811e-01,1.37156e+00) (-5.97006e+00,-5.00794e+00)

(2.88144e+00,3.54299e+00) (-8.14760e+00,-1.03789e+00)
(6.33361e-01,3.35209e+00) (-4.81787e+00,-8.53964e+00)
(5.44811e-01,1.37156e+00) (-5.97006e+00,-5.00794e+00)
```

2.7.60 gerc

Function

```
cmatrix&
cmatrix::gerc (TC dAlpha, const cvector& vCol, const cvector& vRow)
throw (cvmexception);
```

calls one of ?GERC routines of the BLAS library performing a rank-1 update (conjugated) matrix-vector operation defined as $M = \alpha x \cdot y' + M$, where α is a complex number (parameter dAlpha), M is a calling matrix and x and y are complex vectors (parameters vCol and vRow respectively). The function returns a reference to the matrix changed and throws an exception of type cvmexception in case of inappropriate sizes of the operands. The function is *inherited* in the the class scmatrix and *not applicable* to objects of the classes scbmatrix and schmatrix (i.e. an exception of type cvmexception would be thrown in case of using it for objects of those classes). See also cvector, cmatrix. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (5);

std::complex<double> alpha = std::complex<double>(1.2,4.11);
cmatrix m(3,2);
cvector vc(3);
cvector vr(2);
m.randomize_real(-1., 2.); m.randomize_imag(-3., 2.);
vc.randomize_real(-1., 3.); vc.randomize_imag(1., 3.);
vr.randomize_real(0., 2.); vr.randomize_imag(-1., 2.);

std::cout << m + vc.rank1update_c (vr) * alpha << std::endl;
std::cout << m.gerc(alpha, vc, vr);
```

prints

```
(1.27138e+01,1.58049e+01) (1.00616e+01,2.21197e+01)
(1.93326e+01,1.41763e+01) (1.74769e+01,2.49013e+01)
(8.09961e+00,1.36259e+01) (5.86738e+00,1.97800e+01)

(1.27138e+01,1.58049e+01) (1.00616e+01,2.21197e+01)
(1.93326e+01,1.41763e+01) (1.74769e+01,2.49013e+01)
(8.09961e+00,1.36259e+01) (5.86738e+00,1.97800e+01)
```

2.7.61 gemm

Function

```
cmatrix& cmatrix::gemm (const cmatrix& m1, bool bTrans1,
                        const cmatrix& m2, bool bTrans2,
                        TC dAlpha, TC dBeta) throw (cvmexception);
```

calls one of ?GEMM routines of the BLAS library performing a matrix-matrix operation defined as

$$M = \alpha\,\mathcal{C}(M_1)\cdot\mathcal{C}(M_2) + \beta M,$$

where α and β are complex numbers (parameters dAlpha and dBeta), M is a calling matrix and M_1 and M_2 are matrices (parameters m1 and m2 respectively). Function $\mathcal{C}(M_i)$ congugates matrix M_i if appropriate boolean parameter bTrans* is equal to true and does nothing otherwise. The function returns a reference to the matrix changed and throws an exception of type cvmexception in case of inappropriate sizes of the operands. The function is *inherited* in the the class scmatrix and *not applicable* to objects of the classes scbmatrix and schmatrix (i.e. an exception of type cvmexception would be thrown in case of using it for objects of those classes). See also cmatrix. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (3);

try {
    std::complex<double> alpha = std::complex<double>(1.1,2.1);
    std::complex<double> beta = std::complex<double>(0.71,0.12);
    cmatrix m1(4,3); cmatrix m2(4,3);
    cmatrix m(3,3);
    m.randomize_real(-1., 2.); m.randomize_imag(1., 3.);
    m1.randomize_real(-1., 3.); m1.randomize_imag(-2., 4.);
    m2.randomize_real(0., 2.); m2.randomize_imag(-3., 2.);

    std::cout << ~m1 * m2 * alpha + m * beta << std::endl;
    std::cout << m.gemm(m1, true, m2, false, alpha, beta);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(3.225e+01,3.611e+01) (2.042e+01,1.206e+01) (5.065e+01,-2.261e+01)
(3.009e+01,3.665e+00) (2.167e+01,-3.327e+00) (4.305e+01,-1.960e+01)
(1.156e+01,-4.966e+00) (4.067e+00,-1.181e+01) (1.121e+01,-2.684e+01)
```

```
(3.225e+01,3.611e+01) (2.042e+01,1.206e+01) (5.065e+01,-2.261e+01)
(3.009e+01,3.665e+00) (2.167e+01,-3.327e+00) (4.305e+01,-1.960e+01)
(1.156e+01,-4.966e+00) (4.067e+00,-1.181e+01) (1.121e+01,-2.684e+01)
```

2.7.62 hemm

Function

```
cmatrix& cmatrix::hemm (bool bLeft, const schmatrix& ms,
                        const cmatrix& m, TC dAlpha, TC dBeta)
                        throw (cvmexception);
```

calls one of ?HEMM routines of the BLAS library performing one of matrix-matrix operations defined as

$$M = \alpha M_h \cdot M_1 + \beta M \quad \text{or} \quad M = \alpha M_1 \cdot M_h + \beta M,$$

where α and β are complex numbers (parameters dAlpha and dBeta), M is a calling matrix, M_h is a hermitian matrix and M_1 is a complex matrix (parameters ms and m respectively). First operation is performed if bLeft passed is true and second one otherwise. The function returns a reference to the matrix changed and throws an exception of type cvmexception in case of inappropriate sizes of the operands. The function is *inherited* in the the classes scmatrix and schmatrix and *not applicable* to objects of the class scbmatrix (i.e. an exception of type cvmexception would be thrown in case of using it for objects of that class). See also schmatrix, cmatrix. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (3);

try {
    std::complex<double> alpha = std::complex<double>(1.3,0.21);
    std::complex<double> beta = std::complex<double>(0.5,-0.1);
    cmatrix m1(2,3);
    cmatrix m2(3,2);
    schmatrix ms(2);
    cmatrix m(2,3);
    m.randomize_real(-1., 2.); m.randomize_imag(1., 3.);
    m1.randomize_real(-1., 3.); m1.randomize_imag(1., 2.);
    m2.randomize_real(0., 2.); m2.randomize_imag(-3., -1.);
    ms.randomize_real(-3., 1.); ms.randomize_imag(-1.3, 4.);

    std::cout << ms * m1 * alpha + m * beta << std::endl;
    std::cout << m.hemm (true, ms, m1, alpha, beta) << std::endl;

    m.resize(3,2);
    m.randomize_real(-1.4, 1.3); m.randomize_imag(1.1, 3.);
    std::cout << m2 * ms * alpha + m * beta << std::endl;
    std::cout << m.hemm (false, ms, m2, alpha, beta);
}
```

```
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(1.096e+00,-7.692e+00) (-7.923e+00,-3.909e+00) (-1.324e+01,-5.264e+00)
(2.415e+00,1.240e+00) (4.384e-01,-1.771e+00) (7.495e-01,-2.740e+00)

(1.096e+00,-7.692e+00) (-7.923e+00,-3.909e+00) (-1.324e+01,-5.264e+00)
(2.415e+00,1.240e+00) (4.384e-01,-1.771e+00) (7.495e-01,-2.740e+00)

(-5.007e+00,1.010e+01) (2.341e+00,3.248e+00)
(-8.753e+00,7.854e+00) (3.152e+00,4.491e+00)
(-9.162e+00,6.401e+00) (-1.168e+00,3.973e+00)

(-5.007e+00,1.010e+01) (2.341e+00,3.248e+00)
(-8.753e+00,7.854e+00) (3.152e+00,4.491e+00)
(-9.162e+00,6.401e+00) (-1.168e+00,3.973e+00)
```

2.7.63 randomize_real

Function

```
cmatrix& cmatrix::randomize_real (TR dFrom, TR dTo);
```

fills a real part of a calling matrix with pseudo-random numbers distributed between dFrom and dTo. The function returns a reference to the matrix changed. The function is *redefined* in the classes scmatrix, scbmatrix and schmatrix. See also cmatrix. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (3);

cmatrix m(2,3);
m.randomize_real(-1., 2.);
std::cout << m;
```

prints

```
(1.090e+00,0.000e+00) (-6.375e-01,0.000e+00) (1.248e+00,0.000e+00)
(-1.272e-01,0.000e+00) (-8.557e-01,0.000e+00) (4.848e-01,0.000e+00)
```

2.7.64 randomize_imag

Function

```
cmatrix& cmatrix::randomize_imag (TR dFrom, TR dTo);
```

fills an imaginary part of a calling matrix with pseudo-random numbers distributed between dFrom and dTo. The function returns a reference to the matrix changed. The function is *redefined* in the classes scmatrix, scbmatrix and schmatrix. See also cmatrix. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (3);

cmatrix m(2,3);
m.randomize_imag(-1., 2.);
std::cout << m;
```

prints

```
(0.000e+00,1.113e+00) (0.000e+00,6.615e-01) (0.000e+00,1.017e+00)
(0.000e+00,-3.397e-01) (0.000e+00,1.577e+00) (0.000e+00,8.071e-01)
```

2.8 srmatrix

This is end-user class encapsulating a square matrix in Euclidean space of real numbers.

```
template <typename TR>
class srmatrix : public rmatrix <TR>, public SqMatrix <TR,TR> {
public:
    srmatrix ();
    explicit srmatrix (int nMN);
    srmatrix (TR* pD, int nMN);
    srmatrix (const srmatrix& m);
    srmatrix (const rmatrix& m);
    explicit srmatrix (const rvector& v);
    srmatrix (rmatrix& m, int nRow, int nCol, int nSize);
    TR& operator () (int im, int in) throw (cvmexception);
    TR operator () (int im, int in) const throw (cvmexception);
    rvector operator () (int i) throw (cvmexception);
    const rvector operator () (int i) const throw (cvmexception);
    rvector operator [] (int i) throw (cvmexception);
    const rvector operator [] (int i) const throw (cvmexception);
    srmatrix& operator = (const srmatrix& m) throw (cvmexception);
    srmatrix& assign (const rvector& v);
    srmatrix& assign (const TR* pD);
    srmatrix& assign (int nRow, int nCol, const rmatrix& m)
                      throw (cvmexception);
    srmatrix& set (TR x);
    srmatrix& resize (int nNewMN) throw (cvmexception);
    srmatrix& operator << (const srmatrix& m) throw (cvmexception);
    srmatrix operator + (const srmatrix& m) const
                         throw (cvmexception);
    srmatrix operator - (const srmatrix& m) const
                         throw (cvmexception);
    srmatrix& sum (const srmatrix& m1,
                   const srmatrix& m2) throw (cvmexception);
    srmatrix& diff (const srmatrix& m1,
                    const srmatrix& m2) throw (cvmexception);
    srmatrix& operator += (const srmatrix& m) throw (cvmexception);
    srmatrix& operator -= (const srmatrix& m) throw (cvmexception);
    srmatrix operator - () const;
    srmatrix& operator ++ ();
    srmatrix& operator ++ (int);
    srmatrix& operator -- ();
    srmatrix& operator -- (int);
```

```
srmatrix operator * (TR d) const;
srmatrix operator / (TR d) const throw (cvmexception);
srmatrix& operator *= (TR d);
srmatrix& operator /= (TR d) throw (cvmexception);
srmatrix& normalize ();
srmatrix operator ~ () const throw (cvmexception);
srmatrix& transpose (const srmatrix& m) throw (cvmexception);
srmatrix& transpose ();
rvector operator * (const rvector& v) const throw (cvmexception);
rmatrix operator * (const rmatrix& m) const throw (cvmexception);
srmatrix operator * (const srmatrix& m) const throw (cvmexception);
srmatrix& operator *= (const srmatrix& m) throw (cvmexception);
srmatrix& swap_rows (int n1, int n2) throw (cvmexception);
srmatrix& swap_cols (int n1, int n2) throw (cvmexception);
rvector solve (const rvector& vB) const throw (cvmexception);
rmatrix solve (const rmatrix& mB) const throw (cvmexception);
rvector solve (const rvector& vB, TR& dErr) const
               throw (cvmexception);
rmatrix solve (const rmatrix& mB, TR& dErr) const
               throw (cvmexception);
rvector solve_lu (const srmatrix& mLU, const int* pPivots,
                  const rvector& vB, TR& dErr) throw (cvmexception);
rvector solve_lu (const srmatrix& mLU, const int* pPivots,
                  const rvector& vB) throw (cvmexception);
rmatrix solve_lu (const srmatrix& mLU, const int* pPivots,
                  const rmatrix& mB, TR& dErr) throw (cvmexception);
rmatrix solve_lu (const srmatrix& mLU, const int* pPivots,
                  const rmatrix& mB) throw (cvmexception);
TR det () const throw (cvmexception);
srmatrix& low_up (const srmatrix& m,
                  int* nPivots) throw (cvmexception);
srmatrix low_up (int* nPivots) const throw (cvmexception);
TR cond () const throw (cvmexception);
srmatrix& inv (const srmatrix& mArg) throw (cvmexception);
srmatrix inv () const throw (cvmexception);
srmatrix& exp (const srmatrix& m,
               TR tol = cvmMachSp ()) throw (cvmexception);
srmatrix exp (TR tol = cvmMachSp ()) const throw (cvmexception);
srmatrix& polynom (const srmatrix& m, const rvector& v)
                   throw (cvmexception);
srmatrix polynom (const rvector& v) const throw (cvmexception);
cvector eig (scmatrix& mEigVect,
             bool bRightVect = true) const throw (cvmexception);
```

```
    cvector eig () const throw (cvmexception);
    srmatrix& cholesky (const srsmatrix& m) throw (cvmexception);
    srmatrix& bunch_kaufman (const srsmatrix& m,
                             int* pivots) throw (cvmexception);
    void qr(srmatrix& mQ, srmatrix& mR) const throw (cvmexception);
    srmatrix& identity ();
    srmatrix& vanish ();
    srmatrix& randomize (TR dFrom, TR dTo);
};
```

2.8.1 srmatrix ()

Constructor

```
srmatrix::srmatrix ();
```

creates an empty `srmatrix` object. See also `srmatrix`. Example:

```
using namespace cvm;

srmatrix m;
std::cout << m.msize() << std::endl << m.nsize() << std::endl;
std::cout << m.size() << std::endl;

m.resize (3);
std::cout << m;
```

prints

```
0
0
0
0 0 0
0 0 0
0 0 0
```

2.8.2 srmatrix (int)

Constructor

```
explicit srmatrix::srmatrix (int nMN);
```

creates an $n \times n$ `srmatrix` object where n is passed in `nMN` parameter. The constructor throws an exception of type `cvmexception` in case of non-positive size passed or memory allocation failure. See also `srmatrix`. Example:

```
using namespace cvm;

srmatrix m (4);
std::cout << m.msize() << std::endl << m.nsize()
          << std::endl << m.size() << std::endl << m;
```

prints

```
4
4
16
0 0 0 0
0 0 0 0
0 0 0 0
0 0 0 0
```

2.8.3 srmatrix (TR*,int)

Constructor

```
srmatrix::srmatrix (TR* pD, int nMN);
```

creates an $n \times n$ `srmatrix` object where n is passed in `nMN` parameter. Unlike others, this constructor *does not allocate a memory*. It just shares a memory with an array pointed to by `pD`. See also `srmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
srmatrix m (a, 3);

std::cout << m << std::endl;

a[1] = 7.77;
std::cout << m;
```

prints

```
1 4 7
2 5 8
3 6 9

1 4 7
7.77 5 8
3 6 9
```

2.8.4 srmatrix (const srmatrix&)

Copy constructor

```
srmatrix::srmatrix (const srmatrix& m);
```

creates a `srmatrix` object as a copy of `m`. The constructor throws an exception of type `cvmexception` in case of memory allocation failure. See also `srmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
srmatrix m(a, 3);
srmatrix mc(m);

m(1,1) = 7.77;
std::cout << m << std::endl << mc;
```

prints

```
7.77 4 7
2 5 8
3 6 9

1 4 7
2 5 8
3 6 9
```

2.8.5 srmatrix (const rmatrix&)

Constructor

```
srmatrix::srmatrix (const rmatrix& m);
```

creates a `srmatrix` object as a copy of matrix `m`. It's assumed that $m \times n$ matrix `m` must have equal sizes, i.e. $m = n$ is satisfied. The constructor throws an exception of type `cvmexception` if this is not true or in case of memory allocation failure. Please note that this constructor is *not explicit* anymore. See also `srmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6.};
rmatrix m(a, 2, 3);
std::cout << m << std::endl;

m.resize(3, 3);
srmatrix ms (m);
std::cout << ms;
```

prints

```
1 3 5
2 4 6

1 3 5
2 4 6
0 0 0
```

2.8.6 srmatrix (const rvector&)

Constructor

```
explicit srmatrix::srmatrix (const rvector& v);
```

creates a `srmatrix` object of size `v.size()` by `v.size()` and assigns vector `v` to its main diagonal. The constructor throws an exception of type `cvmexception` in case of memory allocation failure. See also `srmatrix`, `rvector`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5.};
rvector v(a, 5);
srmatrix m(v);
std::cout << m;
```

prints

```
1 0 0 0 0
0 2 0 0 0
0 0 3 0 0
0 0 0 4 0
0 0 0 0 5
```

2.8.7 submatrix

Submatrix constructor

```
srmatrix::srmatrix (rmatrix& m, int nRow, int nCol, int nSize);
```

creates a `srmatrix` object as a *submatrix* of `m`. It means that the matrix object created shares a memory with some part of `m`. This part is defined by its upper left corner (parameters `nRow` and `nCol`, both are 1-based) and its size (parameter `nSize`). See also `srmatrix`. Example:

```
using namespace cvm;

rmatrix m(4,5);
srmatrix subm(m, 2, 2, 2);
subm.set(1.);

std::cout << m;
```

prints

```
0 0 0 0 0
0 1 1 0 0
0 1 1 0 0
0 0 0 0 0
```

2.8.8 operator (,)

Indexing operators

```
TR& srmatrix::operator () (int im, int in) throw (cvmexception);
TR srmatrix::operator () (int im, int in) const throw (cvmexception);
```

provide access to an element of a matrix. The first version of the operator is applicable to a non-constant object. This version returns an *l-value* in order to make possible write access to an element. Both operators are 1-based. The operators throw an exception of type `cvmexception` if some of parameters passed is outside of `[1,msize()]` range. The operators are *inherited* in the the class `srbmatrix` and *redefined* in the the class `srsmatrix`. See also `srmatrix`, `Matrix::msize()`, `Matrix::nsize()`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    const srmatrix m (a, 3);
    srmatrix ms(m);

    std::cout << m(1,1) << " " << m(2,3) << std::endl << std::endl;

    ms(2,2) = 7.77;
    std::cout << ms;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1 8

1 4 7
2 7.77 8
3 6 9
```

2.8.9 operator ()

Indexing operators

```
rvector srmatrix::operator () (int i) throw (cvmexception);
const rvector srmatrix::operator () (int i) const throw (cvmexception);
```

provide access to an i-th column of a matrix. The first version of the operator is applicable to a non-constant object and *returns an l-value*, i.e. the vector returned shares a memory with the i-th column of the matrix in order to make possible write access to it. The second version creates a *copy* of a column and therefore it's *not an l-value*. Both operators are 1-based. The operators throw an exception of type `cvmexception` if the parameter `i` is outside of `[1,nsize()]` range. The operators are *redefined* in the the classes `srbmatrix` and `srsmatrix`. See also `srmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    const srmatrix m (a, 3);
    srmatrix ms(3);

    std::cout << m(2) << std::endl;

    ms(2) = m(3);
    std::cout << ms;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
4.00e+00 5.00e+00 6.00e+00

0.00e+00 7.00e+00 0.00e+00
0.00e+00 8.00e+00 0.00e+00
0.00e+00 9.00e+00 0.00e+00
```

2.8.10 operator []

Indexing operators

```
rvector srmatrix::operator [] (int i) throw (cvmexception);
const rvector srmatrix::operator [] (int i) const throw (cvmexception);
```

provide access to an i-th row of a matrix. The first version of the operator is applicable to a non-constant object and *returns an l-value*, i.e. the vector returned shares a memory with the i-th row of the matrix in order to make possible write access to it. The second version creates a *copy* of a row and therefore it's *not an l-value*. Both operators are 1-based. The operators throw an exception of type `cvmexception` if the parameter `i` is outside of `[1,msize()]` range. The operators are *redefined* in the the classes `srbmatrix` and `srsmatrix`. See also `srmatrix`, `Matrix::msize()`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    const srmatrix m (a, 3);
    srmatrix ms(3);

    std::cout << m[2] << std::endl;

    ms[2] = m[3];
    std::cout << ms;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
2.00e+00 5.00e+00 8.00e+00

0.00e+00 0.00e+00 0.00e+00
3.00e+00 6.00e+00 9.00e+00
0.00e+00 0.00e+00 0.00e+00
```

2.8.11 operator = (const srmatrix&)

Operator

```
srmatrix& srmatrix::operator = (const srmatrix& m)
throw (cvmexception);
```

sets an every element of a calling matrix to a value of appropriate element of a matrix `m` and returns a reference to the matrix changed. The operator throws an exception of type `cvmexception` in case of different sizes of the operands. The operator is *redefined* in the the classes `srbmatrix` and `srsmatrix`. See also `srmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    const srmatrix m1(a, 3);
    srmatrix m2(3);

    m2 = m1;
    std::cout << m2;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1.00e+00 4.00e+00 7.00e+00
2.00e+00 5.00e+00 8.00e+00
3.00e+00 6.00e+00 9.00e+00
```

2.8.12 assign (const TR*)

Function

```
srmatrix& srmatrix::assign (const rvector& v);
srmatrix& srmatrix::assign (const TR* pD);
```

sets every element of a calling matrix to a value of appropriate element of a vector `v` or an array pointed to by `pD` and returns a reference to the matrix changed. The function is *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

const double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
srmatrix m(3);

m.assign(a);
std::cout << m;
```

prints

```
1.00e+00 4.00e+00 7.00e+00
2.00e+00 5.00e+00 8.00e+00
3.00e+00 6.00e+00 9.00e+00
```

2.8.13 assign (int, int, const rmatrix&)

Function

```
srmatrix& srmatrix::assign (int nRow, int nCol, const rmatrix& m)
throw (cvmexception);
```

sets sub-matrix of a calling matrix beginning with 1-based row `nRow` and column `nCol` to a matrix `m` and returns a reference to the matrix changed. The function throws an exception of type `cvmexception` if `nRow` or `nCol` are not positive or matrix `m` doesn't fit. The function is *redefined* in the class `srsmatrix`. See also `rmatrix`, `srmatrix`. Example:

```
using namespace cvm;

srmatrix m1(5);
rmatrix m2(2,3);
m1.set(1.);
m2.set(2.);
m1.assign(2,3,m2);
std::cout << m1;
```

prints

```
1 1 1 1 1
1 1 2 2 2
1 1 2 2 2
1 1 1 1 1
1 1 1 1 1
```

2.8.14 set (TR)

Function

```
srmatrix& srmatrix::set (TR x);
```

sets every element of a calling matrix to a value of parameter x and returns a reference to the matrix changed. Use vanish to set every element of a calling matrix to be equal to zero. The function is *redefined* in the classes srbmatrix and srsmatrix. See also srmatrix. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
srmatrix m(3);

m.set(3.);
std::cout << m;
```

prints

```
3.00e+00 3.00e+00 3.00e+00
3.00e+00 3.00e+00 3.00e+00
3.00e+00 3.00e+00 3.00e+00
```

2.8.15 resize

Function

```
srmatrix& srmatrix::resize (int nNewMN) throw (cvmexception);
```

changes a size of a calling matrix to `nNewMN` by `nNewMN` and returns a reference to the matrix changed. In case of increasing of its size, the matrix is filled up with zeroes. The function throws an exception of type `cvmexception` in case of negative size passed or memory allocation failure. The function is *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4.};
    srmatrix m(a, 2);

    std::cout << m << std::endl;

    m.resize (3);
    std::cout << m;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1 3
2 4

1 3 0
2 4 0
0 0 0
```

2.8.16 operator <<

Operator

```
srmatrix& srmatrix::operator << (const srmatrix& m)
throw (cvmexception);
```

destroys a calling matrix, creates a new one as a copy of `m` and returns a reference to the matrix changed. The operator throws an exception of type `cvmexception` in case of memory allocation failure. The operator is *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    srmatrix m(3);
    srmatrix mc(1);
    m(1,2) = 1.;
    m(2,3) = 2.;
    std::cout << m << std::endl << mc << std::endl;

    mc << m;
    std::cout << mc;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
0.00e+00 1.00e+00 0.00e+00
0.00e+00 0.00e+00 2.00e+00
0.00e+00 0.00e+00 0.00e+00

0.00e+00

0.00e+00 1.00e+00 0.00e+00
0.00e+00 0.00e+00 2.00e+00
0.00e+00 0.00e+00 0.00e+00
```

2.8.17 operator +

Operator

```
srmatrix srmatrix::operator + (const srmatrix& m) const
throw (cvmexception);
```

creates an object of type `srmatrix` as a sum of a calling matrix and a matrix `m`. It throws an exception of type `cvmexception` in case of different sizes of the operands. The operator is *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix::sum`, `srmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    double b[] = {10., 20., 30., 40., 50., 60., 70., 80., 90.};
    srmatrix m1(a, 3);
    srmatrix m2(b, 3);

    std::cout << m1 + m2 << std::endl << m1 + m1;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
11 44 77
22 55 88
33 66 99

2 8 14
4 10 16
6 12 18
```

2.8.18 operator -

Operator

```
srmatrix srmatrix::operator - (const srmatrix& m) const
throw (cvmexception);
```

creates an object of type `srmatrix` as a difference of a calling matrix and a matrix `m`. It throws an exception of type `cvmexception` in case of different sizes of the operands. The operator is *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix::diff`, `srmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    double b[] = {10., 20., 30., 40., 50., 60., 70., 80., 90.};
    srmatrix m1(a, 3);
    srmatrix m2(b, 3);

    std::cout << m2 - m1 << std::endl << m1 - m1;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
9 36 63
18 45 72
27 54 81

0 0 0
0 0 0
0 0 0
```

2.8.19 sum

Function

```
srmatrix& srmatrix::sum (const srmatrix& m1, const srmatrix& m2)
throw (cvmexception);
```

assigns a result of addition of matrices `m1` and `m2` to a calling matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes of the operands. The function is *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix::operator +` , `srmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    const srmatrix m1(a, 3);
    srmatrix m2(3);
    srmatrix m(3);
    m2.set(1.);

    std::cout << m.sum(m1, m2) << std::endl;
    std::cout << m.sum(m, m2);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
2 5 8
3 6 9
4 7 10

3 6 9
4 7 10
5 8 11
```

2.8.20 diff

Function

```
srmatrix& srmatrix::diff (const srmatrix& m1, const srmatrix& m2)
throw (cvmexception);
```

assigns a result of subtraction of matrices `m1` and `m2` to a calling matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes of the operands. The function is *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix::operator -` , `srmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    const srmatrix m1(a, 3);
    srmatrix m2(3);
    srmatrix m(3);
    m2.set(1.);

    std::cout << m.diff(m1, m2) << std::endl;
    std::cout << m.diff(m, m2);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
0 3 6
1 4 7
2 5 8

-1 2 5
0 3 6
1 4 7
```

2.8.21 operator +=

Operator

`srmatrix& srmatrix::operator += (const srmatrix& m) throw (cvmexception);`

adds a matrix `m` to a calling matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes of the operands. The operator is *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix::operator +` , `srmatrix::sum`, `srmatrix`. Example:

```
using namespace cvm;
try {
    srmatrix m1(3);
    srmatrix m2(3);
    m1.set(1.);
    m2.set(2.);

    m1 += m2;
    std::cout << m1 << std::endl;

    // well, you can do this too, but temporary object would be created
    m2 += m2;
    std::cout << m2;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
3 3 3
3 3 3
3 3 3

4 4 4
4 4 4
4 4 4
```

2.8.22 operator -=

Operator

```
srmatrix& srmatrix::operator -= (const srmatrix& m) throw (cvmexception);
```

subtracts a matrix m from a calling matrix and returns a reference to the matrix changed. It throws an exception of type cvmexception in case of different sizes of the operands. The operator is *redefined* in the classes srbmatrix and srsmatrix. See also srmatrix::operator - , srmatrix::diff, srmatrix. Example:

```
using namespace cvm;

try {
    srmatrix m1(3);
    srmatrix m2(3);
    m1.set(1.);
    m2.set(2.);

    m1 -= m2;
    std::cout << m1 << std::endl;

    // well, you can do this too, but temporary object would be created
    m2 -= m2;
    std::cout << m2;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
-1 -1 -1
-1 -1 -1
-1 -1 -1

0 0 0
0 0 0
0 0 0
```

2.8.23 operator - ()

Operator

```
srmatrix srmatrix::operator - () const throw (cvmexception);
```

creates an object of type `srmatrix` as a calling matrix multiplied by -1. The operator is *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
srmatrix m(a, 3);

std::cout << -m;
```

prints

```
-1 -4 -7
-2 -5 -8
-3 -6 -9
```

2.8.24 operator ++

Operator

```
srmatrix& srmatrix::operator ++ ();
srmatrix& srmatrix::operator ++ (int);
```

adds identity matrix to a calling matrix and returns a reference to the matrix changed. The operator is *redefined* in the classes srbmatrix and srsmatrix. See also srmatrix. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
srmatrix m (a, 3);

m++;
std::cout << m << std::endl;
std::cout << ++m;
```

prints

```
2 4 7
2 6 8
3 6 10

3 4 7
2 7 8
3 6 11
```

2.8.25 operator --

Operator

```
srmatrix& srmatrix::operator -- ();
srmatrix& srmatrix::operator -- (int);
```

subtracts identity matrix from a calling matrix and returns a reference to the matrix changed. The operator is *redefined* in the classes srbmatrix and srsmatrix. See also srmatrix. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
srmatrix m (a, 3);

m--;
std::cout << m << std::endl;
std::cout << --m;
```

prints

```
0 4 7
2 4 8
3 6 8

-1 4 7
2 3 8
3 6 7
```

2.8.26 operator * (TR)

Operator

```
srmatrix srmatrix::operator * (TR d) const;
```

creates an object of type `srmatrix` as a product of a calling matrix and a number `d`. The operator is *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix::operator *=` , `srmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
srmatrix m(a, 3);

std::cout << m * 5.;
```

prints

```
5 20 35
10 25 40
15 30 45
```

2.8.27 operator / (TR)

Operator

```
srmatrix srmatrix::operator / (TR d) const throw (cvmexception);
```

creates an object of type `srmatrix` as a quotient of a calling matrix and a number `d`. It throws an exception of type `cvmexception` if `d` has an absolute value equal or less than the smallest normalized positive number. The operator is *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix::operator /=` , `srmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    srmatrix m(a, 3);

    std::cout << m / 4.;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
0.25 1 1.75
0.5 1.25 2
0.75 1.5 2.25
```

2.8.28 operator *= (TR)

Operator

```
srmatrix& srmatrix::operator *= (TR d);
```

multiplies a calling matrix by a number `d` and returns a reference to the matrix changed. The operator is *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix::operator *` , `srmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
srmatrix m(a, 3);

m *= 2.;
std::cout << m;
```

prints

```
2 8 14
4 10 16
6 12 18
```

2.8.29 operator /= (TR)

Operator

`srmatrix& srmatrix::operator /= (TR d) throw (cvmexception);`

divides a calling matrix by a number d and returns a reference to the matrix changed. It throws an exception of type cvmexception if d has an absolute value equal or less than the smallest normalized positive number. The operator is *redefined* in the classes srbmatrix and srsmatrix. See also srmatrix::operator / , srmatrix. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    srmatrix m(a, 3);

    m /= 2.;
    std::cout << m;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
0.5 2 3.5
1 2.5 4
1.5 3 4.5
```

2.8.30 normalize

Function

```
srmatrix& srmatrix::normalize ();
```

normalizes a calling matrix so its Euclidean norm becomes equal to 1 if it was greater than the smallest normalized positive number before the call (otherwise the function does nothing). The function is *redefined* in the classes srbmatrix and srsmatrix. See also srmatrix. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (3);

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
srmatrix m(a, 3);

m.normalize();
std::cout << m << m.norm() << std::endl;
```

prints

```
5.923e-02 2.369e-01 4.146e-01
1.185e-01 2.962e-01 4.739e-01
1.777e-01 3.554e-01 5.331e-01
1.000e+00
```

2.8.31 transposition

Operator and functions

```
srmatrix srmatrix::operator ~ () const throw (cvmexception);
srmatrix& srmatrix::transpose (const srmatrix& m) throw (cvmexception);
srmatrix& srmatrix::transpose ();
```

encapsulate matrix transposition. First operator creates an object of type `srmatrix` as a transposed calling matrix (it throws an exception of type `cvmexception` in case of memory allocation failure). Second function sets a calling matrix to be equal to a matrix `m` transposed (it throws an exception of type `cvmexception` in case of not appropriate sizes of the operands), third one makes it to be equal to transposed itself. The functions are *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix`. Example:

```
using namespace cvm;
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    srmatrix m(a,3);
    srmatrix mt(3);
    std::cout << ~m << std::endl ;
    mt.transpose(m);
    std::cout << mt << std::endl;
    mt.transpose();
    std::cout << mt;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1 2 3
4 5 6
7 8 9

1 2 3
4 5 6
7 8 9

1 4 7
2 5 8
3 6 9
```

2.8.32 operator * (const rvector&)

Operator

```
rvector srmatrix::operator * (const rvector& v) const
throw (cvmexception);
```

creates an object of type `rvector` as a product of a calling matrix and a vector `v`. It throws an exception of type `cvmexception` if the number of columns of the calling matrix differs from the size of the vector `v`. Use `rvector::mult` in order to get rid of a new object creation. The function is *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix`, `rvector`. Example:

```
using namespace cvm;

try {
    srmatrix m(3);
    rvector v(3);
    m.set(1.);
    v.set(1.);

    std::cout << m * v;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
3 3 3
```

2.8.33 operator * (const rmatrix&)

Operator

```
rmatrix srmatrix::operator * (const rmatrix& m) const
throw (cvmexception);
```

creates an object of type `rmatrix` as a product of a calling matrix and a matrix `m`. It throws an exception of type `cvmexception` if the number of columns of the calling matrix differs from the number of rows of the matrix `m`. Use `rmatrix::mult` in order to get rid of a new object creation. The operator is *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `rmatrix`, `srmatrix`. Example:

```
using namespace cvm;

try {
    srmatrix ms(3);
    rmatrix m(3,2);
    ms.set(1.);
    m.set(1.);

    std::cout << ms * m;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
3 3
3 3
3 3
```

2.8.34 operator * (const srmatrix&)

Operator

```
srmatrix srmatrix::operator * (const srmatrix& m) const
throw (cvmexception);
```

creates an object of type `srmatrix` as a product of a calling matrix and a matrix `m`. It throws an exception of type `cvmexception` if the operands have different sizes. Use `rmatrix::mult` in order to get rid of a new object creation. The operator is *inherited* in the class `srbmatrix` and *redefined* in `srsmatrix`. See also `srmatrix`. Example:

```
using namespace cvm;

try {
    srmatrix m1(3);
    srmatrix m2(3);
    m1.set(1.);
    m2.set(1.);

    std::cout << m1 * m2;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
3 3 3
3 3 3
3 3 3
```

2.8.35 operator *= (const srmatrix&)

Operator

```
srmatrix& srmatrix::operator *= (const srmatrix& m)
throw (cvmexception);
```

sets a calling matrix to be equal to a product of itself by a matrix `m` and returns a reference to the object it changes. The operator throws an exception of type `cvmexception` in case of different sizes of the operands. The operator is *inherited* in the class `srbmatrix` and *redefined* in `srsmatrix`. See also `srmatrix`. Example:

```
using namespace cvm;

try {
    srmatrix m1(3);
    srmatrix m2(3);
    m1.set(1.);
    m2.set(1.);

    m1 *= m2;
    std::cout << m1 << std::endl;
    m1 *= m1;
    std::cout << m1;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
3 3 3
3 3 3
3 3 3

27 27 27
27 27 27
27 27 27
```

2.8.36 swap_rows

Function

```
srmatrix& srmatrix::swap_rows (int n1, int n2) throw (cvmexception);
```

swaps two rows of a calling matrix and returns a reference to the matrix changed. `n1` and `n2` are the numbers of rows to be swapped, both are 1-based). The function throws an exception of type `cvmexception` if one of the parameters is outside of the range `[1,msize()]`. The function is *not applicable* to objects of the classes `srbmatrix` and `srsmatrix` (i.e. an exception of type `cvmexception` would be thrown in case of using it for objects of those classes). See also `srmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    srmatrix m (a, 3);

    std::cout << m << std::endl;
    std::cout << m.swap_rows(2,3);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1 4 7
2 5 8
3 6 9

1 4 7
3 6 9
2 5 8
```

2.8.37 swap_cols

Function

```
srmatrix& srmatrix::swap_cols (int n1, int n2) throw (cvmexception);
```

swaps two columns of a calling matrix and returns a reference to the matrix changed. `n1` and `n2` are the numbers of columns to be swapped, both are 1-based). The function throws an exception of type `cvmexception` if one of the parameters is outside of the range `[1,nsize()]`. The function is *not applicable* to objects of the classes `srbmatrix` and `srsmatrix` (i.e. an exception of type `cvmexception` would be thrown in case of using it for objects of those classes). See also `srmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    srmatrix m (a, 3);

    std::cout << m << std::endl;
    std::cout << m.swap_cols(2,3);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1 4 7
2 5 8
3 6 9

1 7 4
2 8 5
3 9 6
```

2.8.38 solve

Functions

```
rvector srmatrix::solve (const rvector& vB) const throw (cvmexception);
rmatrix srmatrix::solve (const rmatrix& mB) const throw (cvmexception);
rvector srmatrix::solve (const rvector& vB, TR& dErr) const
throw (cvmexception);
rmatrix srmatrix::solve (const rmatrix& mB, TR& dErr) const
throw (cvmexception);
```

return a solution of a linear equation of kind $Ax = b$ or $AX = B$ where A is a calling matrix. The first and the third versions solve the equation $Ax = b$ where vector b is passed in the parameter vB and the second and fourth versions solve the equation $AX = B$ where matrix B is passed in the parameter mB. The last two versions also set output parameter dErr to be equal to a norm of computation error. The functions throw an exception of type cvmexception in case of inappropriate sizes of the operands. The function is *inherited* in the classes srbmatrix and srsmatrix. See also rvector::solve, rmatrix::solve, srmatrix. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (3);
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 10.};
    srmatrix ma(a, 3);
    rmatrix  mb(3,4);
    rmatrix  mx(3,4);
    double dErr;

    mb(1).set(1.);
    mb(2).set(2.);
    mb(3).set(3.);
    mb(1,4) = 1.; mb(2,4) = 2.; mb(3,4) = 3.;

    mx = ma.solve (mb, dErr);
    std::cout << mx << dErr
              << std::endl << ma * mx - mb << std::endl;

    rvector vb(3), vx(3);
    vb = mb(2);
    vx = ma.solve (vb, dErr);
    std::cout << vx << dErr << std::endl << ma * vx - vb;
```

```
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
-3.333e-01 -6.667e-01 -1.000e+00 1.000e+00
3.333e-01 6.667e-01 1.000e+00 0.000e+00
6.661e-16 1.332e-15 0.000e+00 0.000e+00
3.301e-14
0.000e+00 0.000e+00 0.000e+00 0.000e+00
-1.110e-16 -2.220e-16 0.000e+00 0.000e+00
2.220e-16 4.441e-16 0.000e+00 0.000e+00

-6.667e-01 6.667e-01 1.332e-15
3.301e-14
0.000e+00 -2.220e-16 4.441e-16
```

2.8.39 solve_lu

Functions

```
rvector
srmatrix::solve_lu (const srmatrix& mLU, const int* pPivots,
                    const rvector& vB, TR& dErr) throw (cvmexception);
rvector
srmatrix::solve_lu (const srmatrix& mLU, const int* pPivots,
                    const rvector& vB) throw (cvmexception);

rmatrix
srmatrix::solve_lu (const srmatrix& mLU, const int* pPivots,
                    const rmatrix& mB, TR& dErr) throw (cvmexception);
rmatrix
srmatrix::solve_lu (const srmatrix& mLU, const int* pPivots,
                    const rmatrix& mB) throw (cvmexception);
```

create an object of type `rvector` or `rmatrix` as a solution x or X of the matrix linear equation $Ax = b$ or $AX = B$ respectively. Here A is a calling matrix, parameter `mLU` is LU factorization of the matrix A, parameter `pPivots` is an array of pivot numbers created while factorizing the matrix A and parameters `vB` and `mB` are the vector b and matrix B respectively. The first and third version also set output parameter `dErr` to be equal to a norm of computation error. These functions are useful when you need to solve few linear equations of kind $Ax = b$ or $AX = B$ with the same matrix A and different vectors b or matrices B. In such case you save on matrix A factorization since it's needed to be performed just one time. The functions throw exception of type `cvmexception` in case of inappropriate sizes of the operands or when the matrix A is close to cingular. The function is *inherited* in the classes `srbmatrix` and `srsmatrix`. See also `rvector::solve`, `rmatrix`, `srmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (3);
try {
    double a[] = {1., -1., 1., 2., -2., 1., 3., -2., 1.};
    srmatrix ma(a,3);
    srmatrix mLU(3);
    rmatrix  mb1(3,2); rvector vb1(3);
    rmatrix  mb2(3,2); rvector vb2(3);
    rmatrix  mx1(3,2); rvector vx1(3);
    rmatrix  mx2(3,2); rvector vx2(3);
    iarray   nPivots(3);
```

```
    double   dErr = 0.;
    mb1.randomize(-1.,3.); vb1.randomize(-2.,4.);
    mb2.randomize(-2.,5.); vb2.randomize(-3.,1.);

    mLU.low_up(ma, nPivots);
    mx1 = ma.solve_lu (mLU, nPivots, mb1, dErr);
    std::cout << mx1 << dErr << std::endl;
    mx2 = ma.solve_lu (mLU, nPivots, mb2);
    std::cout << mx2 << std::endl;;
    std::cout << ma * mx1 - mb1 << std::endl << ma * mx2 - mb2;

    vx1 = ma.solve_lu (mLU, nPivots, vb1, dErr);
    std::cout << vx1 << dErr << std::endl;
    vx2 = ma.solve_lu (mLU, nPivots, vb2);
    std::cout << vx2 << std::endl;;
    std::cout << ma * vx1 - vb1 << std::endl << ma * vx2 - vb2;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
2.807e+00 1.107e+00
-3.651e-01 -4.843e+00
-5.412e-01 3.095e+00
6.438e-15
-7.639e-01 1.082e+01
-2.869e-01 -1.110e+01
4.890e-01 3.443e+00

0.000e+00 -4.441e-16
1.110e-16 -4.441e-16
-4.441e-16 4.441e-16

0.000e+00 -4.441e-16
0.000e+00 8.882e-16
0.000e+00 -4.441e-16
-1.651e+00 2.361e-01 -6.384e-02
3.828e-15
-5.886e+00 7.038e+00 -3.125e+00

0.000e+00 0.000e+00 0.000e+00
```

```
0.000e+00 0.000e+00 2.220e-16
```

2.8.40 det

Function

```
TR srmatrix::det () const throw (cvmexception);
```

returns a determinant of a calling matrix. It uses the LU factorization inside and may throw the same exceptions as the factorizer. The function is *inherited* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (3);
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 10.};
    srmatrix m(a, 3);

    std::cout << m << std::endl << m.det() << std::endl;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1.000e+00 4.000e+00 7.000e+00
2.000e+00 5.000e+00 8.000e+00
3.000e+00 6.000e+00 1.000e+01

-3.000e+00
```

2.8.41 low_up

Functions

```
srmatrix&
srmatrix::low_up (const srmatrix& m, int* nPivots) throw (cvmexception);
srmatrix
srmatrix::low_up (int* nPivots) const throw (cvmexception);
```

compute the LU factorization of a calling matrix as

$$A = PLU$$

where P is a permutation matrix, L is a lower triangular matrix with unit diagonal elements and U is an upper triangular matrix. All the functions store the result as the matrix L without main diagonal combined with U. All the functions return pivot indices as an array of integers (it should support at least `msize()` elements) pointed to by `nPivots` so i-th row was interchanged with `nPivots[i]`-th row. The first version sets a calling matrix to be equal to the `m`'s LU factorization and the second one creates an object of type `srmatrix` as the calling matrix's LU factorization. The functions throw exception of type `cvmexception` in case of inappropriate sizes of the operands or when the matrix to be factorized is close to cingular. It is recommended to use `iarray` for pivot values. The function is *redefined* in the class `srbmatrix` and *inherited* in `srsmatrix`. See also `srmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (3);
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 10.};
    srmatrix m(a, 3);
    srmatrix mLU(3), mLo(3), mUp(3);
    iarray naPivots(3);

    mLU.low_up (m, naPivots);

    mLo.identity ();

    mLo.diag(-2) = mLU.diag(-2);
    mLo.diag(-1) = mLU.diag(-1);
    mUp.diag(0) = mLU.diag(0);
    mUp.diag(1) = mLU.diag(1);
    mUp.diag(2) = mLU.diag(2);
```

```
    std::cout << mLo << std::endl << mUp
              << std::endl << naPivots << std::endl;

    mLU = mLo * mUp;
    for (int i = 3; i >= 1; i--) {
        mLU.swap_rows (i, naPivots[i]);
    }
    std::cout << mLU;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1.000e+00 0.000e+00 0.000e+00
3.333e-01 1.000e+00 0.000e+00
6.667e-01 5.000e-01 1.000e+00

3.000e+00 6.000e+00 1.000e+01
0.000e+00 2.000e+00 3.667e+00
0.000e+00 0.000e+00 -5.000e-01

3 3 3

1.000e+00 4.000e+00 7.000e+00
2.000e+00 5.000e+00 8.000e+00
3.000e+00 6.000e+00 1.000e+01
```

2.8.42 cond

Function

```
TR srmatrix::cond () const throw (cvmexception);
```

returns a reciprocal of a condition number of a calling matrix A in the infinity-norm:

$$\kappa_\infty = \|A\|_\infty \|A^{-1}\|_\infty.$$

Less value returned means that matrix A is closer to cingular. Zero value returned means estimation underflow or that matrix A is cingular. The function throws exception of type `cvmexception` in case of LAPACK subroutines failure. The function is *inherited* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix::solve`, `srmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (3);
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    srmatrix m(a, 3);
    std::cout << m.cond() << std::endl
              << m.det() << std::endl << std::endl;
    m(3,3) = 10.;
    std::cout << m.cond() << std::endl << m.det() << std::endl;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
0.000e+00
0.000e+00

7.519e-03
-3.000e+00
```

2.8.43 inv

Functions

```
srmatrix& srmatrix::inv (const srmatrix& m) throw (cvmexception);
srmatrix srmatrix::inv () const throw (cvmexception);
```

implement matrix inversion. The first version sets a calling matrix to be equal to `m` inverted and the second one creates an object of type `srmatrix` as inverted calling matrix. The functions throw exception of type `cvmexception` in case of inappropriate sizes of the operands or when the matrix to be inverted is close to cingular. The function is *redefined* in the class `srsmatrix` and *inherited* in `srbmatrix`. See also `srmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (10);
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 10.};
    srmatrix m(a, 3);
    srmatrix mi(3);
    mi.inv (m);
    std::cout << mi << std::endl << mi * m - eye_real(3);
    std::cout << std::endl << mi.inv() * mi - eye_real(3);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
-6.6666666667e-01 -6.6666666667e-01 1.0000000000e+00
-1.3333333333e+00 3.6666666667e+00 -2.0000000000e+00
1.0000000000e+00 -2.0000000000e+00 1.0000000000e+00

0.0000000000e+00 0.0000000000e+00 1.7763568394e-15
1.7763568394e-15 3.5527136788e-15 0.0000000000e+00
0.0000000000e+00 0.0000000000e+00 1.7763568394e-15

0.0000000000e+00 1.7763568394e-15 -1.7763568394e-15
-8.8817841970e-16 3.5527136788e-15 -3.5527136788e-15
0.0000000000e+00 0.0000000000e+00 -1.7763568394e-15
```

2.8.44 exp

Functions

```
srmatrix& srmatrix::exp (const srmatrix& m, TR tol = cvmMachSp ())
throw (cvmexception);

srmatrix srmatrix::exp (TR tol = cvmMachSp ()) const
throw (cvmexception);
```

compute an exponent of a calling matrix using Padé approximation defined as

$$R_{pq}(z) = D_{pq}(z)^{-1} N_{pq}(z) = 1 + z + \cdots + z^p/p!\,,$$

where

$$N_{pq}(z) = \sum_{k=0}^{p} \frac{(p+q-k)!p!}{(p+q)!k!(q-k)!} z^k,$$
$$D_{pq}(z) = \sum_{k=0}^{q} \frac{(p+q-k)!p!}{(p+q)!k!(q-k)!} (-z)^k$$

along with the matrix normalizing as described in [2], p. 572. The functions use `DMEXP` (or `SMEXP` for `float` version) FORTRAN subroutine implementing the algorithm. The first version sets the calling matrix to be equal to the exponent of `m` and returns a reference to the matrix changed. The second version creates an object of type `srmatrix` as the exponent of the calling matrix. The algorithm uses parameter `tol` as $\varepsilon(p, q)$ in order to choose constants p and q so that

$$\varepsilon(p, q) \geqslant 2^{3-(p+q)} \frac{p!q!}{(p+q)!(p+q+1)!}.$$

This parameter is equal to the largest relative spacing by default. The functions throw an exception of type `cvmexception` in case of inappropriate sizes of the operands or when LAPACK subroutine fails. The functions are *inherited* in the classes `srbmatrix` and `srsmatrix`. The second version is *redefined* in `srbmatrix`. See also `srmatrix`. Example (see [2], p. 567, example 11.2.2):

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (15);
try {
    srmatrix m(2);
    m(1,1) = -49.;
    m(1,2) = 24.;
```

```
    m(2,1) = -64.;
    m(2,2) = 31.;

    std::cout << m << std::endl << m.exp();
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
-4.900000000000000e+01 2.400000000000000e+01
-6.400000000000000e+01 3.100000000000000e+01

-7.357587581448284e-01 5.518190996581556e-01
-1.471517599088415e+00 1.103638240715692e+00
```

Matlab output:

```
-7.357587581446907e-001    5.518190996580505e-001
-1.471517599088136e+000    1.103638240715478e+000
```

2.8.45 polynomial

Functions

```
srmatrix& srmatrix::polynom (const srmatrix& m, const rvector& v)
throw (cvmexception);

srmatrix srmatrix::polynom (const rvector& v) const
throw (cvmexception);
```

compute a matrix polynomial defined as

$$p(A) = b_0 I + b_1 A + \cdots + b_q A^q$$

using the Horner's rule:

$$p(A) = \sum_{k=0}^{r} B_k (A^s)^k, \quad s = \mathrm{floor}(\sqrt{q}\,),\ r = \mathrm{floor}(q/s)$$

where

$$B_k = \begin{cases} \sum\limits_{i=0}^{s-1} b_{sk+i} A^i, & k = 0, 1, \ldots, r-1 \\ \sum\limits_{i=0}^{q-sr} b_{sr+i} A^i, & k = r. \end{cases}$$

See also [2], p. 568. The coefficients $b_0, b_1, \ldots, b_q$ are passed in the parameter v, where q is equal to `v.size()-1`, so the functions compute matrix polynomial equal to

$$\mathtt{v}[1] * \mathtt{I} + \mathtt{v}[2] * \mathtt{m} + \cdots + \mathtt{v}[\mathtt{v.size}()] * \mathtt{m}^{\mathtt{v.size}()-1}$$

The first version sets a calling matrix to be equal to the polynomial of `m` and the second one creates an object of type `srmatrix` as the polynomial of a calling matrix. The functions use `DPOLY` (or `SPOLY` for `float` version) FORTRAN subroutine implementing the Horner's algorithm. The functions throw an exception of type `cvmexception` in case of inappropriate sizes of the operands. The functions are *inherited* in the class `srbmatrix` and *redefined* in `srsmatrix`. See also `srmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (15);
try {
    double a[] = {2.2, 1.3, 1.1, -0.9, 0.2,
                  -0.45, 45, -30, 10, 3, 3.2};
    const rvector v(a, 11);
    srmatrix m(2), mp(2);
```

```
    m(1,1) = 1.;
    m(1,2) = 0.5;
    m(2,1) = -1.;
    m(2,2) = 0.3;

    mp.polynom (m, v);
    std::cout << mp;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
-7.963641665999998e+00 -7.551532476200001e+00
1.510306495240000e+01 2.608503800680002e+00
```

Matlab output:

```
-7.963641665999999e+000   -7.551532476200002e+000
 1.510306495240000e+001    2.608503800680002e+000
```

2.8.46 eig

Functions

```
cvector srmatrix::eig (scmatrix& mEigVect, bool bRightVect = true) const
throw (cvmexception);

cvector srmatrix::eig () const throw (cvmexception);
```

solve a nonsymmetric eigenvalue problem and return a complex vector with eigenvalues of a calling matrix. The first version sets the output parameter `mEigVect` to be equal to the square matrix containing right (if parameter `bRightVect` is true, which is default value) or left (if parameter `bRightVect` is false) eigenvectors as columns. All the functions throw an exception of type `cvmexception` in case of inappropriate sizes of the operands or in case of convergence error. The functions are *inherited* in the class `srbmatrix` and *redefined* in `srsmatrix`. See also `cvector`, `scmatrix` and `srmatrix`. Example:

```
using namespace cvm;

try {
    scmatrix m(3), me(3);
    cvector vl(3);

    m(1,1) = 0.1;  m(1,2) = 0.2;  m(1,3) = 0.1;
    m(2,1) = 0.11; m(2,2) = -2.9; m(2,3) = -8.4;
    m(3,1) = 0.;   m(3,2) = 2.91; m(3,3) = 8.2;

    vl = m.eig (me);
    std::cout << vl;

    m(2,2) = 2.9;
    vl = m.eig (me);
    std::cout << vl << std::endl;

    std::cout.setf (std::ios::scientific | std::ios::showpos);
    std::cout.precision (1);

    std::cout << m * me(1) - me(1) * v(1);
    std::cout << m * me(2) - me(2) * v(2);
    std::cout << m * me(3) - me(3) * v(3);
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(-0.0555784,0) (0.285327,0) (5.17025,0)
(0.0968985,3.19501e-018) (5.55155,4.1733) (5.55155,-4.1733)

(+2.1e-002,-7.6e-002) (-3.9e-004,+1.4e-003) (+1.4e-004,-5.2e-004)
(+1.5e-001,-6.2e-002) (+5.4e+000,+3.3e+000) (-6.7e-002,-3.7e+000)
(+7.7e-002,-1.2e-001) (-2.1e+000,-5.3e+000) (+3.3e+000,+6.1e-001)
```

2.8.47 Cholesky

Function

```
srmatrix& srmatrix::cholesky (const srsmatrix& m)
throw (cvmexception);
```

forms the Cholesky factorization of a symmetric positive-definite matrix A defined as

$$A = U^T U,$$

where U is upper triangular matrix. It utilizes one of ?POTRF routines of the LAPACK library. The function sets a calling matrix to be equal to the factorization of a symmetric positive-definite matrix m. The function throws an exception of type cvmexception in case of inappropriate sizes of the operands or in case of convergence error. See also srmatrix and srsmatrix. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 1., 2., 5., -1., 1., -1., 20.};
    const srsmatrix m(a, 3);
    srmatrix h(3);

    h.cholesky(m);
    std::cout << h << std::endl;
    std::cout << ~h * h - m;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
1 2 1
0 1 -3
0 0 3.16228

0 0 0
0 0 0
0 0 0
```

2.8.48 Bunch-Kaufman

Function

```
srmatrix& srmatrix::bunch_kaufman (const srsmatrix& m, int* pivots)
throw (cvmexception);
```

forms the Bunch-Kaufman factorization of a symmetric matrix (cited from the MKL library documentation):

$$A = PUDU^T P^T,$$

where A is the input matrix passed in parameter `m`, P is a permutation matrix, U and L are upper and lower triangular matrices with unit diagonal, and D is a symmetric block-diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks. U and L have 2-by-2 unit diagonal blocks corresponding to the 2-by-2 blocks of D. It utilizes one of `?SYTRF` routines of the LAPACK library. The function sets a calling matrix to be equal to the factorization of a symmetric positive-definite matrix `m`. The function throws an exception of type `cvmexception` in case of inappropriate sizes of the operands or in case of convergence error. See also `srmatrix` and `srsmatrix`. The function is mostly designed to be used for subsequent calls of `?SYTRS`, `?SYCON` and `?SYTRI` routines of the LAPACK library. Currently it's used internally in `srmatrix::det` flow when argument is symmetric but not positive-definite.

2.8.49 qr

Function

```
void srmatrix::qr (srmatrix& mQ, srmatrix& mR) const throw (cvmexception);
```

computes QR factorization as

$$M = QR$$

where M is a calling square matrix, orthogonal matrix Q and upper triangular matrix R are `mQ` and `mR` respectively. The function throws an exception of type `cvmexception` in case of inappropriate sizes of the operands. See also `rmatrix`, `rmatrix::qr`, `srmatrix`. Example:

```
using namespace cvm;

treal a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
const cvm::srmatrix m(a, 3);
cvm::srmatrix q(3), r(3);

m.qr(q,r);
std::cout << (eye_real(3) - ~q * q).norm()
          << " " << (m - q * r).norm() << std::endl;
```

prints

```
+5.2889959e-16 +7.0854500e-15
```

2.8.50 identity

Function

```
srmatrix& srmatrix::identity();
```

sets a calling matrix to be equal to identity matrix and returns a reference to the matrix changed. The function is *redefined* in the classes srbmatrix and srsmatrix. See also srmatrix. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (3);
srmatrix m(3);
m.randomize(0.,1.);

std::cout << m << std::endl;
std::cout << m.identity();
```

prints

```
9.423e-01 2.950e-01 8.429e-01
2.013e-01 3.250e-01 2.904e-01
7.920e-01 2.405e-02 7.801e-01

1.000e+00 0.000e+00 0.000e+00
0.000e+00 1.000e+00 0.000e+00
0.000e+00 0.000e+00 1.000e+00
```

2.8.51 vanish

Function

```
srmatrix& srmatrix::vanish();
```

sets every element of a calling matrix to be equal to zero and returns a reference to the matrix changed. This function is faster than `srmatrix::set(TR)` with zero operand passed. The function is *redefined* in the classes `srsmatrix` and `srbmatrix`. See also `srmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (3);
srmatrix m(3);
m.randomize(0.,1.);

std::cout << m << std::endl;
std::cout << m.vanish ();
```

prints

```
1.747e-01 7.563e-01 5.163e-01
9.657e-01 6.619e-01 8.036e-01
6.392e-01 6.658e-01 6.495e-01

0.000e+00 0.000e+00 0.000e+00
0.000e+00 0.000e+00 0.000e+00
0.000e+00 0.000e+00 0.000e+00
```

2.8.52 randomize

Function

```
srmatrix& srmatrix::randomize (TR dFrom, TR dTo);
```

fills a calling matrix with pseudo-random numbers distributed between `dFrom` and `dTo`. The function returns a reference to the matrix changed. See also `srmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (7);

srmatrix m(3);
m.randomize(-2.,3.);
std::cout << m;
```

prints

```
-1.6790979e+00 5.0233467e-02 -1.9559008e+00
-1.7987609e-01 -5.2092044e-01 -1.8211615e+00
6.8242439e-01 9.0688803e-01 -1.7891171e+00
```

2.9 scmatrix

This is end-user class encapsulating a square matrix in Euclidean space of complex numbers.

```
template <typename TR, typename TC>
class scmatrix : public cmatrix <TR,TC>, public SqMatrix <TR,TC> {
public:
    scmatrix ();
    explicit scmatrix (int nMN);
    scmatrix (TC* pD, int nMN);
    scmatrix (const scmatrix& m);
    explicit scmatrix (const cmatrix& m);
    explicit scmatrix (const cvector& v);
    explicit scmatrix (const srmatrix& m, bool bRealPart = true);
    scmatrix (const TR* pRe, const TR* pIm, int nMN);
    scmatrix (const srmatrix& mRe, const srmatrix& mIm);
    scmatrix (cmatrix& m, int nRow, int nCol, int nSize);
    TC& operator () (int im, int in) throw (cvmexception);
    TC operator () (int im, int in) const throw (cvmexception);
    cvector operator () (int i) throw (cvmexception);
    const cvector operator () (int i) const throw (cvmexception);
    cvector operator [] (int i) throw (cvmexception);
    const cvector operator [] (int i) const throw (cvmexception);
    const srmatrix real () const;
    const srmatrix imag () const;
    scmatrix& operator = (const scmatrix& m) throw (cvmexception);
    scmatrix& assign (const cvector& v);
    scmatrix& assign (const TC* pD);
    scmatrix& assign (int nRow, int nCol, const cmatrix& m)
                      throw (cvmexception);
    scmatrix& set (TC x);
    scmatrix& assign_real (const srmatrix& mRe) throw (cvmexception);
    scmatrix& assign_imag (const srmatrix& mIm) throw (cvmexception);
    scmatrix& set_real (TR d);
    scmatrix& set_imag (TR d);
    scmatrix& resize (int nNewMN) throw (cvmexception);
    scmatrix& operator << (const scmatrix& m) throw (cvmexception);
    scmatrix operator + (const scmatrix& m) const
                         throw (cvmexception);
    scmatrix operator - (const scmatrix& m) const
                         throw (cvmexception);
    scmatrix& sum (const scmatrix& m1,
```

```
               const scmatrix& m2) throw (cvmexception);
scmatrix& diff (const scmatrix& m1,
                const scmatrix& m2) throw (cvmexception);
scmatrix& operator += (const scmatrix& m) throw (cvmexception);
scmatrix& operator -= (const scmatrix& m) throw (cvmexception);
scmatrix operator - () const;
scmatrix& operator ++ ();
scmatrix& operator ++ (int);
scmatrix& operator -- ();
scmatrix& operator -- (int);
scmatrix operator * (TR d) const;
scmatrix operator / (TR d) const throw (cvmexception);
scmatrix operator * (TC z) const;
scmatrix operator / (TC z) const throw (cvmexception);
scmatrix& operator *= (TR d);
scmatrix& operator /= (TR d) throw (cvmexception);
scmatrix& operator *= (TC z);
scmatrix& operator /= (TC z) throw (cvmexception);
scmatrix& normalize ();
scmatrix operator ~ () const;
scmatrix& conj (const scmatrix& m) throw (cvmexception);
scmatrix& conj ();
cvector operator * (const cvector& v) const
                    throw (cvmexception);
cmatrix operator * (const cmatrix& m) const
                    throw (cvmexception);
scmatrix operator * (const scmatrix& m) const
                     throw (cvmexception);
scmatrix& operator *= (const scmatrix& m)
                       throw (cvmexception);
scmatrix& swap_rows (int n1, int n2) throw (cvmexception);
scmatrix& swap_cols (int n1, int n2) throw (cvmexception);
cvector solve (const cvector& vB) const throw (cvmexception);
cmatrix solve (const cmatrix& mB) const throw (cvmexception);
cvector solve (const cvector& vB, TR& dErr) const
               throw (cvmexception);
cmatrix solve (const cmatrix& mB, TR& dErr) const
               throw (cvmexception);
cvector solve_lu (const scmatrix& mLU, const int* pPivots,
                  const cvector& vB, TR& dErr) throw (cvmexception);
cvector solve_lu (const scmatrix& mLU, const int* pPivots,
                  const cvector& vB) throw (cvmexception);
cmatrix solve_lu (const scmatrix& mLU, const int* pPivots,
```

```
                  const cmatrix& mB, TR& dErr) throw (cvmexception);
    cmatrix solve_lu (const scmatrix& mLU, const int* pPivots,
                  const cmatrix& mB) throw (cvmexception);
    TC det () const throw (cvmexception);
    scmatrix& low_up (const scmatrix& m,
                  int* nPivots) throw (cvmexception);
    scmatrix low_up (int* nPivots) const throw (cvmexception);
    TR cond () const throw (cvmexception);
    scmatrix& inv (const scmatrix& mArg) throw (cvmexception);
    scmatrix inv () const throw (cvmexception);
    scmatrix& exp (const scmatrix& mArg, TR tol = cvmMachSp ())
               throw (cvmexception);
    scmatrix exp (TR tol = cvmMachSp ()) const throw (cvmexception);
    scmatrix& polynom (const scmatrix& m, const cvector& v)
                   throw (cvmexception);
    scmatrix polynom (const cvector& v) const
                  throw (cvmexception);
    cvector eig (scmatrix& mEigVect, bool bRightVect = true) const
             throw (cvmexception);
    cvector eig () const throw (cvmexception);
    scmatrix& cholesky (const schmatrix& m) throw (cvmexception);
    scmatrix& bunch_kaufman (const schmatrix& m,
                         int* pivots) throw (cvmexception);
    void qr(scmatrix& mQ, scmatrix& mR) const throw (cvmexception);
    scmatrix& identity ();
    scmatrix& vanish ();
    scmatrix& randomize_real (TR dFrom, TR dTo);
    scmatrix& randomize_imag (TR dFrom, TR dTo);
};
```

2.9.1 scmatrix ()

Constructor

```
scmatrix::scmatrix ();
```

creates an empty `scmatrix` object. See also `scmatrix`. Example:

```
using namespace cvm;

scmatrix m;
std::cout << m.msize() << std::endl
          << m.nsize() << std::endl
          << m.size() << std::endl;
m.resize(3);
std::cout << m;
```

prints

```
0
0
0
(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
```

2.9.2 scmatrix (int)

Constructor

```
explicit scmatrix::scmatrix (int nMN);
```

creates an $n \times n$ `scmatrix` object where n is passed in `nMN` parameter. The constructor throws an exception of type `cvmexception` in case of non-positive size passed or memory allocation failure. See also `scmatrix`. Example:

```
using namespace cvm;

scmatrix m (4);
std::cout << m.msize() << std::endl
          << m.nsize() << std::endl
          << m.size() << std::endl << m;
```

prints

```
4
4
16
(0,0) (0,0) (0,0) (0,0)
(0,0) (0,0) (0,0) (0,0)
(0,0) (0,0) (0,0) (0,0)
(0,0) (0,0) (0,0) (0,0)
```

2.9.3 scmatrix (TC*,int)

Constructor

```
scmatrix::scmatrix (TC* pD, int nMN);
```

creates an $n \times n$ `scmatrix` object where n is passed in `nMN` parameter. Unlike others, this constructor *does not allocate a memory*. It just shares a memory with an array pointed to by `pD`. See also `scmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
scmatrix m ((std::complex<double>*) a, 2);

std::cout << m << std::endl;
a[1] = 7.77;
std::cout << m;
```

prints

```
(1,2) (5,6)
(3,4) (7,8)

(1,7.77) (5,6)
(3,4) (7,8)
```

2.9.4 scmatrix (const scmatrix&)

Copy constructor

```
scmatrix::scmatrix (const scmatrix& m)
```

creates a `scmatrix` object as a copy of `m`. The constructor throws an exception of type `cvmexception` in case of memory allocation failure. See also `scmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
scmatrix m ((std::complex<double>*) a, 2);
scmatrix mc(m);

m(1,1) = std::complex<double>(7.77,7.77);
std::cout << m << std::endl << mc;
```

prints

```
(7.77,7.77) (5,6)
(3,4) (7,8)

(1,2) (5,6)
(3,4) (7,8)
```

2.9.5 scmatrix (const cmatrix&)

Constructor

```
explicit scmatrix::scmatrix (const cmatrix& m)
```

creates a `scmatrix` object as a copy of matrix `m`. It's assumed that $m \times n$ matrix `m` must have equal sizes, i.e. $m = n$ is satisfied. The constructor throws an exception of type `cvmexception` if this is not true or in case of memory allocation failure. Please note that this constructor is *not explicit* anymore. See also `scmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6.,
              7., 8., 9., 10., 11., 12.};
cmatrix m((std::complex<double>*) a, 2, 3);
std::cout << m << std::endl;

m.resize(3, 3);
scmatrix ms (m);
std::cout << ms;
```

prints

```
(1,2) (5,6) (9,10)
(3,4) (7,8) (11,12)

(1,2) (5,6) (9,10)
(3,4) (7,8) (11,12)
(0,0) (0,0) (0,0)
```

2.9.6 scmatrix (const cvector&)

Constructor

```
explicit scmatrix::scmatrix (const cvector& v);
```

creates a `scmatrix` object of size `v.size()` by `v.size()` and assigns vector `v` to its main diagonal. The constructor throws an exception of type `cvmexception` in case of memory allocation failure. See also `scmatrix`, `cvector`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
cvector v((std::complex<double>*) a, 4);
scmatrix m(v);
std::cout << m;
```

prints

```
(1,2) (0,0) (0,0) (0,0)
(0,0) (3,4) (0,0) (0,0)
(0,0) (0,0) (5,6) (0,0)
(0,0) (0,0) (0,0) (7,8)
```

2.9.7 scmatrix (const srmatrix&,bool)

Constructor

```
explicit scmatrix::scmatrix (const srmatrix& m, bool bRealPart = true);
```

creates a `scmatrix` object having the same dimension as real matrix `m` and copies the matrix `m` to its real part if `bRealPart` is `true` or to its imaginary part otherwise. See also `scmatrix`, `srmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
const srmatrix m(a, 3);
scmatrix mr(m), mi(m, false);
std::cout << mr << std::endl << mi;
```

prints

```
(1,0) (4,0) (7,0)
(2,0) (5,0) (8,0)
(3,0) (6,0) (9,0)

(0,1) (0,4) (0,7)
(0,2) (0,5) (0,8)
(0,3) (0,6) (0,9)
```

2.9.8 scmatrix (const TR*,const TR*,int)

Constructor

```
scmatrix::scmatrix (const TR* pRe, const TRl* pIm, int nMN);
```

creates a `scmatrix` object of size `nMN` by `nMN` and copies every element of arrays pointed to by `pRe` and `pIm` to a real and imaginary part of the matrix created respectively. Use `NULL` pointer to fill up appropriate part with zero values. The constructor throws an exception of type `cvmexception` in case of memory allocation failure. See also `scmatrix`. Example:

```
using namespace cvm;

double re[] = {1., 2., 3., 4.};
double im[] = {4., 3., 2., 1.};
scmatrix m(re, im, 2);
std::cout << m << std::endl;
re[0] = 7.777;
std::cout << m << std::endl;

const double rec[] = {1., 2., 3., 4.};
const scmatrix mc (rec, NULL, 2);
std::cout << mc;
```

prints

```
(1,4) (3,2)
(2,3) (4,1)

(1,4) (3,2)
(2,3) (4,1)

(1,0) (3,0)
(2,0) (4,0)
```

2.9.9 scmatrix (const srmatrix&, const srmatrix&)

Constructor

```
scmatrix::scmatrix (const srmatrix& mRe, const srmatrix& mIm);
```

creates a `scmatrix` object of the same size as `mRe` and `mIm` has (the constructor throws an exception of type `cvmexception` if `mRe` and `mIm` have different sizes) and copies matrices `mRe` and `mIm` to a real and imaginary part of the matrix created respectively. The constructor throws an exception of type `cvmexception` in case of memory allocation failure. See also `scmatrix`, `srmatrix`. Example:

```
using namespace cvm;

srmatrix mr(3), mi(3);
mr.set(1.);
mi.set(2.);
const scmatrix mc(mr, mi);
std::cout << mc;
```

prints

```
(1,2) (1,2) (1,2)
(1,2) (1,2) (1,2)
(1,2) (1,2) (1,2)
```

2.9.10 submatrix

Submatrix constructor

```
scmatrix::scmatrix (cmatrix& m, int nRow, int nCol, int nSize);
```

creates a `scmatrix` object as a *submatrix* of `m`. It means that the matrix object created shares a memory with some part of `m`. This part is defined by its upper left corner (parameters `nRow` and `nCol`, both are 1-based) and its size (parameter `nSize`). See also `scmatrix`. Example:

```
using namespace cvm;

cmatrix m(4,5);
scmatrix subm(m,2,2,2);
subm.set(std::complex<double>(1.,2.));
std::cout << m;
```

prints

```
(0,0) (0,0) (0,0) (0,0) (0,0)
(0,0) (1,2) (1,2) (0,0) (0,0)
(0,0) (1,2) (1,2) (0,0) (0,0)
(0,0) (0,0) (0,0) (0,0) (0,0)
```

2.9.11 operator (,)

Indexing operators

```
TC& scmatrix::operator () (int im, int in) throw (cvmexception);
TC scmatrix::operator () (int im, int in) const throw (cvmexception);
```

provide access to an element of a matrix. The first version of the operator is applicable to a non-constant object. This version returns an *l-value* in order to make possible write access to an element. Both operators are 1-based. The operators throw an exception of type `cvmexception` if some of parameters passed is outside of `[1,msize()]` range. The operators are *inherited* in the the class `scbmatrix` and *redefined* in the the class `schmatrix`. See also `scmatrix`, `Matrix::msize()`, `Matrix::nsize()`. Example:

```
using namespace cvm;

scmatrix m (3);
m.set(std::complex<double>(1.,2.));
std::cout << m(1,1) << std::endl;

m(2,2) = std::complex<double>(7.77,7.77);
std::cout << m;
```

prints

```
(1,2)
(1,2) (1,2) (1,2)
(1,2) (7.77,7.77) (1,2)
(1,2) (1,2) (1,2)
```

2.9.12 operator ()

Indexing operators

```
cvector scmatrix::operator () (int i) throw (cvmexception);
const rcector scmatrix::operator () (int i) const throw (cvmexception);
```

provide access to an i-th column of a matrix. The first version of the operator is applicable to a non-constant object and *returns an l-value*, i.e. the vector returned shares a memory with the i-th column of the matrix in order to make possible write access to it. The second version creates a *copy* of a column and therefore is *not an l-value*. Both operators are 1-based. The operators throw an exception of type `cvmexception` if the parameter `i` is outside of `[1,nsize()]` range. The operators are *redefined* in the the classes `scbmatrix` and `schmatrix`. See also `scmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
                  10., 11., 12., 13., 14., 15., 16., 17., 18.};
    const scmatrix m ((std::complex<double>*)a, 3);
    scmatrix ms(3);
    std::cout << m(2) << std::endl;

    ms(2) = m(3);
    std::cout << ms;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(7,8) (9,10) (11,12)

(0,0) (13,14) (0,0)
(0,0) (15,16) (0,0)
(0,0) (17,18) (0,0)
```

2.9.13 operator []

Indexing operators

```
cvector scmatrix::operator [] (int i) throw (cvmexception);
const cvector scmatrix::operator [] (int i) const throw (cvmexception);
```

provide access to an i-th row of a matrix. The first version of the operator is applicable to a non-constant object and *returns an l-value*, i.e. the vector returned shares a memory with the i-th row of the matrix in order to make possible write access to it. The second version creates a *copy* of a row and therefore is *not an l-value*. Both operators are 1-based. The operators throw an exception of type `cvmexception` if the parameter `i` is outside of `[1,msize()]` range. The operators are *redefined* in the the classes `scbmatrix` and `schmatrix`. See also `scmatrix`, `Matrix::msize()`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
                  10., 11., 12., 13., 14., 15., 16., 17., 18.};
    const scmatrix m ((std::complex<double>*)a, 3);
    scmatrix ms(3);
    std::cout << m[2] << std::endl;

    ms[2] = m[3];
    std::cout << ms;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(3,4) (9,10) (15,16)

(0,0) (0,0) (0,0)
(5,6) (11,12) (17,18)
(0,0) (0,0) (0,0)
```

2.9.14 real

Function

```
const srmatrix scmatrix::real () const;
```

creates an object of type `const srmatrix` as a real part of a calling matrix. Please note that, unlike cvector::real, this function creates new object *not sharing* a memory with a real part of the calling matrix, i.e. the matrix returned is *not an l-value*. The function is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `srmatrix`, `scmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
scmatrix m((std::complex<double>*) a, 2);
std::cout << m << std::endl << m.real();
```

prints

```
(1,2) (5,6)
(3,4) (7,8)

1 5
3 7
```

2.9.15 imag

Function

```
const srmatrix scmatrix::imag () const;
```

creates an object of type `const srmatrix` as an imaginary part of a calling matrix. Please note that, unlike cvector::imag, this function creates new object *not sharing* a memory with an imaginary part of the calling matrix, i.e. the matrix returned is *not an l-value*. The function is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `srmatrix`, `scmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
scmatrix m((std::complex<double>*) a, 2);
std::cout << m << std::endl << m.imag();
```

prints

```
(1,2) (5,6)
(3,4) (7,8)

2 6
4 8
```

2.9.16 operator = (const scmatrix&)

Operator

```
scmatrix& scmatrix::operator = (const scmatrix& m)
throw (cvmexception);
```

sets an every element of a calling matrix to a value of appropriate element of a matrix `m` and returns a reference to the matrix changed. The operator throws an exception of type `cvmexception` in case of different matrix sizes. The operator is *redefined* in the the classes `scbmatrix` and `schmatrix`. See also `scmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
    const scmatrix m1((std::complex<double>*) a, 2);
    scmatrix m2(2);

    m2 = m1;
    std::cout << m2;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(1,2) (5,6)
(3,4) (7,8)
```

2.9.17 assign (const TC*)

Function

```
scmatrix& scmatrix::assign (const cvector& v);
scmatrix& scmatrix::assign (const TC* pD);
```

sets every element of a calling matrix to a value of appropriate element of a vector `v` or an array pointed to by `pD` and returns a reference to the matrix changed. The function is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12., 13., 14., 15., 16., 17., 18.};
scmatrix m(3);
m.assign((std::complex<double>*) a);
std::cout << m;
```

prints

```
(1,2) (7,8) (13,14)
(3,4) (9,10) (15,16)
(5,6) (11,12) (17,18)
```

2.9.18 assign (int, int, const cmatrix&)

Function

```
scmatrix& scmatrix::assign (int nRow, int nCol, const cmatrix& m)
throw (cvmexception);
```

sets sub-matrix of a calling matrix beginning with 1-based row `nRow` and column `nCol` to a matrix `m` and returns a reference to the matrix changed. The function throws an exception of type `cvmexception` if `nRow` or `nCol` are not positive or matrix `m` doesn't fit. The function is *redefined* in the class `schmatrix`. See also `cmatrix`, `scmatrix`. Example:

```
using namespace cvm;

scmatrix m1(5);
cmatrix m2(2,3);
m1.set(std::complex<double>(1.,1.));
m2.set(std::complex<double>(2.,2.));
m1.assign(2,3,m2);
std::cout << m1;
```

prints

```
(1,1) (1,1) (1,1) (1,1) (1,1)
(1,1) (1,1) (2,2) (2,2) (2,2)
(1,1) (1,1) (2,2) (2,2) (2,2)
(1,1) (1,1) (1,1) (1,1) (1,1)
(1,1) (1,1) (1,1) (1,1) (1,1)
```

2.9.19 set (TC)

Function

```
scmatrix& scmatrix::set (TC x);
```

sets every element of a calling matrix to a value of parameter `x` and returns a reference to the matrix changed. Use `vanish` to set every element of a calling matrix to be equal to zero. The function is *redefined* in the classes `scbmatrix` and *not applicable* to objects of the class `srsmatrix` (i.e. an exception of type `cvmexception` would be thrown in case of using it for objects of this class). See also `scmatrix`. Example:

```
using namespace cvm;

scmatrix m(3);
m.set(std::complex<double>(1.,2.));
std::cout << m;
```

prints

```
(1,2) (1,2) (1,2)
(1,2) (1,2) (1,2)
(1,2) (1,2) (1,2)
```

2.9.20 assign_real

Function

```
scmatrix& scmatrix::assign_real (const srmatrix& mRe)
throw (cvmexception);
```

sets real part of every element of a calling matrix to a value of appropriate element of a matrix mRe and returns a reference to the matrix changed. The function throws an exception of type cvmexception in case of different sizes of the operands. The function is *redefined* in the classes scbmatrix and schmatrix. See also scmatrix and srmatrix.
Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

srmatrix m (3);
scmatrix mc(3);
m.randomize (0., 1.);

mc.assign_real(m);
std::cout << mc;
```

prints

```
(0.126835,0) (0.57271,0) (0.28312,0)
(0.784417,0) (0.541673,0) (0.663869,0)
```

2.9.21 assign_imag

Function

```
scmatrix& scmatrix::assign_imag (const srmatrix& mIm)
throw (cvmexception);
```

sets imaginary part of every element of a calling matrix to a value of appropriate element of a matrix `mIm` and returns a reference to the matrix changed. The function throws an exception of type `cvmexception` in case of different sizes of the operands. The function is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix` and `srmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

srmatrix m (3);
scmatrix mc(3);
m.randomize (0., 1.);

mc.assign_imag(m);
std::cout << mc;
```

prints

```
(0.00e+00,6.27e-01) (0.00e+00,1.57e-01) (0.00e+00,9.31e-01)
(0.00e+00,8.50e-01) (0.00e+00,6.11e-01) (0.00e+00,1.00e+00)
(0.00e+00,9.75e-01) (0.00e+00,7.38e-01) (0.00e+00,2.29e-01)
```

2.9.22 set_real

Function

```
scmatrix& scmatrix::set_real (TR d);
```

sets real part of every element of a calling matrix to a value of parameter `d` and returns a reference to the matrix changed. The function is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix`. Example:

```
using namespace cvm;

scmatrix m(3);
m.set_real(1.);
std::cout << m;
```

prints

```
(1,0) (1,0) (1,0)
(1,0) (1,0) (1,0)
(1,0) (1,0) (1,0)
```

2.9.23 set_imag

Function

```
scmatrix& scmatrix::set_imag (TR d);
```

sets imaginary part of every element of a calling matrix to a value of parameter `d` and returns a reference to the matrix changed. The function is *redefined* in the class `scbmatrix` and *not allowed* in `schmatrix`. See also `scmatrix`. Example:

```
using namespace cvm;

scmatrix m(3);
m.set_imag(1.);
std::cout << m;
```

prints

```
(0,1) (0,1) (0,1)
(0,1) (0,1) (0,1)
(0,1) (0,1) (0,1)
```

2.9.24 resize

Function

```
scmatrix& scmatrix::resize (int nNewMN);
throw (cvmexception);
```

changes a size of a calling matrix to `nNewMN` by `nNewMN` and returns a reference to the matrix changed. In case of increasing of its size, the matrix is filled up with zeroes. The function throws an exception of type `cvmexception` in case of negative size passed or memory allocation failure. The function is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
    scmatrix m((std::complex<double>*) a, 2);

    std::cout << m << std::endl;
    m.resize (3);
    std::cout << m;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(1,2) (5,6)
(3,4) (7,8)

(1,2) (5,6) (0,0)
(3,4) (7,8) (0,0)
(0,0) (0,0) (0,0)
```

2.9.25 operator <<

Operator

```
scmatrix& scmatrix::operator << (const scmatrix& m)
throw (cvmexception);
```

destroys a calling matrix, creates a new one as a copy of `m` and returns a reference to the matrix changed. The operator throws an exception of type `cvmexception` in case of memory allocation failure. The operator is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix`. Example:

```
using namespace cvm;

try {
    scmatrix m(3);
    scmatrix mc(1);
    m(1,2) = 1.;
    m(2,3) = 2.;
    std::cout << m << std::endl << mc << std::endl;

    mc << m;
    std::cout << mc;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(0,0) (1,0) (0,0)
(0,0) (0,0) (2,0)
(0,0) (0,0) (0,0)

(0,0)

(0,0) (1,0) (0,0)
(0,0) (0,0) (2,0)
(0,0) (0,0) (0,0)
```

2.9.26 operator +

Operator

```
scmatrix scmatrix::operator + (const scmatrix& m) const
throw (cvmexception);
```

creates an object of type `scmatrix` as a sum of a calling matrix and a matrix `m`. It throws an exception of type `cvmexception` in case of different sizes of the operands. The operator is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix::sum`, `scmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
    double b[] = {10., 20., 30., 40., 50., 60., 70., 80.};
    scmatrix m1((std::complex<double>*) a, 2);
    scmatrix m2((std::complex<double>*) b, 2);

    std::cout << m1 + m2 << std::endl << m1 + m1;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(11,22) (55,66)
(33,44) (77,88)

(2,4) (10,12)
(6,8) (14,16)
```

2.9.27 operator -

Operator

```
scmatrix scmatrix::operator - (const scmatrix& m) const
throw (cvmexception);
```

creates an object of type `scmatrix` as a difference of a calling matrix and a matrix `m`. It throws an exception of type `cvmexception` in case of different sizes of the operands. The operator is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix::diff`, `scmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
    double b[] = {10., 20., 30., 40., 50., 60., 70., 80.};
    scmatrix m1((std::complex<double>*) a, 2);
    scmatrix m2((std::complex<double>*) b, 2);

    std::cout << m2 - m1 << std::endl << m1 - m1;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(9,18) (45,54)
(27,36) (63,72)

(0,0) (0,0)
(0,0) (0,0)
```

2.9.28 sum

Function

```
scmatrix& scmatrix::sum (const scmatrix& m1, const scmatrix& m2)
throw (cvmexception);
```

assigns a result of addition of matrices m1 and m2 to a calling matrix and returns a reference to the matrix changed. It throws an exception of type cvmexception in case of different sizes of the operands. The function is *redefined* in the classes scbmatrix and schmatrix. See also scmatrix::operator + , scmatrix. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
                  10., 11., 12., 13., 14., 15., 16., 17., 18.};
    const scmatrix m1((std::complex<double>*)a, 3);
    scmatrix m2(3);
    scmatrix m(3);
    m2.set(std::complex<double>(1.,1.));

    std::cout << m.sum(m1, m2) << std::endl;
    std::cout << m.sum(m, m2);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(2,3) (8,9) (14,15)
(4,5) (10,11) (16,17)
(6,7) (12,13) (18,19)

(3,4) (9,10) (15,16)
(5,6) (11,12) (17,18)
(7,8) (13,14) (19,20)
```

2.9.29 diff

Function

```
scmatrix& scmatrix::diff (const scmatrix& m1, const scmatrix& m2)
throw (cvmexception);
```

assigns a result of subtraction of matrices `m1` and `m2` to a calling matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes of the operands. The function is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix::operator -` , `scmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
                  10., 11., 12., 13., 14., 15., 16., 17., 18.};
    const scmatrix m1((std::complex<double>*)a, 3);
    scmatrix m2(3);
    scmatrix m(3);
    m2.set(std::complex<double>(1.,1.));

    std::cout << m.diff(m1, m2) << std::endl;
    std::cout << m.diff(m, m2);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(0,1) (6,7) (12,13)
(2,3) (8,9) (14,15)
(4,5) (10,11) (16,17)

(-1,0) (5,6) (11,12)
(1,2) (7,8) (13,14)
(3,4) (9,10) (15,16)
```

2.9.30 operator +=

Operator

```
scmatrix& scmatrix::operator += (const scmatrix& m) throw (cvmexception);
```

adds a matrix `m` to a calling matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes of the operands. The operator is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix::operator +` , `scmatrix::sum`, `scmatrix`. Example:

```
using namespace cvm;

try {
    scmatrix m1(3);
    scmatrix m2(3);
    m1.set(std::complex<double>(1.,2.));
    m2.set(std::complex<double>(3.,4.));

    m1 += m2;
    std::cout << m1 << std::endl;

    // well, you can do this too, but temporary object would be created
    m2 += m2;
    std::cout << m2;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(4,6) (4,6) (4,6)
(4,6) (4,6) (4,6)
(4,6) (4,6) (4,6)

(6,8) (6,8) (6,8)
(6,8) (6,8) (6,8)
(6,8) (6,8) (6,8)
```

2.9.31 operator -=

Operator

```
scmatrix& scmatrix::operator -= (const scmatrix& m) throw (cvmexception);
```

subtracts a matrix m from a calling matrix and returns a reference to the matrix changed. It throws an exception of type cvmexception in case of different sizes of the operands. The operator is *redefined* in the classes scbmatrix and schmatrix. See also scmatrix::operator - , scmatrix::diff, scmatrix. Example:

```
using namespace cvm;

try {
    scmatrix m1(3);
    scmatrix m2(3);
    m1.set(std::complex<double>(1.,2.));
    m2.set(std::complex<double>(3.,4.));

    m1 -= m2;
    std::cout << m1 << std::endl;

    // well, you can do this too, but temporary object would be created
    m2 -= m2;
    std::cout << m2;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(-2,-2) (-2,-2) (-2,-2)
(-2,-2) (-2,-2) (-2,-2)
(-2,-2) (-2,-2) (-2,-2)

(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
```

2.9.32 operator - ()

Operator

```
scmatrix scmatrix::operator - () const throw (cvmexception);
```

creates an object of type scmatrix as a calling matrix multiplied by -1. The operator is *redefined* in the classes scbmatrix and schmatrix. See also scmatrix. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
scmatrix m((std::complex<double>*) a, 2);
std::cout << -m;
```

prints

```
(-1,-2) (-5,-6)
(-3,-4) (-7,-8)
```

2.9.33 operator ++

Operator

```
scmatrix& scmatrix::operator ++ ();
scmatrix& scmatrix::operator ++ (int);
```

adds identity matrix to a calling matrix and returns a reference to the matrix changed. The operator is *redefined* in the classes scbmatrix and schmatrix. See also scmatrix. Example:

```
using namespace cvm;

scmatrix m(3);
m.set(std::complex<double>(1.,1.));
m++;
std::cout << m << std::endl;
std::cout << ++m;
```

prints

```
(2,1) (1,1) (1,1)
(1,1) (2,1) (1,1)
(1,1) (1,1) (2,1)

(3,1) (1,1) (1,1)
(1,1) (3,1) (1,1)
(1,1) (1,1) (3,1)
```

2.9.34 operator --

Operator

```
scmatrix& scmatrix::operator -- ();
scmatrix& scmatrix::operator -- (int);
```

subtracts identity matrix from a calling matrix and returns a reference to the matrix changed. The operator is *redefined* in the classes scbmatrix and schmatrix. See also scmatrix. Example:

```
using namespace cvm;

scmatrix m(3);
m.set(std::complex<double>(1.,1.));
m--;
std::cout << m << std::endl;
std::cout << --m;
```

prints

```
(0,1) (1,1) (1,1)
(1,1) (0,1) (1,1)
(1,1) (1,1) (0,1)

(-1,1) (1,1) (1,1)
(1,1) (-1,1) (1,1)
(1,1) (1,1) (-1,1)
```

2.9.35 operator * (TR)

Operator

```
scmatrix scmatrix::operator * (TR d) const;
```

creates an object of type `scmatrix` as a product of a calling matrix and a real number `d`. The operator is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix::operator *=` , `scmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12., 13., 14., 15., 16., 17., 18.};
scmatrix m((std::complex<double>*) a, 3);
std::cout << m * 5.;
```

prints

```
(5,10) (35,40) (65,70)
(15,20) (45,50) (75,80)
(25,30) (55,60) (85,90)
```

2.9.36 operator / (TR)

Operator

```
scmatrix scmatrix::operator / (TR d) const throw (cvmexception);
```

creates an object of type `scmatrix` as a quotient of a calling matrix and a real number `d`. It throws an exception of type `cvmexception` if `d` has an absolute value equal or less than the smallest normalized positive number. The operator is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix::operator /=` , `scmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12., 13., 14., 15., 16., 17., 18.};
scmatrix m((std::complex<double>*) a, 3);
std::cout << m / 2.;
```

prints

```
(0.5,1) (3.5,4) (6.5,7)
(1.5,2) (4.5,5) (7.5,8)
(2.5,3) (5.5,6) (8.5,9)
```

2.9.37 operator * (TC)

Operator

```
scmatrix scmatrix::operator * (TC z) const;
```

creates an object of type scmatrix as a product of a calling matrix and a complex number z. The operator is *redefined* in the classes scbmatrix and schmatrix. See also scmatrix::operator *= , scmatrix. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12., 13., 14., 15., 16., 17., 18.};
scmatrix m((std::complex<double>*) a, 3);
std::cout << m * std::complex<double>(1.,1.);
```

prints

```
(-1,3) (-1,15) (-1,27)
(-1,7) (-1,19) (-1,31)
(-1,11) (-1,23) (-1,35)
```

2.9.38 operator / (TC)

Operator

```
scmatrix scmatrix::operator / (TC z) const throw (cvmexception);
```

creates an object of type `scmatrix` as a quotient of a calling matrix and a complex number `z`. It throws an exception of type `cvmexception` if `z` has an absolute value equal or less than the smallest normalized positive number. The operator is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix::operator /=` , `scmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12., 13., 14., 15., 16., 17., 18.};
scmatrix m((std::complex<double>*) a, 3);
std::cout << m / std::complex<double>(1.,1.);
```

prints

```
(1.5,0.5) (7.5,0.5) (13.5,0.5)
(3.5,0.5) (9.5,0.5) (15.5,0.5)
(5.5,0.5) (11.5,0.5) (17.5,0.5)
```

2.9.39 operator *= (TR)

Operator

```
scmatrix& scmatrix::operator *= (TR d);
```

multiplies a calling matrix by a real number `d` and returns a reference to the matrix changed. The operator is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix::operator *` , `scmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12., 13., 14., 15., 16., 17., 18.};
scmatrix m((std::complex<double>*) a, 3);
m *= 5.;
std::cout << m;
```

prints

```
(5,10) (35,40) (65,70)
(15,20) (45,50) (75,80)
(25,30) (55,60) (85,90)
```

2.9.40 operator /= (TR)

Operator

```
scmatrix& scmatrix::operator /= (TR d) throw (cvmexception);
```

divides a calling matrix by a real number d and returns a reference to the matrix changed. It throws an exception of type cvmexception if d has an absolute value equal or less than the smallest normalized positive number. The operator is *redefined* in the classes scbmatrix and schmatrix. See also scmatrix::operator / , scmatrix. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12., 13., 14., 15., 16., 17., 18.};
scmatrix m((std::complex<double>*) a, 3);
m /= 2.;
std::cout << m;
```

prints

```
(0.5,1) (3.5,4) (6.5,7)
(1.5,2) (4.5,5) (7.5,8)
(2.5,3) (5.5,6) (8.5,9)
```

2.9.41 operator *= (TC)

Operator

```
scmatrix& scmatrix::operator *= (TC z);
```

multiplies a calling matrix by a complex number z and returns a reference to the matrix changed. The operator is *redefined* in the classes scbmatrix and schmatrix. See also scmatrix::operator * , scmatrix. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12., 13., 14., 15., 16., 17., 18.};
scmatrix m((std::complex<double>*) a, 3);
m *= std::complex<double>(2.,1.);
std::cout << m;
```

prints

```
(0,5) (6,23) (12,41)
(2,11) (8,29) (14,47)
(4,17) (10,35) (16,53)
```

2.9.42 operator /= (TC)

Operator

```
scmatrix& scmatrix::operator /= (TC z) throw (cvmexception);
```

divides a calling matrix by a complex number z and returns a reference to the matrix changed. It throws an exception of type cvmexception if z has an absolute value equal or less than the smallest normalized positive number. The operator is *redefined* in the classes scbmatrix and schmatrix. See also scmatrix::operator / , scmatrix. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12., 13., 14., 15., 16., 17., 18.};
scmatrix m((std::complex<double>*) a, 3);
m /= std::complex<double>(2.,1.);
std::cout << m;
```

prints

```
(0.8,0.6) (4.4,1.8) (8,3)
(2,1) (5.6,2.2) (9.2,3.4)
(3.2,1.4) (6.8,2.6) (10.4,3.8)
```

2.9.43 normalize

Function

```
scmatrix& scmatrix::normalize ();
```

normalizes a calling matrix so its Euclidean norm becomes equal to 1 if it was greater than the smallest normalized positive number before the call (otherwise the function does nothing). The function is *redefined* in the classes scbmatrix and schmatrix. See also scmatrix. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (5);

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
scmatrix m((std::complex<double>*) a, 2);
m.normalize();
std::cout << m << m.norm() << std::endl;
```

prints

```
(7.00140e-002,1.40028e-001) (3.50070e-001,4.20084e-001)
(2.10042e-001,2.80056e-001) (4.90098e-001,5.60112e-001)
1.00000e+000
```

2.9.44 conjugation

Operator and functions

```
scmatrix scmatrix::operator ~ () const throw (cvmexception);
scmatrix& scmatrix::conj (const scmatrix& m) throw (cvmexception);
scmatrix& scmatrix::conj () throw (cvmexception);
```

encapsulate complex matrix conjugation. First operator creates an object of type `scmatrix` as a conjugated calling matrix (it throws an exception of type `cvmexception` in case of memory allocation failure). Second function sets a calling matrix to be equal to a matrix `m` conjugated (it throws an exception of type `cvmexception` in case of not appropriate sizes of the operands), third one makes it to be equal to conjugated itself (it also throws an exception of type `cvmexception` in case of memory allocation failure). The functions are *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12., 13., 14., 15., 16., 17., 18.};
scmatrix m((std::complex<double>*) a, 3);
scmatrix mc(3);
std::cout << m << std::endl << ~m << std::endl ;
mc.conj(m);
std::cout << mc << std::endl;
mc.conj();
std::cout << mc;
```

prints

```
(1,2) (7,8) (13,14)
(3,4) (9,10) (15,16)
(5,6) (11,12) (17,18)

(1,-2) (3,-4) (5,-6)
(7,-8) (9,-10) (11,-12)
(13,-14) (15,-16) (17,-18)

(1,-2) (3,-4) (5,-6)
(7,-8) (9,-10) (11,-12)
(13,-14) (15,-16) (17,-18)

(1,2) (7,8) (13,14)
(3,4) (9,10) (15,16)
(5,6) (11,12) (17,18)
```

2.9.45 operator * (const cvector&)

Operator

```
cvector scmatrix::operator * (const cvector& v) const
throw (cvmexception);
```

creates an object of type `cvector` as a product of a calling matrix and a vector `v`. It throws an exception of type `cvmexception` if the number of columns of the calling matrix differs from the size of the vector `v`. Use `cvector::mult` in order to get rid of a new object creation. The function is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix`, `cvector`. Example:

```
using namespace cvm;

try {
    scmatrix m(3);
    cvector v(3);
    m.set(std::complex<double>(1.,1.));
    v.set(std::complex<double>(1.,1.));
    std::cout << m * v;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(0,6) (0,6) (0,6)
```

2.9.46 operator * (const cmatrix&)

Operator

```
cmatrix scmatrix::operator * (const cmatrix& m) const
throw (cvmexception);
```

creates an object of type `cmatrix` as a product of a calling matrix and a matrix `m`. It throws an exception of type `cvmexception` if the number of columns of the calling matrix differs from the number of rows of the matrix `m`. Use `cmatrix::mult` in order to get rid of a new object creation. The operator is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `cmatrix`, `scmatrix`. Example:

```
using namespace cvm;

try {
    scmatrix ms(3);
    cmatrix m(3,2);
    ms.set(std::complex<double>(1.,1.));
    m.set(std::complex<double>(1.,1.));
    std::cout << ms * m;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(0,6) (0,6)
(0,6) (0,6)
(0,6) (0,6)
```

2.9.47 operator * (const scmatrix&)

Operator

```
scmatrix scmatrix::operator * (const scmatrix& m) const
throw (cvmexception);
```

creates an object of type `scmatrix` as a product of a calling matrix and a matrix `m`. It throws an exception of type `cvmexception` if the operands have different sizes. Use `cmatrix::mult` in order to get rid of a new object creation. The operator is *inherited* in the class `scbmatrix` and *redefined* in `schmatrix`. See also `scmatrix`. Example:

```
using namespace cvm;

try {
    scmatrix m1(3), m2(3);
    m1.set(std::complex<double>(1.,1.));
    m2.set(std::complex<double>(1.,1.));
    std::cout << m1 * m2;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(0,6) (0,6) (0,6)
(0,6) (0,6) (0,6)
(0,6) (0,6) (0,6)
```

2.9.48 operator *= (const scmatrix&)

Operator

```
scmatrix& scmatrix::operator *= (const scmatrix& m)
throw (cvmexception);
```

sets a calling matrix to be equal to a product of itself by a matrix `m` and returns a reference to the object it changes. The operator throws an exception of type `cvmexception` in case of different sizes of the operands. The operator is *inherited* in the class `scbmatrix` and *redefined* in `schmatrix`. See also `scmatrix`. Example:

```
using namespace cvm;

try {
    scmatrix m1(3), m2(3);
    m1.set(std::complex<double>(1.,2.));
    m2.set(std::complex<double>(2.,1.));
    m1 *= m2;
    m2 *= m2;
    std::cout << m1 << std::endl << m2;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(0,15) (0,15) (0,15)
(0,15) (0,15) (0,15)
(0,15) (0,15) (0,15)

(9,12) (9,12) (9,12)
(9,12) (9,12) (9,12)
(9,12) (9,12) (9,12)
```

2.9.49 swap_rows

Function

```
scmatrix& scmatrix::swap_rows (int n1, int n2) throw (cvmexception);
```

swaps two rows of a calling matrix and returns a reference to the matrix changed. `n1` and `n2` are the numbers of rows to be swapped, both are 1-based). The function throws an exception of type `cvmexception` if one of the parameters is outside of the range `[1,msize()]`. The function is *not applicable* to objects of the classes `scbmatrix` and `schmatrix` (i.e. an exception of type `cvmexception` would be thrown in case of using it for objects of those classes). See also `scmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
                  10., 11., 12., 13., 14., 15., 16., 17., 18.};
    scmatrix m ((std::complex<double>*)a, 3);

    std::cout << m << std::endl;
    std::cout << m.swap_rows(2,3);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(1,2) (7,8) (13,14)
(3,4) (9,10) (15,16)
(5,6) (11,12) (17,18)

(1,2) (7,8) (13,14)
(5,6) (11,12) (17,18)
(3,4) (9,10) (15,16)
```

2.9.50 swap_cols

Function

```
scmatrix& scmatrix::swap_cols (int n1, int n2) throw (cvmexception);
```

swaps two columns of a calling matrix and returns a reference to the matrix changed. `n1` and `n2` are the numbers of columns to be swapped, both are 1-based). The function throws an exception of type `cvmexception` if one of the parameters is outside of the range `[1,nsize()]`. The function is *not applicable* to objects of the classes `scbmatrix` and `schmatrix` (i.e. an exception of type `cvmexception` would be thrown in case of using it for objects of those classes). See also `scmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
                  10., 11., 12., 13., 14., 15., 16., 17., 18.};
    scmatrix m ((std::complex<double>*)a, 3);

    std::cout << m << std::endl;
    std::cout << m.swap_cols(2,3);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(1,2) (7,8) (13,14)
(3,4) (9,10) (15,16)
(5,6) (11,12) (17,18)

(1,2) (13,14) (7,8)
(3,4) (15,16) (9,10)
(5,6) (17,18) (11,12)
```

2.9.51 solve

Functions

```
cvector scmatrix::solve (const cvector& vB) const throw (cvmexception);
cmatrix scmatrix::solve (const cmatrix& mB) const throw (cvmexception);
cvector scmatrix::solve (const cvector& vB, TR& dErr) const
throw (cvmexception);
cmatrix scmatrix::solve (const cmatrix& mB, TR& dErr) const
throw (cvmexception);
```

return a solution of a linear equation of kind $Ax = b$ or $AX = B$ where A is a calling matrix. The first and the third versions solve the equation $Ax = b$ where vector b is passed in the parameter `vB` and the second and fourth versions solve the equation $AX = B$ where matrix B is passed in the parameter `mB`. The last two versions also set output parameter `dErr` to be equal to a norm of computation error. The functions throw an exception of type `cvmexception` in case of inappropriate sizes of the operands. The function is *inherited* in the classes `scbmatrix` and `schmatrix`. See also `cvector::solve`, `cmatrix::solve`, `scmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (5);
try {
    double re[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    double im[] = {-1., 2., -3., -4., 5., -6., 7., -8., 9.};
    scmatrix ma(re, im, 3);
    cmatrix  mb(3,2);
    cmatrix  mx(3,2);
    double dErr;

    mb(1).set(std::complex<double>(1.,1.));
    mb(1,2) = std::complex<double>(1.,1.);
    mb(2,2) = std::complex<double>(2.,2.);
    mb(3,2) = std::complex<double>(3.,3.);

    mx.solve (ma, mb, dErr);

    std::cout << mx << std::endl << ma * mx - mb;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(-6.25000e-002,-2.70833e-001) (-1.25000e-001,+1.12500e+000)
(+1.25000e-001,+2.08333e-001) (+2.50000e-001,-2.50000e-001)
(+6.25000e-002,+6.25000e-002) (+1.25000e-001,+1.25000e-001)

(+0.00000e+000,+0.00000e+000) (+0.00000e+000,+0.00000e+000)
(-1.11022e-016,+0.00000e+000) (-2.22045e-016,+0.00000e+000)
(+0.00000e+000,+0.00000e+000) (+0.00000e+000,+0.00000e+000)
```

2.9.52 solve_lu

Functions

```
cvector
scmatrix::solve_lu (const scmatrix& mLU, const int* pPivots,
                    const cvector& vB, TR& dErr) throw (cvmexception);
cvector
scmatrix::solve_lu (const scmatrix& mLU, const int* pPivots,
                    const cvector& vB) throw (cvmexception);
cmatrix
scmatrix::solve_lu (const scmatrix& mLU, const int* pPivots,
                    const cmatrix& mB, TR& dErr) throw (cvmexception);
cmatrix
scmatrix::solve_lu (const scmatrix& mLU, const int* pPivots,
                    const cmatrix& mB) throw (cvmexception);
```

create an object of type `cvector` or `cmatrix` as a solution x or X of the matrix linear equation $Ax = b$ or $AX = B$ respectively. Here A is a calling matrix, parameter `mLU` is LU factorization of the matrix A, parameter `pPivots` is an array of pivot numbers created while factorizing the matrix A and parameters `vB` and `mB` are the vector b and matrix B respectively. The first and third version also set output parameter `dErr` to be equal to a norm of computation error. These functions are useful when you need to solve few linear equations of kind $Ax = b$ or $AX = B$ with the same matrix A and different vectors b or matrices B. In such case you save on matrix A factorization since it's needed to be performed just one time. The functions throw exception of type `cvmexception` in case of inappropriate sizes of the operands or when the matrix A is close to cingular. The function is *inherited* in the classes `scbmatrix` and `schmatrix`. See also `cvector::solve`, `cmatrix`, `scmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    scmatrix ma(3);
    scmatrix mLU(3);
    cmatrix  mb1(3,2); cvector vb1(3);
    cmatrix  mb2(3,2); cvector vb2(3);
    cmatrix  mx1(3,2); cvector vx1(3);
    cmatrix  mx2(3,2); cvector vx2(3);
    iarray   nPivots(3);
    double   dErr = 0.;
    ma.randomize_real(-1.1,3.); ma.randomize_imag(-3.7,3.);
    mb1.randomize_real(-1.,3.); mb1.randomize_imag(-1.,3.);
```

```
    vb1.randomize_real(-2.,3.); vb1.randomize_imag(-3.,1.);
    mb2.randomize_real(-5.,1.); mb2.randomize_imag(-4.,1.);
    vb2.randomize_real(-1.,6.); vb1.randomize_imag(-4.,4.);
    mLU.low_up(ma, nPivots);
    mx1 = ma.solve_lu (mLU, nPivots, mb1, dErr);
    std::cout << mx1 << dErr << std::endl;
    mx2 = ma.solve_lu (mLU, nPivots, mb2);
    std::cout << mx2 << std::endl;;
    std::cout << ma * mx1 - mb1 << std::endl << ma * mx2 - mb2;
    vx1 = ma.solve_lu (mLU, nPivots, vb1, dErr);
    std::cout << vx1 << dErr << std::endl;
    vx2 = ma.solve_lu (mLU, nPivots, vb2);
    std::cout << vx2 << std::endl;;
    std::cout << ma * vx1 - vb1 << std::endl << ma * vx2 - vb2;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(-3.91e-001,-1.62e-001) (-3.17e-001,2.39e-001)
(-3.00e-001,-7.91e-001) (4.71e-001,-9.13e-001)
(-2.34e-001,1.09e+000) (1.10e-001,2.72e-001)
2.78e-015
(-2.60e-001,-5.48e-001) (-3.09e-002,-9.62e-001)
(8.77e-001,8.41e-001) (-6.02e-001,1.87e+000)
(4.20e-003,-9.72e-001) (6.18e-001,-5.64e-001)

(0.00e+000,-2.64e-016) (-1.11e-016,1.11e-016)
(0.00e+000,-2.22e-016) (2.22e-016,-4.44e-016)
(0.00e+000,0.00e+000) (-5.55e-017,0.00e+000)

(6.66e-016,-2.22e-016) (6.18e-016,0.00e+000)
(0.00e+000,1.11e-016) (0.00e+000,0.00e+000)
(0.00e+000,0.00e+000) (-4.44e-016,0.00e+000)
(2.61e-001,2.97e-001) (1.95e+000,-1.07e-001) (-5.51e-001,-1.03e-001)
1.96e-015
(1.26e-001,4.07e-001) (-4.82e-001,-1.14e-002) (2.59e-001,1.60e-001)

(1.11e-016,0.00e+000) (5.55e-017,0.00e+000) (1.11e-016,-2.22e-016)

(-1.11e-016,-7.61e-017) (-2.22e-016,-8.94e-017) (0.00e+000,4.07e-017)
```

2.9.53 det

Function

```
TC scmatrix::det () const throw (cvmexception);
```

returns a determinant of a calling matrix. It uses the LU factorization inside and may throw the same exceptions as the factorizer. The function is *inherited* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix`. Example:

```
using namespace cvm;

try {
    double re[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    double im[] = {-1., 2., -3., -4., 5., -6., 7., -8., 9.};
    const scmatrix m(re, im, 3);

    std::cout << m << std::endl << m.det() << std::endl;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(1,-1) (4,-4) (7,7)
(2,2) (5,5) (8,-8)
(3,-3) (6,-6) (9,9)

(-192,-192)
```

2.9.54 low_up

Functions

```
scmatrix& scmatrix::low_up (const scmatrix& m, int* nPivots)
throw (cvmexception);

scmatrix scmatrix::low_up (int* nPivots) const
throw (cvmexception);
```

compute the LU factorization of a calling matrix as

$$A = PLU$$

where P is a permutation matrix, L is a lower triangular matrix with unit diagonal elements and U is an upper triangular matrix. All the functions store the result as the matrix L without main diagonal combined with U. All the functions return pivot indices as an array of integers (it should support at least `msize()` elements) pointed to by `nPivots` so i-th row was interchanged with `nPivots[i]`-th row. The first version sets a calling matrix to be equal to the `m`'s LU factorization and the second one creates an object of type `scmatrix` as the calling matrix's LU factorization. The functions throw exception of type `cvmexception` in case of inappropriate sizes of the operands or when the matrix to be factorized is close to cingular. It is recommended to use `iarray` for pivot values. The function is *redefined* in the class `scbmatrix` and *inherited* in `schmatrix`. See also `scmatrix`. Example:

```
using namespace cvm;

try {
    double re[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    double im[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.1};
    scmatrix m(re, im, 3);
    scmatrix mLU(3), mLo(3), mUp(3);
    iarray naPivots(3);

    mLU.low_up (m, naPivots);

    mLo.identity ();
    mLo.diag(-2) = mLU.diag(-2);
    mLo.diag(-1) = mLU.diag(-1);
    mUp.diag(0) = mLU.diag(0);
    mUp.diag(1) = mLU.diag(1);
    mUp.diag(2) = mLU.diag(2);

    std::cout << mLo << std::endl << mUp
```

```
              << std::endl << naPivots << std::endl;

    mLU = mLo * mUp;
    for (int i = 3; i >= 1; i--) {
        mLU.swap_rows (i, naPivots[i]);
    }
    std::cout << mLU << std::endl << m - mLU;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(1,0) (0,0) (0,0)
(0.333333,0) (1,0) (0,0)
(0.666667,0) (0.5,0) (1,0)

(3,3) (6,6) (9,9.1)
(0,0) (2,2) (4,3.96667)
(0,0) (0,0) (-1.11022e-016,-0.05)

3 3 3

(1,1) (4,4) (7,7)
(2,2) (5,5) (8,8)
(3,3) (6,6) (9,9.1)

(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
```

2.9.55 cond

Function

```
TR scmatrix::cond () const throw (cvmexception);
```

returns a reciprocal of a condition number of a calling matrix A in the infinity-norm:

$$\kappa_\infty = \|A\|_\infty \|A^{-1}\|_\infty.$$

Less value returned means that matrix A is closer to cingular. Zero value returned means estimation underflow or that matrix A is cingular. The function is used for error analysis of systems of linear equations. The function throws exception of type `cvmexception` in case of LAPACK subroutines failure. The function is *inherited* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix::solve`, `scmatrix`. Example:

```
using namespace cvm;

try {
    double re[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    double im[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};

    scmatrix m(re, im, 3);
    std::cout << m.cond() << std::endl
              << m.det() << std::endl << std::endl;

    m(3,3) = std::complex<double>(9.,10.);
    std::cout << m.cond() << std::endl << m.det() << std::endl;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1.54198e-018
(1.33227e-015,-1.33227e-015)

0.0050679
(6,-1.33227e-015)
```

2.9.56 inv

Functions

```
scmatrix& scmatrix::inv (const scmatrix& m) throw (cvmexception);
scmatrix scmatrix::inv () const throw (cvmexception);
```

implement matrix inversion. The first version sets a calling matrix to be equal to `m` inverted and the second one creates an object of type `scmatrix` as inverted calling matrix. The functions throw exception of type `cvmexception` in case of inappropriate sizes of the operands or when the matrix to be inverted is close to cingular. The function is *redefined* in the class `schmatrix` and *inherited* in `scbmatrix`. See also `scmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    double re[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    double im[] = {1., 2., 3., 4., 5., 6., 7., 8., 10.};
    scmatrix m(re, im, 3);
    scmatrix mi(3);

    mi.inv (m);
    std::cout << mi << std::endl << mi * m - eye_complex(3);
    std::cout << std::endl << mi.inv() * mi - eye_complex(3);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(-8.33e-001,-1.67e-001) (6.67e-001,1.33e+000) (4.81e-016,-1.00e+000)
(3.33e-001,1.67e+000) (-1.67e-001,-3.83e+000) (-5.37e-016,2.00e+000)
(2.22e-016,-1.00e+000) (-4.44e-016,2.00e+000) (2.22e-016,-1.00e+000)

(-1.11e-016,1.11e-016) (0.00e+000,2.22e-016) (8.33e-017,1.26e-015)
(4.44e-016,-5.00e-016) (0.00e+000,7.77e-016) (7.22e-016,-1.15e-015)
(-1.11e-016,1.11e-016) (2.22e-016,-6.66e-016) (0.00e+000,1.11e-016)

(4.44e-016,4.44e-016) (-1.18e-015,1.33e-015) (6.66e-016,-7.77e-016)
(-6.85e-016,8.88e-016) (1.33e-015,3.16e-030) (-8.33e-016,-1.77e-030)
(-5.09e-016,6.66e-016) (7.96e-016,-1.78e-015) (-4.44e-016,1.11e-016)
```

2.9.57 exp

Functions

```
scmatrix& scmatrix::exp (const scmatrix& m, TR tol = cvmMachSp ())
throw (cvmexception);
scmatrix scmatrix::exp (TR tol = cvmMachSp ()) const
throw (cvmexception);
```

compute an exponent of a calling matrix using Padé approximation defined as

$$R_{pq}(z) = D_{pq}(z)^{-1} N_{pq}(z) = 1 + z + \cdots + z^p/p!\,,$$

where

$$N_{pq}(z) = \sum_{k=0}^{p} \frac{(p+q-k)!p!}{(p+q)!k!(q-k)!} z^k,$$
$$D_{pq}(z) = \sum_{k=0}^{q} \frac{(p+q-k)!p!}{(p+q)!k!(q-k)!} (-z)^k$$

along with the matrix normalizing as described in [2], p. 572. The functions use `ZMEXP` (or `CMEXP` for `float` version) FORTRAN subroutine implementing the algorithm. The first version sets the calling matrix to be equal to the exponent of `m` and returns the reference to the matrix changed. The second version creates an object of type `scmatrix` as the exponent of the calling matrix. The algorithm uses parameter `tol` as $\varepsilon(p, q)$ in order to choose constants p and q so that

$$\varepsilon(p, q) \geqslant 2^{3-(p+q)} \frac{p!q!}{(p+q)!(p+q+1)!}.$$

This parameter is equal to the largest relative spacing by default. The functions throw an exception of type `cvmexception` in case of inappropriate sizes of the operands or when LAPACK subroutine fails. The functions are *inherited* in the classes `scbmatrix` and `schmatrix`. The second version is *redefined* in `scbmatrix`. See also `scmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (15);
try {
    scmatrix m(2);
    m(1,1) = std::complex<double>(-49.,1.);
    m(1,2) = std::complex<double>(24.,1.);
    m(2,1) = std::complex<double>(-64.,1.);
    m(2,2) = std::complex<double>(31.,1.);
```

```
    m = m.exp();
    std::cout << m(1,1) << std::endl << "    "
              << m(1,2) << std::endl;
    std::cout << m(2,1) << std::endl << "    "
              << m(2,2) << std::endl;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(-4.508497580070061e-001,7.900659666739228e-001)
   (3.199576050798058e-001,-6.081804753524478e-001)
(-7.584316151932173e-001,1.666747485117903e+000)
   (5.295040786048336e-001,-1.278050361026397e+000)
```

MATLAB output:

```
  Column 1

    -4.508497580070262e-001 +7.900659666739607e-001i
    -7.584316151932523e-001 +1.666747485117982e+000i

  Column 2

     3.199576050798204e-001 -6.081804753524764e-001i
     5.295040786048589e-001 -1.278050361026457e+000i
```

2.9.58 polynomial

Functions

```
scmatrix& scmatrix::polynom (const scmatrix& m, const cvector& v)
throw (cvmexception);

scmatrix scmatrix::polynom (const cvector& v) const
throw (cvmexception);
```

compute a matrix polynomial defined as

$$p(A) = b_0 I + b_1 A + \cdots + b_q A^q$$

using the Horner's rule:

$$p(A) = \sum_{k=0}^{r} B_k (A^s)^k, \quad s = \text{floor}(\sqrt{q}\,),\ r = \text{floor}(q/s)$$

where

$$B_k = \begin{cases} \sum\limits_{i=0}^{s-1} b_{sk+i} A^i, & k = 0, 1, \ldots, r-1 \\ \sum\limits_{i=0}^{q-sr} b_{sr+i} A^i, & k = r. \end{cases}$$

See also [2], p. 568. The coefficients $b_0, b_1, \ldots, b_q$ are passed in the parameter v, where q is equal to `v.size()-1`, so the functions compute matrix polynomial equal to

$$\mathtt{v}[1] * \mathtt{I} + \mathtt{v}[2] * \mathtt{m} + \cdots + \mathtt{v}[\mathtt{v.size}()] * \mathtt{m}^{\mathtt{v.size}()-1}$$

The first version sets a calling matrix to be equal to the polynomial of `m` and the second one creates an object of type `scmatrix` as the polynomial of a calling matrix. The functions use `ZPOLY` (or `CPOLY` for `float` version) FORTRAN subroutine implementing the Horner's algorithm. The functions throw an exception of type `cvmexception` in case of inappropriate sizes of the operands. The functions are *inherited* in the class `scbmatrix` and *redefined* in `schmatrix`. See also `scmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (15);
try {
    const double re[] = {2.2, 1.3, 1.1, -0.9, 0.2,
                         -0.45, 45., -30., 10., 3., 0.};
    const double im[] = {0.5, -2, 0, 1, 3,
                         -3., 30., 0., -9., 0., 1.};
```

```
    const cvector v(re, im, 11);
    scmatrix m(2), mp(2);
    m(1,1) = std::complex<double>(0.1, -0.2);
    m(1,2) = std::complex<double>(0.1, -0.2);
    m(2,1) = std::complex<double>(0.5, -0.6);
    m(2,2) = std::complex<double>(0.3, -0.4);

    mp.polynom (m, v);
    std::cout << mp(1,1) << std::endl << "   "
              << mp(1,2) << std::endl;
    std::cout << mp(2,1) << std::endl << "   "
              << mp(2,2) << std::endl;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(2.485652665600000e+000,3.791263308800001e+000)
   (2.817786176000004e-001,2.301942860800001e+000)
(-8.835069888000001e-001,8.052028620800002e+000)
   (1.903009862400001e+000,6.666306188800002e+000)
```

MATLAB output:

```
  Column 1

     2.485652665600000e+000 +3.791263308800001e+000i
    -8.835069887999991e-001 +8.052028620800002e+000i

  Column 2

     2.817786176000000e-001 +2.301942860800001e+000i
     1.903009862399999e+000 +6.666306188800003e+000i
```

2.9.59 eig

Functions

```
cvector scmatrix::eig (scmatrix& mEigVect, bool bRightVect = true) const
throw (cvmexception);
```

```
cvector scmatrix::eig () const throw (cvmexception);
```

solve a nonsymmetric eigenvalue problem and return a complex vector with eigenvalues of a calling matrix. The first version sets the output parameter `mEigVect` to be equal to the square matrix containing right (if parameter `bRightVect` is true, which is default value) or left (if parameter `bRightVect` is false) eigenvectors as columns. All the functions throw an exception of type `cvmexception` in case of inappropriate sizes of the operands or in case of convergence error. The functions are *inherited* in the class `scbmatrix` and *redefined* in `schmatrix`. See also `cvector`, `scmatrix`. Example:

```
using namespace cvm;
try {
    double re[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    double im[] = {1., 2., 3., 4., 5., 6., 7., 8., 10.};
    scmatrix m(re, im, 3);
    scmatrix me(3);
    cvector vl(3);

    vl = m.eig (me);
    std::cout << vl << std::endl;
    std::cout.setf (std::ios::scientific | std::ios::left);
    std::cout.precision (2);
    std::cout << m * me(1) - me(1) * vl(1);
    std::cout << m * me(2) - me(2) * vl(2);
    std::cout << m * me(3) - me(3) * vl(3);
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(16.1096,16.7004) (-1.09351,-0.88358) (-0.0161248,0.183218)

(-9.44e-016,-3.55e-015) (-1.11e-016,-1.07e-014) (2.66e-015,-1.42e-014)
(-5.55e-016,-4.44e-016) (-1.80e-015,9.44e-016) (-2.00e-015,7.22e-016)
(9.92e-016,1.24e-015) (1.05e-015,2.78e-017) (1.64e-015,9.30e-016)
```

2.9.60 Cholesky

Function

```
scmatrix& scmatrix::cholesky (const schmatrix& m)
throw (cvmexception);
```

forms the Cholesky factorization of a hermitian positive-definite matrix A defined as

$$A = U^T U,$$

where U is upper triangular matrix. It utilizes one of ?POTRF routines of the LAPACK library. The function sets a calling matrix to be equal to the factorization of a hermitian positive-definite matrix m. The function throws an exception of type cvmexception in case of inappropriate sizes of the operands or in case of convergence error. See also scmatrix and schmatrix. Example:

```
using namespace cvm;

try {
    double r[] = {1., 2., 1., 2., 15., -1., 1., -1., 20.};
    double i[] = {0., -1., 2., 1., 0., 3., -2., -3., 0.};
    const schmatrix m(r, i, 3);
    scmatrix c(3);

    c.cholesky(m);

    std::cout << c << std::endl;
    std::cout << ~c * c - m;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(1,0) (2,1) (1,-2)
(0,-0) (3.16228,0) (-0.316228,0.632456)
(0,0) (-0,0) (3.80789,0)

(0,0) (0,0) (0,0)
(0,0) (1.77636e-015,0) (0,0)
(0,0) (0,0) (0,0)
```

2.9.61 Bunch-Kaufman

Function

```
scmatrix& scmatrix::bunch_kaufman (const schmatrix& m, int* pivots)
throw (cvmexception);
```

forms the Bunch-Kaufman factorization of a hermitian matrix (cited from the MKL library documentation):

$$A = PUDU^T P^T,$$

where A is the input matrix passed in parameter `m`, P is a permutation matrix, U and L are upper and lower triangular matrices with unit diagonal, and D is a symmetric block-diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks. U and L have 2-by-2 unit diagonal blocks corresponding to the 2-by-2 blocks of D. It utilizes one of `?SYTRF` routines of the LAPACK library. The function sets a calling matrix to be equal to the factorization of a hermitian positive-definite matrix `m`. The function throws an exception of type `cvmexception` in case of inappropriate sizes of the operands or in case of convergence error. See also `scmatrix` and `schmatrix`. The function is mostly designed to be used for subsequent calls of `?SYTRS`, `?SYCON` and `?SYTRI` routines of the LAPACK library. Currently it's used internally in `scmatrix::det` flow when argument is hermitian but not positive-definite matrix.

2.9.62 qr

Function

```
void scmatrix::qr (scmatrix& mQ, scmatrix& mR) const throw (cvmexception);
```

computes QR factorization as

$$M = QR$$

where M is a calling square matrix, unitary matrix Q and upper triangular matrix R are mQ and mR respectively. The function throws an exception of type cvmexception in case of inappropriate sizes of the operands. See also cmatrix, cmatrix::qr, scmatrix. Example:

```
using namespace cvm;

treal ar[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
treal ai[] = {1., -1., 2., -2., 3., -3., 4., -4., 5.};
const cvm::scmatrix m(ar, ai, 3);
cvm::scmatrix q(3), r(3);

m.qr(q,r);
std::cout << (eye_complex(3) - ~q * q).norm()
          << " " << (m - q * r).norm() << std::endl;
```

prints

```
+3.7957075e-16 +1.9641850e-15
```

2.9.63 identity

Function

```
scmatrix& scmatrix::identity();
```

sets a calling matrix to be equal to identity matrix and returns a reference to the matrix changed. The function is *redefined* in the classes scbmatrix and schmatrix. See also scmatrix. Example:

```
using namespace cvm;

scmatrix m(3);
m.randomize_real(0.,3.);
m.randomize_imag(-1.,2.);

std::cout << m << std::endl;
std::cout << m.identity();
```

prints

```
(1.31162,-0.52501) (2.8612,-0.531144) (1.31849,0.547838)
(1.19929,1.48253) (0.535417,0.41316) (0.459883,1.7019)
(0.415937,-0.491134) (2.0969,-0.218024) (0.545305,1.17866)

(1,0) (0,0) (0,0)
(0,0) (1,0) (0,0)
(0,0) (0,0) (1,0)
```

2.9.64 vanish

Function

```
scmatrix& scmatrix::vanish();
```

sets every element of a calling matrix to be equal to zero and returns a reference to the matrix changed. This function is faster than `scmatrix::set(TC)` with zero operand passed. The function is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix`. Example:

```
using namespace cvm;

scmatrix m(3);
m.randomize_real(0.,3.);
m.randomize_imag(-1.,2.);

std::cout << m << std::endl;
std::cout << m.vanish();
```

prints

```
(1.34834,-0.758385) (0.837825,-0.225532) (0.367687,0.791833)
(2.23698,-0.183142) (2.6878,0.741111) (0.495865,0.698904)
(0.584124,0.00491348) (1.31574,0.687643) (0.482131,1.66482)

(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
```

2.9.65 randomize_real

Function

```
scmatrix& scmatrix::randomize_real (TR dFrom, TR dTo);
```

fills a real part of a calling matrix with pseudo-random numbers distributed between `dFrom` and `dTo`. The function returns a reference to the matrix changed. The function is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
scmatrix m(3);
m.randomize_real(0.,3.);
std::cout << m;
```

prints

```
(1.56e+000,0.00e+000) (2.39e+000,0.00e+000) (2.41e+000,0.00e+000)
(3.73e-002,0.00e+000) (2.61e+000,0.00e+000) (1.36e+000,0.00e+000)
(2.71e+000,0.00e+000) (1.69e+000,0.00e+000) (2.68e+000,0.00e+000)
```

2.9.66 randomize_imag

Function

```
scmatrix& scmatrix::randomize_imag (TR dFrom, TR dTo);
```

fills an imaginary part of a calling matrix with pseudo-random numbers distributed between `dFrom` and `dTo`. The function returns a reference to the matrix changed. The function is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
scmatrix m(3);
m.randomize_imag(0.,3.);
std::cout << m;
```

prints

```
(0.00e+000,1.58e+000) (0.00e+000,2.38e+000) (0.00e+000,2.64e+000)
(0.00e+000,1.62e-002) (0.00e+000,4.26e-002) (0.00e+000,2.21e+000)
(0.00e+000,2.39e+000) (0.00e+000,6.95e-001) (0.00e+000,4.30e-001)
```

2.10 BandMatrix

This base class contains member functions common for band matrices. This class is not designed to be instantiated.

```
template <typename TR, typename TC>
class BandMatrix {
public:
    int lsize () const;
    int usize () const;
};
```

2.10.1 lsize

Function

```
int BandMatrix<TR,TC>::lsize () const;
```

returns a number lower sub-diagonals a calling matrix. The function is *inherited* in the classes srbmatrix and scbmatrix. See also BandMatrix. Example:

```
using namespace cvm;

srbmatrix m(4,2,1);
m.set(1.);
std::cout << m;
std::cout << m.msize() << " " << m.nsize() << " " << m.size() ;
std::cout << " " << m.lsize() << " " << m.usize() << std::endl;
```

prints

```
1 1 0 0
1 1 1 0
1 1 1 1
0 1 1 1
4 4 16 2 1
```

2.10.2 usize

Function

```
int BandMatrix<TR,TC>::usize () const;
```

returns a number upper sub-diagonals a calling matrix. The function is *inherited* in the classes srbmatrix and scbmatrix. See also BandMatrix. Example:

```
using namespace cvm;

srbmatrix m(4,2,1);
m.set(1.);
std::cout << m;
std::cout << m.msize() << " " << m.nsize() << " " << m.size() ;
std::cout << " " << m.lsize() << " " << m.usize() << std::endl;
```

prints

```
1 1 0 0
1 1 1 0
1 1 1 1
0 1 1 1
4 4 16 2 1
```

2.11 srbmatrix

This is end-user class encapsulating a square band matrix in Euclidean space of real numbers. This class utilizes band storage for its elements.

```
template <typename TR>
class srbmatrix : public srmatrix <TR>, public BandMatrix <TR,TR> {
public:
    srbmatrix ();
    explicit srbmatrix (int nMN);
    srbmatrix (int nMN);
    srbmatrix (TR* pD, int nMN, int nKL, int nKU);
    srbmatrix (const srbmatrix& m);
    srbmatrix (const rmatrix& m, int nKL, int nKU);
    explicit srbmatrix (const rvector& v);
    TR& operator () (int im, int in) throw (cvmexception);
    TR operator () (int im, int in) const throw (cvmexception);
    const rvector operator () (int i) const throw (cvmexception);
    const rvector operator [] (int i) const throw (cvmexception);
    srbmatrix& operator = (const srbmatrix& m) throw (cvmexception);
    srbmatrix& assign (const rvector& v);
    srbmatrix& assign (const TR* pD);
    srbmatrix& set (TR x);
    srbmatrix& resize (int nNewMN) throw (cvmexception);
    srbmatrix& resize_lu (int nNewKL, int nNewKU) throw (cvmexception);
    bool operator == (const srbmatrix& v) const;
    bool operator != (const srbmatrix& v) const;
    srbmatrix& operator << (const srbmatrix& m) throw (cvmexception);
    srbmatrix operator + (const srbmatrix& m) const
                          throw (cvmexception);
    srbmatrix operator - (const srbmatrix& m) const
                          throw (cvmexception);
    srbmatrix& sum (const srbmatrix& m1,
                    const srbmatrix& m2) throw (cvmexception);
    srbmatrix& diff (const srbmatrix& m1,
                     const srbmatrix& m2) throw (cvmexception);
    srbmatrix& operator += (const srbmatrix& m) throw (cvmexception);
    srbmatrix& operator -= (const srbmatrix& m) throw (cvmexception);
    srbmatrix operator - () const;
    srbmatrix& operator ++ ();
    srbmatrix& operator ++ (int);
    srbmatrix& operator -- ();
    srbmatrix& operator -- (int);
```

```
    srbmatrix operator * (TR d) const;
    srbmatrix operator / (TR d) const throw (cvmexception);
    srbmatrix& operator *= (TR d);
    srbmatrix& operator /= (TR d) throw (cvmexception);
    srbmatrix& normalize ();
    srbmatrix operator ~ () const throw (cvmexception);
    srbmatrix& transpose (const srbmatrix& m) throw (cvmexception);
    srbmatrix& transpose () throw (cvmexception);
    rvector operator * (const rvector& v) const throw (cvmexception);
    rmatrix operator * (const rmatrix& m) const throw (cvmexception);
    srmatrix operator * (const srmatrix& m) const throw (cvmexception);
    srbmatrix operator * (const srbmatrix& m) const throw (cvmexception);
    srbmatrix& low_up (const srbmatrix& m,
                       int* nPivots) throw (cvmexception);
    srbmatrix low_up (int* nPivots) const throw (cvmexception);
    srbmatrix& identity ();
    srbmatrix& vanish ();
    srbmatrix& randomize (TR dFrom, TR dTo);
};
```

2.11.1 srbmatrix ()

Constructor

```
srbmatrix::srbmatrix ();
```

creates an empty `srbmatrix` object. See also `srbmatrix`. Example:

```
using namespace cvm;

srbmatrix m;
std::cout << m.msize() << " " << m.nsize() << " " << m.size();
std::cout << " " << m.lsize() << " " << m.usize() << std::endl;

m.resize (3);
m.resize_lu(1,0);
m.set(1.);
std::cout << m;
```

prints

```
0 0 0 0 0
1 0 0
1 1 0
0 1 1
```

2.11.2 srbmatrix (int)

Constructor

```
explicit srbmatrix::srbmatrix (int nMN);
```

creates an $n \times n$ `srbmatrix` object where n is passed in `nMN` parameter. The matrix created is diagonal, i.e. $k_l = k_u = 0$. The constructor throws an exception of type `cvmexception` in case of non-positive size passed or memory allocation failure. See also `srbmatrix`. Example:

```
using namespace cvm;

srbmatrix m(4);
std::cout << m.msize() << " " << m.nsize() << " " << m.size();
std::cout << " " << m.lsize() << " " << m.usize() << std::endl;

m.set(1.);
std::cout << m;
```

prints

```
4 4 4 0 0
1 0 0 0
0 1 0 0
0 0 1 0
0 0 0 1
```

2.11.3 srbmatrix (int,int,int)

Constructor

```
srbmatrix::srbmatrix (int nMN, int nKL, int nKU);
```

creates an $n \times n$ `srbmatrix` object where n is passed in `nMN` parameter. The matrix created has `nKL` sub-diagonals and `nKU` super-diagonals. The constructor throws an exception of type `cvmexception` in case of non-positive size or negative number of sub-diagonals or super-diagonals passed or in case of memory allocation failure. See also `srbmatrix`. Example:

```
using namespace cvm;

srbmatrix m(5,1,1);
m.set(1.);
std::cout << m << std::endl
          << m.msize() << " " << m.nsize() << " " << m.size()
          << " " << m.lsize() << " " << m.usize() << std::endl;
```

prints

```
1 1 0 0 0
1 1 1 0 0
0 1 1 1 0
0 0 1 1 1
0 0 0 1 1

5 5 15 1 1
```

2.11.4 srbmatrix (TR*,int,int,int)

Constructor

```
srbmatrix::srbmatrix (TR* pD, int nMN, int nKL, int nKU);
```

creates an $n \times n$ `srbmatrix` object where n is passed in `nMN` parameter. The matrix created has `nKL` sub-diagonals and `nKU` super-diagonals. Unlike others, this constructor *does not allocate a memory*. It just shares a memory with an array pointed to by `pD`. Please note that this array must contain at least $(k_l + k_u + 1)n$ elements. The constructor throws an exception of type `cvmexception` in case of non-positive size or negative number of sub-diagonals passed or in case of memory allocation failure. See also `srbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
srbmatrix ml(a,4,1,0);
srbmatrix mu(a,4,0,1);
std::cout << ml << std::endl << mu;
```

prints

```
1 0 0 0
2 3 0 0
0 4 5 0
0 0 6 7

2 3 0 0
0 4 5 0
0 0 6 7
0 0 0 8
```

2.11.5 srbmatrix (const srbmatrix&)

Copy constructor

```
srbmatrix::srbmatrix (const srbmatrix& m);
```

creates a `srbmatrix` object as a copy of `m`. The constructor throws an exception of type `cvmexception` in case of memory allocation failure. See also `srbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
srbmatrix m(a,4,1,0);
srbmatrix mc(m);
m(1,1) = 7.77;
std::cout << m << std::endl << mc;
```

prints

```
7.77 0 0 0
2 3 0 0
0 4 5 0
0 0 6 7

1 0 0 0
2 3 0 0
0 4 5 0
0 0 6 7
```

2.11.6 srbmatrix (const rmatrix&,int,int)

Constructor

```
srbmatrix::srbmatrix (const rmatrix& m, int nKL, int nKU);
```

creates a `srbmatrix` object as a copy of "sliced" matrix `m`, i.e. it copies main diagonal, `nKL` sub-diagonals and `nKU` super-diagonals of the matrix `m`. It's assumed that $m \times n$ matrix `m` must have equal sizes, i.e. $m = n$ is satisfied. The constructor throws an exception of type `cvmexception` if this is not true or in case of memory allocation failure. See also `srbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12., 13., 14., 15., 16.};
srmatrix m(a,4);
srbmatrix mb(m,1,2);
std::cout << m << std::endl << mb;
```

prints

```
1 5 9 13
2 6 10 14
3 7 11 15
4 8 12 16

1 5 9 0
2 6 10 14
0 7 11 15
0 0 12 16
```

2.11.7 srbmatrix (const rvector&)

Constructor

```
explicit srbmatrix::srbmatrix (const rvector& v);
```

creates a `srbmatrix` object of size `v.size()` by `v.size()` and assigns vector `v` to its main diagonal. The constructor throws an exception of type `cvmexception` in case of memory allocation failure. See also `srbmatrix`, `rvector`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5.};
rvector v(a, 5);
srbmatrix m(v);
std::cout << m;
```

prints

```
1 0 0 0 0
0 2 0 0 0
0 0 3 0 0
0 0 0 4 0
0 0 0 0 5
```

2.11.8 operator (,)

Indexing operators

```
TR& srbmatrix::operator () (int im, int in) throw (cvmexception);
TR srbmatrix::operator () (int im, int in) const throw (cvmexception);
```

provide access to an element of a band matrix. The first version of the operator is applicable to a non-constant object. This version returns an *l-value* in order to make possible write access to an element. Only elements located on main diagonal or on non-zero sub- or super-diagonals are l-values. All other values located outside this area are not writable. Both operators are 1-based. The operators throw an exception of type `cvmexception` if some of parameters passed is outside of `[1,msize()]` range or in case of attempt to write to a non-writable element[11]. See also `srbmatrix`, `BandMatrix::lsize()` and `BandMatrix::usize()`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
    srbmatrix m (a,4,1,0);

    m(2,2) = 7.77;
    std::cout << m << std::endl;
    std::cout << m(3,2) << " " << m(1,4) << std::endl;

    m(1,3) = 7.77;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
1.00e+000 0.00e+000 0.00e+000 0.00e+000
2.00e+000 7.77e+000 0.00e+000 0.00e+000
0.00e+000 4.00e+000 5.00e+000 0.00e+000
0.00e+000 0.00e+000 6.00e+000 7.00e+000

4.00e+000 0.00e+000
Exception: Attempt to change a read-only element
```

[11] Here I use `type_proxy<T>` class originally described in [4], p. 217.

2.11.9 operator ()

Indexing operator

```
const rvector srbmatrix::operator () (int i) const throw (cvmexception);
```

provides access to an i-th column of a band matrix. Unlike srmatrix::operator (), this operator creates only a *copy* of a column and therefore it returns *not an l-value*. The operator is 1-based. The operator throws an exception of type `cvmexception` if the parameter `i` is outside of `[1,nsize()]` range. See also `srbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
srbmatrix m (a,4,1,0);

std::cout << m << std::endl;
std::cout << m(2);
```

prints

```
1 0 0 0
2 3 0 0
0 4 5 0
0 0 6 7

0 3 4 0
```

2.11.10 operator []

Indexing operator

```
const rvector srbmatrix::operator [] (int i) const throw (cvmexception);
```

provides access to an i-th row of a band matrix. Unlike srmatrix::operator [], this operator creates only a *copy* of a column and therefore it returns *not an l-value*. The operator is 1-based. The operator throws an exception of type `cvmexception` if the parameter `i` is outside of `[1,nsize()]` range. See also `srbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
srbmatrix m (a,4,1,0);

std::cout << m << std::endl;
std::cout << m[2];
```

prints

```
1 0 0 0
2 3 0 0
0 4 5 0
0 0 6 7

2 3 0 0
```

2.11.11 operator = (const srbmatrix&)

Operator

```
srbmatrix& srbmatrix::operator = (const srbmatrix& m)
throw (cvmexception);
```

sets an every element of a calling band matrix to a value of appropriate element of a band matrix m and returns a reference to the matrix changed. The operator throws an exception of type cvmexception in case of different matrix sizes or in case of different numbers of sub- or super-diagonals. See also srbmatrix. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
    const srbmatrix m1(a,4,1,0);
    srbmatrix m2(4,1,0);

    m2 = m1;
    std::cout << m2;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
1 0 0 0
2 3 0 0
0 4 5 0
0 0 6 7
```

2.11.12 assign (const TR*)

Function

```
srbmatrix& srbmatrix::assign (const rvector& v);
srbmatrix& srbmatrix::assign (const TR* pD);
```

sets every element of a calling band matrix to a value of appropriate element of a vector v or an array pointed to by pD and returns a reference to the matrix changed. Please note that this array must contain at least $(k_l + k_u + 1)n$ elements. See also srbmatrix. Example:

```
using namespace cvm;

const double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
srbmatrix m(4,0,1);

m.assign(a);
std::cout << m;
```

prints

```
2 3 0 0
0 4 5 0
0 0 6 7
0 0 0 8
```

2.11.13 set (TR)

Function

```
srbmatrix& srbmatrix::set (TR x);
```

sets every element of a calling band matrix to a value of parameter x and returns a reference to the matrix changed. Use vanish to set every element of a calling matrix to be equal to zero. See also srbmatrix. Example:

```
using namespace cvm;

srbmatrix m(8,2,1);
m.set(3.);
std::cout << m;
```

prints

```
3 3 0 0 0 0 0 0
3 3 3 0 0 0 0 0
3 3 3 3 0 0 0 0
0 3 3 3 3 0 0 0
0 0 3 3 3 3 0 0
0 0 0 3 3 3 3 0
0 0 0 0 3 3 3 3
0 0 0 0 0 3 3 3
```

2.11.14 resize

Function

```
srbmatrix& srbmatrix::resize (int nNewMN) throw (cvmexception);
```

changes a size of a calling band matrix to nNewMN by nNewMN and returns a reference to the matrix changed. In case of increasing of its size, the matrix is filled up with zeroes. This function doesn't change a number of sub- ore super-diagonals. Like any band matrix class member function, this function doesn't change non-referred elements. See number 8 appearing after resize in example below. The function throws an exception of type cvmexception in case of non-positive size passed or memory allocation failure. See also srbmatrix.resize_lu, srbmatrix. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
    srbmatrix m(a,4,1,0);
    std::cout << m << std::endl;
    m.resize (5);
    std::cout << m;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
1 0 0 0
2 3 0 0
0 4 5 0
0 0 6 7

1 0 0 0 0
2 3 0 0 0
0 4 5 0 0
0 0 6 7 0
0 0 0 8 0
```

2.11.15 resize_lu

Function

```
srbmatrix& srbmatrix::resize_lu (int nNewKL, int nNewKU)
throw (cvmexception);
```

changes a number of sub- and super-diagonals of a calling band matrix to `nNewKL` by `nNewKU` respectively and returns a reference to the matrix changed. In case of increasing of the numbers, the matrix is filled up with zeroes. The function throws an exception of type `cvmexception` in case of negative number passed or memory allocation failure. See also `srbmatrix::resize`, `srbmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
    srbmatrix m(a,4,1,0);
    std::cout << m << std::endl;
    m.resize_lu (0,1);
    m.diag(1).set(9.);
    std::cout << m;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
1 0 0 0
2 3 0 0
0 4 5 0
0 0 6 7

1 9 0 0
0 3 9 0
0 0 5 9
0 0 0 7
```

2.11.16 operator ==

Operator

```
bool srbmatrix::operator == (const srbmatrix& m) const;
```

compares a calling band matrix with a band matrix `m` and returns `true` if they have the same sizes, tha same numbers of sub- and super-diagonals and their appropriate elements differ by not more than the smallest normalized positive number. Returns `false` otherwise. See also `srbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4.};
srbmatrix m1(a,2,1,0);
srbmatrix m2(2,1,0);

std::cout << m1 << std::endl;

m2(1,1) = 1.;
m2(2,1) = 2.; m2(2,2) = 3.;

std::cout << (m1 == m2) << std::endl;
```

prints

```
1 0
2 3

1
```

2.11.17 operator !=

Operator

```
bool srbmatrix::operator != (const srbmatrix& m) const;
```

compares a calling band matrix with a band matrix `m` and returns `true` if they have different sizes, different numbers of sub- or super-diagonals or at least one of their appropriate elements differs by more than the smallest normalized positive number. Returns `false` otherwise. See also `srbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4.};
srbmatrix m1(a,2,1,0);
srbmatrix m2(2,1,0);

std::cout << m1 << std::endl;

m2(1,1) = 1.;
m2(2,1) = 2.; m2(2,2) = 3.;

std::cout << (m1 != m2) << std::endl;
```

prints

```
1 0
2 3

0
```

2.11.18 operator <<

Operator

```
srbmatrix& srbmatrix::operator << (const srbmatrix& m)
throw (cvmexception);
```

destroys a calling band matrix, creates a new one as a copy of `m` and returns a reference to the matrix changed. The operator throws an exception of type `cvmexception` in case of memory allocation failure. See also `srbmatrix`. Example:

```
using namespace cvm;

try {
    srbmatrix m(3,1,0);
    srmatrix mc(1);
    m(2,1) = 1.;
    m(2,2) = 2.;
    std::cout << m << std::endl << mc << std::endl;

    mc << m;
    std::cout << mc;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
0 0 0
1 2 0
0 0 0

0

0 0 0
1 2 0
0 0 0
```

2.11.19 operator +

Operator

```
srbmatrix srbmatrix::operator + (const srbmatrix& m) const
throw (cvmexception);
```

creates an object of type `srbmatrix` as a sum of a calling band matrix and a band matrix `m`. It throws an exception of type `cvmexception` in case of different sizes or different numbers of sub- or super-diagonals of the operands. See also `srbmatrix::sum`, `srbmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (1);
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
    double b[] = {10., 20., 30., 40., 50., 60., 70., 80.};
    srbmatrix m1(a,4,1,0);
    srbmatrix m2(b,4,1,0);

    std::cout << m1 + m2 << std::endl << m1 + m1;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
1.1e+01 0.0e+00 0.0e+00 0.0e+00
2.2e+01 3.3e+01 0.0e+00 0.0e+00
0.0e+00 4.4e+01 5.5e+01 0.0e+00
0.0e+00 0.0e+00 6.6e+01 7.7e+01

2.0e+00 0.0e+00 0.0e+00 0.0e+00
4.0e+00 6.0e+00 0.0e+00 0.0e+00
0.0e+00 8.0e+00 1.0e+01 0.0e+00
0.0e+00 0.0e+00 1.2e+01 1.4e+01
```

2.11.20 operator -

Operator

```
srbmatrix srbmatrix::operator - (const srbmatrix& m) const
throw (cvmexception);
```

creates an object of type `srbmatrix` as a difference of a calling band matrix and a band matrix `m`. It throws an exception of type `cvmexception` in case of different sizes or different numbers of sub- or super-diagonals of the operands. See also `srbmatrix::diff`, `srbmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (1);
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
    double b[] = {10., 20., 30., 40., 50., 60., 70., 80.};
    srbmatrix m1(a,4,1,0);
    srbmatrix m2(b,4,1,0);

    std::cout << m2 - m1 << std::endl << m1 - m1;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
9.0e+00 0.0e+00 0.0e+00 0.0e+00
1.8e+01 2.7e+01 0.0e+00 0.0e+00
0.0e+00 3.6e+01 4.5e+01 0.0e+00
0.0e+00 0.0e+00 5.4e+01 6.3e+01

0.0e+00 0.0e+00 0.0e+00 0.0e+00
0.0e+00 0.0e+00 0.0e+00 0.0e+00
0.0e+00 0.0e+00 0.0e+00 0.0e+00
0.0e+00 0.0e+00 0.0e+00 0.0e+00
```

2.11.21 sum

Function

```
srbmatrix& srbmatrix::sum (const srbmatrix& m1, const srbmatrix& m2)
throw (cvmexception);
```

assigns a result of addition of band matrices `m1` and `m2` to a calling band matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes or different numbers of sub- or super-diagonals of the operands. See also `srbmatrix::operator +` , `srbmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6.};
    const srbmatrix m1(a,3,1,0);
    srbmatrix m2(3,1,0);
    srbmatrix m(3,1,0);
    m2.set(1.);
    std::cout << m1 << std::endl << m2 << std::endl;
    std::cout << m.sum(m1, m2) << std::endl;
    std::cout << m.sum(m, m2);
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
1 0 0
2 3 0
0 4 5

1 0 0
1 1 0
0 1 1

2 0 0
3 4 0
0 5 6

3 0 0
4 5 0
0 6 7
```

2.11.22 diff

Function

```
srbmatrix& srbmatrix::diff (const srbmatrix& m1, const srbmatrix& m2)
throw (cvmexception);
```

assigns a result of subtraction of band matrices `m1` and `m2` to a calling band matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes or different numbers of sub- or super-diagonals of the operands. See also `srbmatrix::operator -` , `srbmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6.};
    const srbmatrix m1(a,3,1,0);
    srbmatrix m2(3,1,0);
    srbmatrix m(3,1,0);
    m2.set(1.);
    std::cout << m1 << std::endl << m2 << std::endl;
    std::cout << m.diff(m1, m2) << std::endl;
    std::cout << m.diff(m, m2);
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
1 0 0
2 3 0
0 4 5

1 0 0
1 1 0
0 1 1

0 0 0
1 2 0
0 3 4

-1 0 0
0 1 0
0 2 3
```

2.11.23 operator +=

Operator

```
srbmatrix& srbmatrix::operator += (const srbmatrix& m)
throw (cvmexception);
```

adds a band matrix `m` to a calling band matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes or different numbers of sub- or super-diagonals of the operands. See also `srbmatrix::operator +` , `srbmatrix::sum`, `srbmatrix`. Example:

```
using namespace cvm;

try {
    srbmatrix m1(4,0,1);
    srbmatrix m2(4,0,1);
    m1.set(1.);
    m2.set(2.);

    m1 += m2;
    std::cout << m1 << std::endl;

    // well, you can do this too, but temporary object would be created
    m2 += m2;
    std::cout << m2;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
3 3 0 0
0 3 3 0
0 0 3 3
0 0 0 3

4 4 0 0
0 4 4 0
0 0 4 4
0 0 0 4
```

2.11.24 operator -=

Operator

```
srbmatrix& srbmatrix::operator -= (const srbmatrix& m)
throw (cvmexception);
```

subtracts a band matrix `m` from a calling band matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes or different numbers of sub- or super-diagonals of the operands. See also `srbmatrix::operator -` , `srbmatrix::diff`, `srbmatrix`. Example:

```
using namespace cvm;

try {
    srbmatrix m1(4,0,1);
    srbmatrix m2(4,0,1);
    m1.set(1.);
    m2.set(4.);

    m2 -= m1;
    std::cout << m2 << std::endl;

    // well, you can do this too, but temporary object would be created
    m2 -= m2;
    std::cout << m2;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
3 3 0 0
0 3 3 0
0 0 3 3
0 0 0 3

0 0 0 0
0 0 0 0
0 0 0 0
0 0 0 0
```

2.11.25 operator - ()

Operator

```
srbmatrix srbmatrix::operator - () const throw (cvmexception);
```

creates an object of type `srbmatrix` as a calling band matrix multiplied by -1. See also `srbmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific |
                std::ios::left |
                std::ios::showpos);
std::cout.precision (1);
double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
srbmatrix m(a,4,1,0);

std::cout << -m;
```

prints

```
-1.0e+00 +0.0e+00 +0.0e+00 +0.0e+00
-2.0e+00 -3.0e+00 +0.0e+00 +0.0e+00
+0.0e+00 -4.0e+00 -5.0e+00 +0.0e+00
+0.0e+00 +0.0e+00 -6.0e+00 -7.0e+00
```

2.11.26 operator ++

Operator

```
srbmatrix& srbmatrix::operator ++ ();
srbmatrix& srbmatrix::operator ++ (int);
```

adds identity matrix to a calling band matrix and returns a reference to the matrix changed. See also srbmatrix. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
srbmatrix m (a,4,1,0);

m++;
std::cout << m << std::endl;
std::cout << ++m;
```

prints

```
2 0 0 0
2 4 0 0
0 4 6 0
0 0 6 8

3 0 0 0
2 5 0 0
0 4 7 0
0 0 6 9
```

2.11.27 operator --

Operator

```
srbmatrix& srbmatrix::operator -- ();
srbmatrix& srbmatrix::operator -- (int);
```

subtracts identity matrix from a calling band matrix and returns a reference to the matrix changed. See also srbmatrix. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
srbmatrix m (a,4,1,0);

m--;
std::cout << m << std::endl;
std::cout << --m;
```

prints

```
0 0 0 0
2 2 0 0
0 4 4 0
0 0 6 6

-1 0 0 0
2 1 0 0
0 4 3 0
0 0 6 5
```

2.11.28 operator * (TR)

Operator

```
srbmatrix srbmatrix::operator * (TR d) const;
```

creates an object of type `srbmatrix` as a product of a calling band matrix and a number `d`. See also `srbmatrix::operator *=` , `srbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
srbmatrix m (a,4,1,0);
std::cout << m * 5. << std::endl;
```

prints

```
5 0 0 0
10 15 0 0
0 20 25 0
0 0 30 35
```

2.11.29 operator / (TR)

Operator

```
srbmatrix srbmatrix::operator / (TR d) const throw (cvmexception);
```

creates an object of type `srbmatrix` as a quotient of a calling band matrix and a number `d`. It throws an exception of type `cvmexception` if `d` has an absolute value equal or less than the smallest normalized positive number. See also `srbmatrix::operator /=` , `srbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
srbmatrix m (a,4,1,0);
std::cout << m / 2. << std::endl;
```

prints

```
0.5 0 0 0
1 1.5 0 0
0 2 2.5 0
0 0 3 3.5
```

2.11.30 operator *= (TR)

Operator

`srbmatrix& srbmatrix::operator *= (TR d);`

multiplies a calling band matrix by a number `d` and returns a reference to the matrix changed. See also `srbmatrix::operator *` , `srbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
srbmatrix m (a,4,1,0);
m *= 2.;
std::cout << m;
```

prints

```
2 0 0 0
4 6 0 0
0 8 10 0
0 0 12 14
```

2.11.31 operator /= (TR)

Operator

```
srbmatrix& srbmatrix::operator /= (TR d) throw (cvmexception);
```

divides a calling matrix by a number `d` and returns a reference to the matrix changed. It throws an exception of type `cvmexception` if `d` has an absolute value equal or less than the smallest normalized positive number. See also `srbmatrix::operator /` , `srbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
srbmatrix m (a,4,1,0);
m /= 2.;
std::cout << m;
```

prints

```
0.5 0 0 0
1 1.5 0 0
0 2 2.5 0
0 0 3 3.5
```

2.11.32 normalize

Function

```
srbmatrix& srbmatrix::normalize ();
```

normalizes a calling band matrix so its Euclidean norm becomes equal to 1 if it was greater than the smallest normalized positive number before the call (otherwise the function does nothing). See also `srbmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (3);
double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
srbmatrix m (a,4,1,0);

m.normalize();
std::cout << m << m.norm() << std::endl;
```

prints

```
8.452e-02 0.000e+00 0.000e+00 0.000e+00
1.690e-01 2.535e-01 0.000e+00 0.000e+00
0.000e+00 3.381e-01 4.226e-01 0.000e+00
0.000e+00 0.000e+00 5.071e-01 5.916e-01
1.000e+00
```

2.11.33 transposition

Operator and functions

```
srbmatrix srbmatrix::operator ~ () const throw (cvmexception);
srbmatrix& srbmatrix::transpose (const srbmatrix& m) throw (cvmexception);
srbmatrix& srbmatrix::transpose () throw (cvmexception);
```

encapsulate band matrix transposition. First operator creates an object of type `srbmatrix` as a transposed calling band matrix (it throws an exception of type `cvmexception` in case of memory allocation failure). Second function sets a calling band matrix to be equal to a band matrix `m` transposed (it throws an exception of type `cvmexception` in case of not appropriate sizes of the operands or in case of memory allocation failure), third one makes it to be equal to transposed itself. See also `srbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
srbmatrix m (a,4,1,0);
srbmatrix mt(4,0,1);
std::cout << ~m << std::endl ;
mt.transpose(m);
std::cout << mt << std::endl;
mt.transpose();
std::cout << mt;
```

prints

```
1 2 0 0
0 3 4 0
0 0 5 6
0 0 0 7

1 2 0 0
0 3 4 0
0 0 5 6
0 0 0 7

1 0 0 0
2 3 0 0
0 4 5 0
0 0 6 7
```

2.11.34 operator * (const rvector&)

Operator

```
rvector srbmatrix::operator * (const rvector& v) const
throw (cvmexception);
```

creates an object of type `rvector` as a product of a calling band matrix and a vector `v`. It throws an exception of type `cvmexception` if the number of columns of the calling matrix differs from the size of the vector `v`. Use `rvector::mult` in order to get rid of a new object creation. See also `srbmatrix` and `rvector`. Example:

```
using namespace cvm;

try {
    srbmatrix m (4,1,0);
    rvector v(4);
    m.set(1.);
    v.set(1.);

    std::cout << m * v;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
1 2 2 2
```

2.11.35 operator * (const rmatrix&)

Operator

```
rmatrix srbmatrix::operator * (const rmatrix& m) const
throw (cvmexception);
```

creates an object of type `rmatrix` as a product of a calling band matrix and a matrix `m`. It throws an exception of type `cvmexception` if the number of columns of the calling matrix differs from the number of rows of the matrix `m`. Use `rmatrix::mult` in order to get rid of a new object creation. See also `rmatrix` and `srbmatrix`. Example:

```
using namespace cvm;

try {
    srbmatrix mb (4,1,0);
    rmatrix m(4,2);
    mb.set(1.);
    m.set(1.);

    std::cout << mb * m;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1 1
2 2
2 2
2 2
```

2.11.36 operator * (const srmatrix&)

Operator

```
srmatrix srbmatrix::operator * (const srmatrix& m) const
throw (cvmexception);
```

creates an object of type `srmatrix` as a product of a calling band matrix and a matrix `m`. It throws an exception of type `cvmexception` if the operands have different sizes. Use `rmatrix::mult` in order to get rid of a new object creation. See also `srmatrix` and `srbmatrix`. Example:

```
using namespace cvm;

try {
    srbmatrix mb(4,1,0);
    srmatrix ms(4);
    mb.set(1.);
    ms.set(1.);

    std::cout << mb * ms;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1 1 1 1
2 2 2 2
2 2 2 2
2 2 2 2
```

2.11.37 operator * (const srbmatrix&)

Operator

```
srbmatrix srbmatrix::operator * (const srbmatrix& m) const
throw (cvmexception);
```

creates an object of type `srbmatrix` as a product of a calling band matrix and band matrix `m`. It throws an exception of type `cvmexception` if the operands have different sizes. Use `rmatrix::mult` in order to get rid of a new object creation. See also `srbmatrix`. Example:

```
using namespace cvm;

try {
    srbmatrix m1(7,1,0);
    srbmatrix m2(7,1,1);
    m1.set(1.);
    m2.set(1.);

    std::cout << m1 * m2;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
1 1 0 0 0 0 0
2 2 1 0 0 0 0
1 2 2 1 0 0 0
0 1 2 2 1 0 0
0 0 1 2 2 1 0
0 0 0 1 2 2 1
0 0 0 0 1 2 2
```

2.11.38 low_up

Functions

```
srbmatrix&
srbmatrix::low_up (const srbmatrix& m, int* nPivots) throw (cvmexception);
srbmatrix
srbmatrix::low_up (int* nPivots) const throw (cvmexception);
```

compute the LU factorization of a calling band matrix as

$$A = PLU$$

where P is a permutation matrix, L is a lower triangular matrix with unit diagonal elements and U is an upper triangular matrix. All the functions store the result as the matrix L without main diagonal combined with U. All the functions return pivot indices as an array of integers (it should support at least `msize()` elements) pointed to by `nPivots` so i-th row was interchanged with `nPivots[i]`-th row. The first version sets a calling matrix to be equal to the `m`'s LU factorization and the second one creates an object of type `srbmatrix` as the calling band matrix's LU factorization. The functions throw exception of type `cvmexception` in case of inappropriate sizes of the operands or when the matrix to be factorized is close to cingular. The first version also changes numbers of super-diagonals to be equal to $k_l + k_u$ in order to keep the result of factorization. It is recommended to use `iarray` for pivot values. This function is provided mostly for solving multiple systems of linear equations using `srmatrix::solve_lu` function, See also `srbmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (4);
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
    srbmatrix ma(a,4,1,0);
    srbmatrix mLU(4,1,0);
    rmatrix  mb1(4,2); rvector vb1(4);
    rmatrix  mb2(4,2); rvector vb2(4);
    rmatrix  mx1(4,2); rvector vx1(4);
    rmatrix  mx2(4,2); rvector vx2(4);
    iarray   nPivots(4);
    double   dErr = 0.;
    mb1.randomize(-1.,3.); vb1.randomize(-2.,4.);
    mb2.randomize(-2.,5.); vb2.randomize(-3.,1.);

    mLU.low_up(ma, nPivots);
    mx1 = ma.solve_lu (mLU, nPivots, mb1, dErr);
```

```
    std::cout << mx1 << dErr << std::endl << std::endl;
    mx2 = ma.solve_lu (mLU, nPivots, mb2);
    std::cout << mx2 << std::endl;;
    std::cout << ma * mx1 - mb1 << std::endl << ma * mx2 - mb2;

    vx1 = ma.solve_lu (mLU, nPivots, vb1, dErr);
    std::cout << vx1 << dErr << std::endl;
    vx2 = ma.solve_lu (mLU, nPivots, vb2);
    std::cout << vx2 << std::endl;;
    std::cout << ma * vx1 - vb1 << std::endl << ma * vx2 - vb2;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
7.6327e-02 -4.7386e-01
-2.9523e-01 9.7577e-01
1.7288e-01 -3.5093e-01
1.0595e-01 4.7363e-01
1.1832e-15

3.1963e+00 4.8622e+00
-4.9904e-01 -2.6575e+00
8.2183e-01 2.3294e+00
-6.1693e-01 -1.8015e+00

0.0000e+00 0.0000e+00
0.0000e+00 -2.2204e-16
0.0000e+00 -4.4409e-16
0.0000e+00 0.0000e+00

0.0000e+00 0.0000e+00
8.8818e-16 0.0000e+00
0.0000e+00 -4.4409e-16
-4.4409e-16 4.4409e-16
7.8933e-01 7.0543e-01 -1.6338e-02 -2.6206e-01
1.4832e-15
-1.5505e+00 5.8987e-01 -8.4977e-01 7.3059e-01

0.0000e+00 0.0000e+00 0.0000e+00 0.0000e+00

-2.2204e-16 0.0000e+00 0.0000e+00 4.4409e-16
```

2.11.39 identity

Function

```
srbmatrix& srbmatrix::identity();
```

sets a calling band matrix to be equal to identity matrix and returns a reference to the matrix changed. The function doesn't change numbers of sub- and super-diagonals. See also srbmatrix. Example:

```
using namespace cvm;

srbmatrix m(4);
m.randomize(0.,1.);
std::cout << m << std::endl;
std::cout << m.identity();
```

prints

```
0.327372 0 0 0
0 0.955718 0 0
0 0 0.0960723 0
0 0 0 0.291818

1 0 0 0
0 1 0 0
0 0 1 0
0 0 0 1
```

2.11.40 vanish

Function

```
srbmatrix& srbmatrix::vanish();
```

sets every element of a calling band matrix to be equal to zero and returns a reference to the matrix changed. This function is faster than `srbmatrix::set(TR)` with zero operand passed. See also `srbmatrix`. Example:

```
using namespace cvm;

srbmatrix m(4,1,0);
m.randomize(0.,1.);
std::cout << m << std::endl;
std::cout << m.vanish();
```

prints

```
0.337138 0 0 0
0.101199 0.522843 0 0
0 0.258522 0.123447 0
0 0 0.591723 0.661489

0 0 0 0
0 0 0 0
0 0 0 0
0 0 0 0
```

2.11.41 randomize

Function

```
srbmatrix& srbmatrix::randomize (TR dFrom, TR dTo);
```

fills a calling band matrix with pseudo-random numbers distributed between `dFrom` and `dTo`. The function returns a reference to the matrix changed. See also `srbmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (7);
srbmatrix m(4,1,0);
m.randomize(0.,1.);
std::cout << m << std::endl;
```

prints

```
+3.4281442e-01 +0.0000000e+00 +0.0000000e+00 +0.0000000e+00
+7.9808954e-01 +5.9761345e-01 +0.0000000e+00 +0.0000000e+00
+0.0000000e+00 +1.1670278e-01 +6.5645314e-02 +0.0000000e+00
+0.0000000e+00 +0.0000000e+00 +4.2225410e-01 +7.5563829e-02
```

2.12 scbmatrix

This is end-user class encapsulating a square band matrix in Euclidean space of complex numbers. This class utilizes band storage for its elements.

```
template <typename TR, typename TC>
class scbmatrix : public scmatrix <TR,TC>, public BandMatrix <TR,TC> {
public:
    scbmatrix ();
    explicit scbmatrix (int nMN);
    scbmatrix (int nMN);
    scbmatrix (TC* pD, int nMN, int nKL, int nKU);
    scbmatrix (const scbmatrix& m);
    scbmatrix (const cmatrix& m, int nKL, int nKU);
    explicit scbmatrix (const cvector& v);
    explicit scbmatrix (const srbmatrix& m, bool bRealPart = true);
    scbmatrix (const srbmatrix& mRe, const srbmatrix& mIm);
    TC& operator () (int im, int in) throw (cvmexception);
    TC operator () (int im, int in) const throw (cvmexception);
    const cvector operator () (int i) const throw (cvmexception);
    const cvector operator [] (int i) const throw (cvmexception);
    const srbmatrix real () const;
    const srbmatrix imag () const;
    scbmatrix& operator = (const scbmatrix& m) throw (cvmexception);
    scbmatrix& assign (const cvector& v);
    scbmatrix& assign (const TC* pD);
    scbmatrix& set (TC z);
    scbmatrix& assign_real (const srbmatrix& mRe) throw (cvmexception);
    scbmatrix& assign_imag (const srbmatrix& mIm) throw (cvmexception);
    scbmatrix& set_real (TR d);
    scbmatrix& set_imag (TR d);
    scbmatrix& resize (int nNewMN) throw (cvmexception);
    scbmatrix& resize_lu (int nNewKL, int nNewKU) throw (cvmexception);
    bool operator == (const scbmatrix& v) const;
    bool operator != (const scbmatrix& v) const;
    scbmatrix& operator << (const scbmatrix& m) throw (cvmexception);
    scbmatrix operator + (const scbmatrix& m) const
                          throw (cvmexception);
    scbmatrix operator - (const scbmatrix& m) const
                          throw (cvmexception);
    scbmatrix& sum (const scbmatrix& m1,
                    const scbmatrix& m2) throw (cvmexception);
    scbmatrix& diff (const scbmatrix& m1,
```

```
                  const scbmatrix& m2) throw (cvmexception);
    scbmatrix& operator += (const scbmatrix& m) throw (cvmexception);
    scbmatrix& operator -= (const scbmatrix& m) throw (cvmexception);
    scbmatrix operator - () const;
    scbmatrix& operator ++ ();
    scbmatrix& operator ++ (int);
    scbmatrix& operator -- ();
    scbmatrix& operator -- (int);
    scbmatrix operator * (TR d) const;
    scbmatrix operator / (TR d) const throw (cvmexception);
    scbmatrix operator * (TC z) const;
    scbmatrix operator / (TC z) const throw (cvmexception);
    scbmatrix& operator *= (TR d);
    scbmatrix& operator /= (TR d) throw (cvmexception);
    scbmatrix& operator *= (TC z);
    scbmatrix& operator /= (TC z) throw (cvmexception);
    scbmatrix& normalize ();
    scbmatrix operator ~ () const;
    scbmatrix& conj (const scbmatrix& m) throw (cvmexception);
    scbmatrix& conj ();
    cvector operator * (const cvector& v) const throw (cvmexception);
    cmatrix operator * (const cmatrix& m) const throw (cvmexception);
    scmatrix operator * (const scmatrix& m) const throw (cvmexception);
    scbmatrix operator * (const scbmatrix& m) const throw (cvmexception);
    scbmatrix& low_up (const scbmatrix& m,
                       int* nPivots) throw (cvmexception);
    scbmatrix low_up (int* nPivots) const throw (cvmexception);
    scbmatrix& identity();
    scbmatrix& vanish ();
    scbmatrix& randomize_real (TR dFrom, TR dTo);
    scbmatrix& randomize_imag (TR dFrom, TR dTo);
};
```

2.12.1 scbmatrix ()

Constructor

```
scbmatrix::scbmatrix ();
```

creates an empty `scbmatrix` object. See also `scbmatrix`. Example:

```
using namespace cvm;

scbmatrix m;
std::cout << m.msize() << " " << m.nsize() << " " << m.size() ;
std::cout << " " << m.lsize() << " " << m.usize() << std::endl;
m.resize(3);
m.resize_lu(1,0);
m.set(std::complex<double>(1.,2.));
std::cout << m;
```

prints

```
0 0 0 0 0
(1,2) (0,0) (0,0)
(1,2) (1,2) (0,0)
(0,0) (1,2) (1,2)
```

2.12.2 scbmatrix (int)

Constructor

```
explicit scbmatrix::scbmatrix (int nMN);
```

creates an $n \times n$ `scbmatrix` object where n is passed in `nMN` parameter. The matrix created is diagonal, i.e. $k_l = k_u = 0$. The constructor throws an exception of type `cvmexception` in case of non-positive size passed or memory allocation failure. See also `scbmatrix`. Example:

```
using namespace cvm;

scbmatrix m(4);
std::cout << m.msize() << " " << m.nsize() << " " << m.size() ;
std::cout << " " << m.lsize() << " " << m.usize() << std::endl;
m.set(std::complex<double>(1.,2.));
std::cout << m;
```

prints

```
4 4 4 0 0
(1,2) (0,0) (0,0) (0,0)
(0,0) (1,2) (0,0) (0,0)
(0,0) (0,0) (1,2) (0,0)
(0,0) (0,0) (0,0) (1,2)
```

2.12.3 scbmatrix (int,int,int)

Constructor

```
scbmatrix::scbmatrix (int nMN, int nKL, int nKU);
```

creates an $n \times n$ `scbmatrix` object where n is passed in `nMN` parameter. The matrix created has `nKL` sub-diagonals and `nKU` super-diagonals. The constructor throws an exception of type `cvmexception` in case of non-positive size or negative number of sub-diagonals or super-diagonals passed or in case of memory allocation failure. See also `scbmatrix`. Example:

```
using namespace cvm;

scbmatrix m(4,1,1);
m.set(std::complex<double>(1.,2.));
std::cout << m << std::endl
          << m.msize() << " " << m.nsize() << " " << m.size()
          << " " << m.lsize() << " " << m.usize() << std::endl;
```

prints

```
(1,2) (1,2) (0,0) (0,0)
(1,2) (1,2) (1,2) (0,0)
(0,0) (1,2) (1,2) (1,2)
(0,0) (0,0) (1,2) (1,2)

4 4 12 1 1
```

2.12.4 scbmatrix (TC*,int,int,int)

Constructor

```
scbmatrix::scbmatrix (TC* pD, int nMN, int nKL, int nKU);
```

creates an $n \times n$ `scbmatrix` object where n is passed in `nMN` parameter. The matrix created has `nKL` sub-diagonals and `nKU` super-diagonals. Unlike others, this constructor *does not allocate a memory*. It just shares a memory with an array pointed to by `pD`. Please note that this array must contain at least $(k_l + k_u + 1)n$ elements. The constructor throws an exception of type `cvmexception` in case of non-positive size or negative number of sub-diagonals passed or in case of memory allocation failure. See also `scbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12., 13., 14., 15., 16.};
scbmatrix ml ((std::complex<double>*)a,4,1,0);
scbmatrix mu ((std::complex<double>*)a,4,0,1);
std::cout << ml << std::endl << mu;
```

prints

```
(1,2) (0,0) (0,0) (0,0)
(3,4) (5,6) (0,0) (0,0)
(0,0) (7,8) (9,10) (0,0)
(0,0) (0,0) (11,12) (13,14)

(3,4) (5,6) (0,0) (0,0)
(0,0) (7,8) (9,10) (0,0)
(0,0) (0,0) (11,12) (13,14)
(0,0) (0,0) (0,0) (15,16)
```

2.12.5 scbmatrix (const scbmatrix&)

Copy constructor

```
scbmatrix::scbmatrix (const scbmatrix& m);
```

creates a `scbmatrix` object as a copy of `m`. The constructor throws an exception of type `cvmexception` in case of memory allocation failure. See also `scbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12., 13., 14., 15., 16.};
scbmatrix m ((std::complex<double>*)a,4,1,0);
scbmatrix mc(m);
m(1,1) = 7.77;
std::cout << m << std::endl << mc;
```

prints

```
(7.77,0) (0,0) (0,0) (0,0)
(3,4) (5,6) (0,0) (0,0)
(0,0) (7,8) (9,10) (0,0)
(0,0) (0,0) (11,12) (13,14)

(1,2) (0,0) (0,0) (0,0)
(3,4) (5,6) (0,0) (0,0)
(0,0) (7,8) (9,10) (0,0)
(0,0) (0,0) (11,12) (13,14)
```

2.12.6 scbmatrix (const cmatrix&,int,int)

Constructor

```
scbmatrix::scbmatrix (const cmatrix& m, int nKL, int nKU);
```

creates a `scbmatrix` object as a copy of "sliced" matrix `m`, i.e. it copies main diagonal, `nKL` sub-diagonals and `nKU` super-diagonals of the matrix `m`. It's assumed that $m \times n$ matrix `m` must have equal sizes, i.e. $m = n$ is satisfied. The constructor throws an exception of type `cvmexception` if this is not true or in case of memory allocation failure. See also `scbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12., 13., 14., 15., 16., 17., 18.};
scmatrix m((std::complex<double>*)a,3);
scbmatrix mb(m,1,0);
std::cout << m << std::endl << mb;
```

prints

```
(1,2) (7,8) (13,14)
(3,4) (9,10) (15,16)
(5,6) (11,12) (17,18)

(1,2) (0,0) (0,0)
(3,4) (9,10) (0,0)
(0,0) (11,12) (17,18)
```

2.12.7 scbmatrix (const cvector&)

Constructor

```
explicit scbmatrix::scbmatrix (const cvector& v);
```

creates a `scbmatrix` object of size `v.size()` by `v.size()` and assigns vector `v` to its main diagonal. The constructor throws an exception of type `cvmexception` in case of memory allocation failure. See also `scbmatrix`, `cvector`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6.};
cvector v((std::complex<double>*)a,3);
scbmatrix m(v);
std::cout << m;
```

prints

```
(1,2) (0,0) (0,0)
(0,0) (3,4) (0,0)
(0,0) (0,0) (5,6)
```

2.12.8 scbmatrix (const srbmatrix&,bool)

Constructor

```
explicit scbmatrix::scbmatrix (const srbmatrix& m, bool bRealPart = true);
```

creates a `scbmatrix` object having the same dimension and the same numbers of sub- and super-diagonals as real matrix `m` and copies the matrix `m` to its real part if `bRealPart` is `true` or to its imaginary part otherwise. See also `scbmatrix`, `srbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
const srbmatrix m(a,4,1,0);
scbmatrix mr(m), mi(m, false);
std::cout << mr << std::endl << mi;
```

prints

```
(1,0) (0,0) (0,0) (0,0)
(2,0) (3,0) (0,0) (0,0)
(0,0) (4,0) (5,0) (0,0)
(0,0) (0,0) (6,0) (7,0)

(0,1) (0,0) (0,0) (0,0)
(0,2) (0,3) (0,0) (0,0)
(0,0) (0,4) (0,5) (0,0)
(0,0) (0,0) (0,6) (0,7)
```

2.12.9 scbmatrix (const srbmatrix&, const srbmatrix&)

Constructor

```
scbmatrix::scbmatrix (const srbmatrix& mRe, const srbmatrix& mIm);
```

creates a `scbmatrix` object of the same size as `mRe` and `mIm` has (the constructor throws an exception of type `cvmexception` if `mRe` and `mIm` have different sizes or different numbers of sub- or super-diagonals) and copies matrices `mRe` and `mIm` to a real and imaginary part of the matrix created respectively. The constructor throws an exception of type `cvmexception` in case of memory allocation failure. See also `scbmatrix`, `srbmatrix`. Example:

```
using namespace cvm;

srbmatrix mr(4,1,0), mi(4,1,0);
mr.set(1.);
mi.set(2.);
const scbmatrix m(mr,mi);
std::cout << m;
```

prints

```
(1,2) (0,0) (0,0) (0,0)
(1,2) (1,2) (0,0) (0,0)
(0,0) (1,2) (1,2) (0,0)
(0,0) (0,0) (1,2) (1,2)
```

2.12.10 operator (,)

Indexing operators

```
TC& scbmatrix::operator () (int im, int in) throw (cvmexception);
TC scbmatrix::operator () (int im, int in) const throw (cvmexception);
```

provide access to an element of a band matrix. The first version of the operator is applicable to a non-constant object. This version returns an *l-value* in order to make possible write access to an element. Only elements located on main diagonal or on non-zero sub- or super-diagonals are l-values. All other values located outside this area are not writable. Both operators are 1-based. The operators throw an exception of type `cvmexception` if some of parameters passed is outside of `[1,msize()]` range or in case of attempt to write to a non-writable element[12]. See also `scbmatrix`, `BandMatrix::lsize()` and `BandMatrix::usize()`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9., 10., 11., 12.};
    srbmatrix m (a,3,1,0);

    m(2,1) = 7.77;
    std::cout << m << std::endl;
    std::cout << m(3,2) << " " << m(1,3) << std::endl;

    m(1,3) = 7.77;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
1.00e+00 0.00e+00 0.00e+00
7.77e+00 3.00e+00 0.00e+00
0.00e+00 4.00e+00 5.00e+00

4.00e+00 0.00e+00
Exception: Attempt to change a read-only element
```

[12] Here I use `type_proxy<T>` class originally described in [4], p. 217.

2.12.11 operator ()

Indexing operator

```
const cvector scbmatrix::operator () (int i) const throw (cvmexception);
```

provides access to an i-th column of a band matrix. Unlike scmatrix::operator (), this operator creates only a *copy* of a column and therefore it returns *not an l-value*. The operator is 1-based. The operator throws an exception of type `cvmexception` if the parameter `i` is outside of `[1,nsize()]` range. See also `scbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9., 10., 11., 12.};
scbmatrix m ((std::complex<double>*)a,3,1,0);
std::cout << m << std::endl;
std::cout << m(2);
```

prints

```
(1,2) (0,0) (0,0)
(3,4) (5,6) (0,0)
(0,0) (7,8) (9,10)

(0,0) (5,6) (7,8)
```

2.12.12 operator []

Indexing operator

```
const cvector scbmatrix::operator [] (int i) const throw (cvmexception);
```

provides access to an i-th row of a band matrix. Unlike scmatrix::operator [], this operator creates only a *copy* of a column and therefore it returns *not an l-value*. The operator is 1-based. The operator throws an exception of type `cvmexception` if the parameter `i` is outside of `[1,nsize()]` range. See also `scbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9., 10., 11., 12.};
scbmatrix m ((std::complex<double>*)a,3,1,0);
std::cout << m << std::endl;
std::cout << m[3];
```

prints

```
(1,2) (0,0) (0,0)
(3,4) (5,6) (0,0)
(0,0) (7,8) (9,10)

(0,0) (7,8) (9,10)
```

2.12.13 real

Function

```
const srbmatrix scbmatrix::real () const;
```

creates an object of type `const srbmatrix` as a real part of a calling band matrix. Please note that, unlike cvector::real, this function creates new object *not sharing* a memory with a real part of the calling matrix, i.e. the matrix returned is *not an l-value*. See also `srbmatrix`, `scbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9., 10., 11., 12.};
scbmatrix m ((std::complex<double>*)a,3,1,0);
std::cout << m << std::endl;
std::cout << m.real();
```

prints

```
(1,2) (0,0) (0,0)
(3,4) (5,6) (0,0)
(0,0) (7,8) (9,10)

1 0 0
3 5 0
0 7 9
```

2.12.14 imag

Function

```
const srbmatrix scbmatrix::imag () const;
```

creates an object of type `const srbmatrix` as an imaginary part of a calling band matrix. Please note that, unlike cvector::imag, this function creates new object *not sharing* a memory with an imaginary part of the calling matrix, i.e. the matrix returned is *not an l-value*. See also `srbmatrix`, `scbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9., 10., 11., 12.};
scbmatrix m ((std::complex<double>*)a,3,1,0);
std::cout << m << std::endl;
std::cout << m.imag();
```

prints

```
(1,2) (0,0) (0,0)
(3,4) (5,6) (0,0)
(0,0) (7,8) (9,10)

2 0 0
4 6 0
0 8 10
```

2.12.15 operator = (const scbmatrix&)

Operator

```
scbmatrix& scbmatrix::operator = (const scbmatrix& m)
throw (cvmexception);
```

sets an every element of a calling band matrix to a value of appropriate element of a band matrix m and returns a reference to the matrix changed. The operator throws an exception of type cvmexception in case of different matrix sizes or in case of different numbers of sub- or super-diagonals. See also scbmatrix. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9., 10., 11., 12.};
    scbmatrix m1((std::complex<double>*)a,3,1,0);
    scbmatrix m2(3,1,0);

    m2 = m1;
    std::cout << m2;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(1.00e+00,2.00e+00) (0.00e+00,0.00e+00) (0.00e+00,0.00e+00)
(3.00e+00,4.00e+00) (5.00e+00,6.00e+00) (0.00e+00,0.00e+00)
(0.00e+00,0.00e+00) (7.00e+00,8.00e+00) (9.00e+00,1.00e+01)
```

2.12.16 assign (const TC*)

Function

```
scbmatrix& scbmatrix::assign (const cvector& v);
scbmatrix& scbmatrix::assign (const TC* pD);
```

sets every element of a calling band matrix to a value of appropriate element of a vector v or an array pointed to by pD and returns a reference to the matrix changed. Please note that this array must contain at least $(k_l + k_u + 1)n$ elements. See also scbmatrix. Example:

```
using namespace cvm;

const double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9., 10., 11., 12.};
scbmatrix m(3,0,1);
m.assign((const std::complex<double>*)a);
std::cout << m;
```

prints

```
(3,4) (5,6) (0,0)
(0,0) (7,8) (9,10)
(0,0) (0,0) (11,12)
```

2.12.17 set (TC)

Function

```
scbmatrix& scbmatrix::set (TC z);
```

sets every element of a calling band matrix to a value of parameter z and returns a reference to the matrix changed. Use vanish to set every element of a calling matrix to be equal to zero. See also scbmatrix. Example:

```
using namespace cvm;

scbmatrix m(4,1,0);
m.set(std::complex<double>(1.,2.));
std::cout << m;
```

prints

```
(1,2) (0,0) (0,0) (0,0)
(1,2) (1,2) (0,0) (0,0)
(0,0) (1,2) (1,2) (0,0)
(0,0) (0,0) (1,2) (1,2)
```

2.12.18 assign_real

Function

```
scbmatrix& scbmatrix::assign_real (const srbmatrix& mRe)
throw (cvmexception);
```

sets real part of every element of a calling band matrix to a value of appropriate element of a band matrix mRe and returns a reference to the matrix changed. The function throws an exception of type cvmexception in case of different sizes of the operands. See also scbmatrix and srbmatrix. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
srbmatrix m (3,0,1);
scbmatrix mc(3,0,1);
m.randomize (0., 1.);

mc.assign_real(m);
std::cout << mc;
```

prints

```
(5.44e-01,0.00e+00) (5.48e-02,0.00e+00) (0.00e+00,0.00e+00)
(0.00e+00,0.00e+00) (3.66e-01,0.00e+00) (3.49e-01,0.00e+00)
(0.00e+00,0.00e+00) (0.00e+00,0.00e+00) (8.00e-01,0.00e+00)
```

2.12.19 assign_imag

Function

```
scbmatrix& scbmatrix::assign_imag (const srbmatrix& mIm)
throw (cvmexception);
```

sets imaginary part of every element of a calling band matrix to a value of appropriate element of a bandmatrix `mIm` and returns a reference to the matrix changed. The function throws an exception of type `cvmexception` in case of different sizes of the operands. See also `scbmatrix` and `srbmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
srbmatrix m (3,0,1);
scbmatrix mc(3,0,1);
m.randomize (0., 1.);

mc.assign_imag(m);
std::cout << mc;
```

prints

```
(0.00e+00,5.53e-01) (0.00e+00,2.16e-01) (0.00e+00,0.00e+00)
(0.00e+00,0.00e+00) (0.00e+00,1.57e-01) (0.00e+00,1.12e-01)
(0.00e+00,0.00e+00) (0.00e+00,0.00e+00) (0.00e+00,7.03e-01)
```

2.12.20 set_real

Function

```
scbmatrix& scbmatrix::set_real (TR d);
```

sets real part of every element of a calling band matrix to a value of parameter `d` and returns a reference to the matrix changed. See also `scbmatrix`. Example:

```
using namespace cvm;

scbmatrix m(4,0,1);
m.set_real(1.);
std::cout << m;
```

prints

```
(1,0) (1,0) (0,0) (0,0)
(0,0) (1,0) (1,0) (0,0)
(0,0) (0,0) (1,0) (1,0)
(0,0) (0,0) (0,0) (1,0)
```

2.12.21 set_imag

Function

```
scbmatrix& scbmatrix::set_imag (TR d);
```

sets imaginary part of every element of a calling band matrix to a value of parameter `d` and returns a reference to the matrix changed. See also `scbmatrix`. Example:

```
using namespace cvm;

scbmatrix m(4,0,1);
m.set_imag(1.);
std::cout << m;
```

prints

```
(0,1) (0,1) (0,0) (0,0)
(0,0) (0,1) (0,1) (0,0)
(0,0) (0,0) (0,1) (0,1)
(0,0) (0,0) (0,0) (0,1)
```

2.12.22 resize

Function

```
scbmatrix& scbmatrix::resize (int nNewMN) throw (cvmexception);
```

changes a size of a calling band matrix to nNewMN by nNewMN and returns a reference to the matrix changed. In case of increasing of its size, the matrix is filled up with zeroes. This function doesn't change a number of sub- ore super-diagonals. Like any band matrix class member function, this function doesn't change non-referred elements. See number (11,12) appearing after resize in example below. The function throws an exception of type cvmexception in case of non-positive size passed or memory allocation failure. See also scbmatrix.resize_lu, scbmatrix. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9., 10., 11., 12.};
    scbmatrix m((std::complex<double>*)a,3,1,0);
    std::cout << m << std::endl;
    m.resize (4);
    std::cout << m;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(1,2) (0,0) (0,0)
(3,4) (5,6) (0,0)
(0,0) (7,8) (9,10)

(1,2) (0,0) (0,0) (0,0)
(3,4) (5,6) (0,0) (0,0)
(0,0) (7,8) (9,10) (0,0)
(0,0) (0,0) (11,12) (0,0)
```

2.12.23 resize_lu

Function

```
scbmatrix& scbmatrix::resize_lu (int nNewKL, int nNewKU)
throw (cvmexception);
```

changes a number of sub- and super-diagonals of a calling band matrix to `nNewKL` by `nNewKU` respectively and returns a reference to the matrix changed. In case of increasing of the numbers, the matrix is filled up with zeroes. The function throws an exception of type `cvmexception` in case of negative number passed or memory allocation failure. See also `scbmatrix::resize`, `scbmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9., 10., 11., 12.};
    scbmatrix m((std::complex<double>*)a,3,1,0);
    std::cout << m << std::endl;
    m.resize_lu (0,1);
    m.diag(1).set(std::complex<double>(9.,9.));
    std::cout << m;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(1,2) (0,0) (0,0)
(3,4) (5,6) (0,0)
(0,0) (7,8) (9,10)

(1,2) (9,9) (0,0)
(0,0) (5,6) (9,9)
(0,0) (0,0) (9,10)
```

2.12.24 operator ==

Operator

```
bool scbmatrix::operator == (const scbmatrix& m) const;
```

compares a calling band matrix with a band matrix `m` and returns `true` if they have the same sizes, the same numbers of sub- and super-diagonals and their appropriate elements differ by not more than the smallest normalized positive number. Returns `false` otherwise. See also `scbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
scbmatrix m1((std::complex<double>*)a,2,1,0);
scbmatrix m2(2,1,0);
std::cout << m1 << std::endl;

m2(1,1) = std::complex<double>(1.,2.);
m2(2,1) = std::complex<double>(3.,4.);
m2(2,2) = std::complex<double>(5.,6.);

std::cout << (m1 == m2) << std::endl;
```

prints

```
(1,2) (0,0)
(3,4) (5,6)

1
```

2.12.25 operator !=

Operator

```
bool scbmatrix::operator != (const scbmatrix& m) const;
```

compares a calling band matrix with a band matrix `m` and returns `true` if they have different sizes, different numbers of sub- or super-diagonals or at least one of their appropriate elements differs by more than the smallest normalized positive number. Returns `false` otherwise. See also `scbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
scbmatrix m1((std::complex<double>*)a,2,1,0);
scbmatrix m2(2,1,0);
std::cout << m1 << std::endl;

m2(1,1) = std::complex<double>(1.,2.);
m2(2,1) = std::complex<double>(3.,4.);
m2(2,2) = std::complex<double>(5.,6.00001);

std::cout << (m1 != m2) << std::endl;
```

prints

```
(1,2) (0,0)
(3,4) (5,6)

1
```

2.12.26 operator <<

Operator

```
scbmatrix& scbmatrix::operator << (const scbmatrix& m)
throw (cvmexception);
```

destroys a calling band matrix, creates a new one as a copy of `m` and returns a reference to the matrix changed. The operator throws an exception of type `cvmexception` in case of memory allocation failure. See also `scbmatrix`. Example:

```
using namespace cvm;

try {
    scbmatrix m(3,1,0);
    scbmatrix mc(1);
    m(2,1) = std::complex<double>(1.,2.);
    m(2,2) = std::complex<double>(3.,4.);
    std::cout << m << std::endl << mc << std::endl;

    mc << m;
    std::cout << mc;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(0,0) (0,0) (0,0)
(1,2) (3,4) (0,0)
(0,0) (0,0) (0,0)

(0,0)

(0,0) (0,0) (0,0)
(1,2) (3,4) (0,0)
(0,0) (0,0) (0,0)
```

2.12.27 operator +

Operator

```
scbmatrix scbmatrix::operator + (const scbmatrix& m) const
throw (cvmexception);
```

creates an object of type `scbmatrix` as a sum of a calling band matrix and a band matrix `m`. It throws an exception of type `cvmexception` in case of different sizes or different numbers of sub- or super-diagonals of the operands. See also `scbmatrix::sum`, `scbmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8.,
                  9., 10., 11., 12.};
    double b[] = {10., 20., 30., 40., 50., 60.,
                  70., 80., 90., 100., 110., 120.};
    scbmatrix m1((std::complex<double>*)a,3,0,1);
    scbmatrix m2((std::complex<double>*)b,3,0,1);

    std::cout << m1 << std::endl << m2 << std::endl;
    std::cout << m1 + m2 << std::endl << m1 + m1;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(3,4) (5,6) (0,0)
(0,0) (7,8) (9,10)
(0,0) (0,0) (11,12)

(30,40) (50,60) (0,0)
(0,0) (70,80) (90,100)
(0,0) (0,0) (110,120)

(33,44) (55,66) (0,0)
(0,0) (77,88) (99,110)
(0,0) (0,0) (121,132)

(6,8) (10,12) (0,0)
(0,0) (14,16) (18,20)
(0,0) (0,0) (22,24)
```

2.12.28 operator -

Operator

```
scbmatrix scbmatrix::operator - (const scbmatrix& m) const
throw (cvmexception);
```

creates an object of type `scbmatrix` as a difference of a calling band matrix and a band matrix `m`. It throws an exception of type `cvmexception` in case of different sizes or different numbers of sub- or super-diagonals of the operands. See also `scbmatrix::diff`, `scbmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8.,
                  9., 10., 11., 12.};
    double b[] = {10., 20., 30., 40., 50., 60.,
                  70., 80., 90., 100., 110., 120.};
    scbmatrix m1((std::complex<double>*)a,3,0,1);
    scbmatrix m2((std::complex<double>*)b,3,0,1);

    std::cout << m1 << std::endl << m2 << std::endl;
    std::cout << m1 - m2 << std::endl << m1 - m1;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(3,4) (5,6) (0,0)
(0,0) (7,8) (9,10)
(0,0) (0,0) (11,12)

(30,40) (50,60) (0,0)
(0,0) (70,80) (90,100)
(0,0) (0,0) (110,120)

(-27,-36) (-45,-54) (0,0)
(0,0) (-63,-72) (-81,-90)
(0,0) (0,0) (-99,-108)

(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
```

2.12.29 sum

Function

```
scbmatrix& scbmatrix::sum (const scbmatrix& m1, const scbmatrix& m2)
throw (cvmexception);
```

assigns a result of addition of band matrices m1 and m2 to a calling band matrix and returns a reference to the matrix changed. It throws an exception of type cvmexception in case of different sizes or different numbers of sub- or super-diagonals of the operands. See also scbmatrix::operator + , scbmatrix. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.,
              9., 10., 11., 12.};
const scbmatrix m1((std::complex<double>*)a,3,1,0);
scbmatrix m2(3,1,0);
scbmatrix m(3,1,0);
m2.set(std::complex<double>(1.,1.));
std::cout << m1 << std::endl << m2 << std::endl;
std::cout << m.sum(m1, m2) << std::endl;
std::cout << m.sum(m, m2);
```

prints

```
(1,2) (0,0) (0,0)
(3,4) (5,6) (0,0)
(0,0) (7,8) (9,10)

(1,1) (0,0) (0,0)
(1,1) (1,1) (0,0)
(0,0) (1,1) (1,1)

(2,3) (0,0) (0,0)
(4,5) (6,7) (0,0)
(0,0) (8,9) (10,11)

(3,4) (0,0) (0,0)
(5,6) (7,8) (0,0)
(0,0) (9,10) (11,12)
```

2.12.30 diff

Function

```
scbmatrix& scbmatrix::diff (const scbmatrix& m1, const scbmatrix& m2)
throw (cvmexception);
```

assigns a result of subtraction of band matrices m1 and m2 to a calling band matrix and returns a reference to the matrix changed. It throws an exception of type cvmexception in case of different sizes or different numbers of sub- or super-diagonals of the operands. See also scbmatrix::operator - , scbmatrix. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.,
              9., 10., 11., 12.};
const scbmatrix m1((std::complex<double>*)a,3,1,0);
scbmatrix m2(3,1,0);
scbmatrix m(3,1,0);
m2.set(std::complex<double>(1.,1.));
std::cout << m1 << std::endl << m2 << std::endl;
std::cout << m.diff(m1, m2) << std::endl;
std::cout << m.diff(m, m2);
```

prints

```
(1,2) (0,0) (0,0)
(3,4) (5,6) (0,0)
(0,0) (7,8) (9,10)

(1,1) (0,0) (0,0)
(1,1) (1,1) (0,0)
(0,0) (1,1) (1,1)

(0,1) (0,0) (0,0)
(2,3) (4,5) (0,0)
(0,0) (6,7) (8,9)

(-1,0) (0,0) (0,0)
(1,2) (3,4) (0,0)
(0,0) (5,6) (7,8)
```

2.12.31 operator +=

Operator

```
scbmatrix& scbmatrix::operator += (const scbmatrix& m)
throw (cvmexception);
```

adds a band matrix `m` to a calling band matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes or different numbers of sub- or super-diagonals of the operands. See also `scbmatrix::operator +` , `scbmatrix::sum`, `scbmatrix`. Example:

```
using namespace cvm;

try {
    scbmatrix m1(4,0,1);
    scbmatrix m2(4,0,1);
    m1.set(std::complex<double>(1.,2.));
    m2.set(std::complex<double>(3.,4.));

    m1 += m2;
    std::cout << m1 << std::endl;

    // well, you can do this too, but temporary object would be created
    m2 += m2;
    std::cout << m2;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(4,6) (4,6) (0,0) (0,0)
(0,0) (4,6) (4,6) (0,0)
(0,0) (0,0) (4,6) (4,6)
(0,0) (0,0) (0,0) (4,6)

(6,8) (6,8) (0,0) (0,0)
(0,0) (6,8) (6,8) (0,0)
(0,0) (0,0) (6,8) (6,8)
(0,0) (0,0) (0,0) (6,8)
```

2.12.32 operator -=

Operator

```
scbmatrix& scbmatrix::operator -= (const scbmatrix& m)
throw (cvmexception);
```

subtracts a band matrix `m` from a calling band matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes or different numbers of sub- or super-diagonals of the operands. See also `scbmatrix::operator -` , `scbmatrix::diff`, `scbmatrix`. Example:

```
using namespace cvm;

try {
    scbmatrix m1(4,0,1);
    scbmatrix m2(4,0,1);
    m1.set(std::complex<double>(1.,2.));
    m2.set(std::complex<double>(3.,4.));

    m1 -= m2;
    std::cout << m1 << std::endl;

    // well, you can do this too, but temporary object would be created
    m2 -= m2;
    std::cout << m2;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(-2,-2) (-2,-2) (0,0) (0,0)
(0,0) (-2,-2) (-2,-2) (0,0)
(0,0) (0,0) (-2,-2) (-2,-2)
(0,0) (0,0) (0,0) (-2,-2)

(0,0) (0,0) (0,0) (0,0)
(0,0) (0,0) (0,0) (0,0)
(0,0) (0,0) (0,0) (0,0)
(0,0) (0,0) (0,0) (0,0)
```

2.12.33 operator - ()

Operator

```
scbmatrix scbmatrix::operator - () const throw (cvmexception);
```

creates an object of type `scbmatrix` as a calling band matrix multiplied by -1. See also `scbmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific |
                std::ios::left |
                std::ios::showpos);
std::cout.precision (2);

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.,
              9., 10., 11., 12.};
scbmatrix m((std::complex<double>*)a,3,1,0);

std::cout << -m;
```

prints

```
(-1.00e+000,-2.00e+000) (+0.00e+000,+0.00e+000) (+0.00e+000,+0.00e+000)
(-3.00e+000,-4.00e+000) (-5.00e+000,-6.00e+000) (+0.00e+000,+0.00e+000)
(+0.00e+000,+0.00e+000) (-7.00e+000,-8.00e+000) (-9.00e+000,-1.00e+001)
```

2.12.34 operator ++

Operator

```
scbmatrix& scbmatrix::operator ++ ();
scbmatrix& scbmatrix::operator ++ (int);
```

adds identity matrix to a calling band matrix and returns a reference to the matrix changed. See also scbmatrix. Example:

```
using namespace cvm;

scbmatrix m(4,1,0);
m.set(std::complex<double>(1.,1.));

m++;
std::cout << m << std::endl;
std::cout << ++m;
```

prints

```
(2,1) (0,0) (0,0) (0,0)
(1,1) (2,1) (0,0) (0,0)
(0,0) (1,1) (2,1) (0,0)
(0,0) (0,0) (1,1) (2,1)

(3,1) (0,0) (0,0) (0,0)
(1,1) (3,1) (0,0) (0,0)
(0,0) (1,1) (3,1) (0,0)
(0,0) (0,0) (1,1) (3,1)
```

2.12.35 operator --

Operator

```
scbmatrix& scbmatrix::operator -- ();
scbmatrix& scbmatrix::operator -- (int);
```

subtracts identity matrix from a calling band matrix and returns a reference to the matrix changed. See also scbmatrix. Example:

```
using namespace cvm;

scbmatrix m(4,1,0);
m.set(std::complex<double>(1.,1.));

m--;
std::cout << m << std::endl;
std::cout << --m;
```

prints

```
(0,1) (0,0) (0,0) (0,0)
(1,1) (0,1) (0,0) (0,0)
(0,0) (1,1) (0,1) (0,0)
(0,0) (0,0) (1,1) (0,1)

(-1,1) (0,0) (0,0) (0,0)
(1,1) (-1,1) (0,0) (0,0)
(0,0) (1,1) (-1,1) (0,0)
(0,0) (0,0) (1,1) (-1,1)
```

2.12.36 operator * (TR)

Operator

```
scbmatrix scbmatrix::operator * (TR d) const;
```

creates an object of type `scbmatrix` as a product of a calling band matrix and a real number `d`. See also `scbmatrix::operator *=` , `scbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12.};
scbmatrix m((std::complex<double>*)a,3,0,1);
std::cout << m * 5.;
```

prints

```
(15,20) (25,30) (0,0)
(0,0) (35,40) (45,50)
(0,0) (0,0) (55,60)
```

2.12.37 operator / (TR)

Operator

`scbmatrix scbmatrix::operator / (TR d) const throw (cvmexception);`

creates an object of type `scbmatrix` as a quotient of a calling band matrix and a real number `d`. It throws an exception of type `cvmexception` if `d` has an absolute value equal or less than the smallest normalized positive number. See also `scbmatrix::operator /=` , `scbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12.};
scbmatrix m((std::complex<double>*)a,3,0,1);
std::cout << m / 2.;
```

prints

```
(1.5,2) (2.5,3) (0,0)
(0,0) (3.5,4) (4.5,5)
(0,0) (0,0) (5.5,6)
```

2.12.38 operator * (TC)

Operator

```
scbmatrix scbmatrix::operator * (TC z) const;
```

creates an object of type `scbmatrix` as a product of a calling band matrix and a complex number z. See also `scbmatrix::operator *=` , `scbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12.};
scbmatrix m((std::complex<double>*)a,3,0,1);
std::cout << m * std::complex<double>(1.,1.);
```

prints

```
(-1,7) (-1,11) (0,0)
(0,0) (-1,15) (-1,19)
(0,0) (0,0) (-1,23)
```

2.12.39 operator / (TC)

Operator

```
scbmatrix scbmatrix::operator / (TC z) const throw (cvmexception);
```

creates an object of type `scbmatrix` as a quotient of a calling band matrix and a complex number z. It throws an exception of type `cvmexception` if z has an absolute value equal or less than the smallest normalized positive number. See also `scbmatrix::operator /=` , `scbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12.};
scbmatrix m((std::complex<double>*)a,3,0,1);
std::cout << m / std::complex<double>(1.,1.);
```

prints

```
(3.5,0.5) (5.5,0.5) (0,0)
(0,0) (7.5,0.5) (9.5,0.5)
(0,0) (0,0) (11.5,0.5)
```

2.12.40 operator *= (TR)

Operator

```
scbmatrix& scbmatrix::operator *= (TR d);
```

multiplies a calling band matrix by a real number d and returns a reference to the matrix changed. See also scbmatrix::operator * , scbmatrix. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12.};
scbmatrix m((std::complex<double>*)a,3,0,1);
m *= 5.;
std::cout << m;
```

prints

```
(15,20) (25,30) (0,0)
(0,0) (35,40) (45,50)
(0,0) (0,0) (55,60)
```

2.12.41 operator /= (TR)

Operator

```
scbmatrix& scbmatrix::operator /= (TR d) throw (cvmexception);
```

divides a calling band matrix by a real number d and returns a reference to the matrix changed. It throws an exception of type cvmexception if d has an absolute value equal or less than the smallest normalized positive number. See also scbmatrix::operator / , scbmatrix. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12.};
scbmatrix m((std::complex<double>*)a,3,0,1);
m /= 2.;
std::cout << m;
```

prints

```
(1.5,2) (2.5,3) (0,0)
(0,0) (3.5,4) (4.5,5)
(0,0) (0,0) (5.5,6)
```

2.12.42 operator *= (TC)

Operator

```
scbmatrix& scbmatrix::operator *= (TC z);
```

multiplies a calling band matrix by a complex number `z` and returns a reference to the matrix changed. See also `scbmatrix::operator *` , `scbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12.};
scbmatrix m((std::complex<double>*)a,3,0,1);
m *= std::complex<double>(1.,1.);
std::cout << m;
```

prints

```
(-1,7) (-1,11) (0,0)
(0,0) (-1,15) (-1,19)
(0,0) (0,0) (-1,23)
```

2.12.43 operator /= (TC)

Operator

```
scbmatrix& scbmatrix::operator /= (TC z) throw (cvmexception);
```

divides a calling band matrix by a complex number z and returns a reference to the matrix changed. It throws an exception of type cvmexception if z has an absolute value equal or less than the smallest normalized positive number. See also scbmatrix::operator / , scbmatrix. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12.};
scbmatrix m((std::complex<double>*)a,3,0,1);
m /= std::complex<double>(1.,1.);
std::cout << m;
```

prints

```
(3.5,0.5) (5.5,0.5) (0,0)
(0,0) (7.5,0.5) (9.5,0.5)
(0,0) (0,0) (11.5,0.5)
```

2.12.44 normalize

Function

```
scbmatrix& scbmatrix::normalize ();
```

normalizes a calling band matrix so its Euclidean norm becomes equal to 1 if it was greater than the smallest normalized positive number before the call (otherwise the function does nothing). See also scbmatrix. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12.};
scbmatrix m((std::complex<double>*)a,3,0,1);

m.normalize();
std::cout << m << m.norm() << std::endl;
```

prints

```
(1.18e-001,1.57e-001) (1.97e-001,2.36e-001) (0.00e+000,0.00e+000)
(0.00e+000,0.00e+000) (2.76e-001,3.15e-001) (3.54e-001,3.94e-001)
(0.00e+000,0.00e+000) (0.00e+000,0.00e+000) (4.33e-001,4.72e-001)
1.00e+000
```

2.12.45 conjugation

Operator and functions

```
scbmatrix scbmatrix::operator ~ () const throw (cvmexception);
scbmatrix& scbmatrix::conj (const scbmatrix& m) throw (cvmexception);
scbmatrix& scbmatrix::conj () throw (cvmexception);
```

encapsulate complex band matrix conjugation. First operator creates an object of type scbmatrix as a conjugated calling band matrix (it throws an exception of type cvmexception in case of memory allocation failure). Second function sets a calling matrix to be equal to a matrix m conjugated (it throws an exception of type cvmexception in case of not appropriate sizes or numbers of sub- or super-diagonals of the operands), third one makes it to be equal to conjugated itself (it also throws an exception of type cvmexception in case of memory allocation failure). See also scbmatrix. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12., 13., 14., 15., 16.};
scbmatrix m((std::complex<double>*)a,4,1,0);
scbmatrix mc(4,0,1);
std::cout << m << std::endl << ~m << std::endl ;
mc.conj(m);
std::cout << mc << std::endl;
mc.conj();
std::cout << mc;
```

prints

```
(1,2) (0,0) (0,0) (0,0)
(3,4) (5,6) (0,0) (0,0)
(0,0) (7,8) (9,10) (0,0)
(0,0) (0,0) (11,12) (13,14)

(1,-2) (3,-4) (0,0) (0,0)
(0,0) (5,-6) (7,-8) (0,0)
(0,0) (0,0) (9,-10) (11,-12)
(0,0) (0,0) (0,0) (13,-14)

(1,-2) (3,-4) (0,0) (0,0)
(0,0) (5,-6) (7,-8) (0,0)
(0,0) (0,0) (9,-10) (11,-12)
(0,0) (0,0) (0,0) (13,-14)
```

```
(1,2) (0,0) (0,0) (0,0)
(3,4) (5,6) (0,0) (0,0)
(0,0) (7,8) (9,10) (0,0)
(0,0) (0,0) (11,12) (13,14)
```

2.12.46 operator * (const cvector&)

Operator

```
cvector scbmatrix::operator * (const cvector& v) const
throw (cvmexception);
```

creates an object of type `cvector` as a product of a calling band matrix and a vector `v`. It throws an exception of type `cvmexception` if the number of columns of the calling matrix differs from the size of the vector `v`. Use `cvector::mult` in order to get rid of a new object creation. See also `scbmatrix` and `cvector`. Example:

```
using namespace cvm;

try {
    scbmatrix m (4,1,0);
    cvector v(4);
    m.set(std::complex<double>(1.,1.));
    v.set(std::complex<double>(1.,1.));

    std::cout << m * v;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(0,2) (0,4) (0,4) (0,4)
```

2.12.47 operator * (const cmatrix&)

Operator

```
cmatrix scbmatrix::operator * (const cmatrix& m) const
throw (cvmexception);
```

creates an object of type `cmatrix` as a product of a calling band matrix and a matrix `m`. It throws an exception of type `cvmexception` if the number of columns of the calling matrix differs from the number of rows of the matrix `m`. Use `cmatrix::mult` in order to get rid of a new object creation. See also `cmatrix` and `scbmatrix`. Example:

```
using namespace cvm;

try {
    scbmatrix mb(4,1,0);
    cmatrix m(4,2);
    mb.set(std::complex<double>(1.,1.));
    m.set(std::complex<double>(1.,1.));

    std::cout << mb * m;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(0,2) (0,2)
(0,4) (0,4)
(0,4) (0,4)
(0,4) (0,4)
```

2.12.48 operator * (const scmatrix&)

Operator

```
scmatrix scbmatrix::operator * (const scmatrix& m) const
throw (cvmexception);
```

creates an object of type scmatrix as a product of a calling band matrix and a matrix m. It throws an exception of type cvmexception if the operands have different sizes. Use cmatrix::mult in order to get rid of a new object creation. See also scmatrix and scbmatrix. Example:

```
using namespace cvm;

try {
    scbmatrix mb(4,1,0);
    scmatrix m(4);
    mb.set(std::complex<double>(1.,1.));
    m.set(std::complex<double>(1.,1.));

    std::cout << mb * m;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(0,2) (0,2) (0,2) (0,2)
(0,4) (0,4) (0,4) (0,4)
(0,4) (0,4) (0,4) (0,4)
(0,4) (0,4) (0,4) (0,4)
```

2.12.49 operator * (const scbmatrix&)

Operator

```
scbmatrix scbmatrix::operator * (const scbmatrix& m) const
throw (cvmexception);
```

creates an object of type scbmatrix as a product of a calling band matrix and a band matrix m. It throws an exception of type cvmexception if the operands have different sizes. Use cmatrix::mult in order to get rid of a new object creation. See also scbmatrix. Example:

```
using namespace cvm;

try {
    scbmatrix m1(5,1,0);
    scbmatrix m2(5,1,1);
    m1.set(std::complex<double>(1.,1.));
    m2.set(std::complex<double>(1.,1.));

    std::cout << m1 * m2;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(0,2) (0,2) (0,0) (0,0) (0,0)
(0,4) (0,4) (0,2) (0,0) (0,0)
(0,2) (0,4) (0,4) (0,2) (0,0)
(0,0) (0,2) (0,4) (0,4) (0,2)
(0,0) (0,0) (0,2) (0,4) (0,4)
```

2.12.50 low_up

Functions

```
scbmatrix&
scbmatrix::low_up (const scbmatrix& m, int* nPivots) throw (cvmexception);
scbmatrix
scbmatrix::low_up (int* nPivots) const throw (cvmexception);
```

compute the LU factorization of a calling band matrix as

$$A = PLU$$

where P is a permutation matrix, L is a lower triangular matrix with unit diagonal elements and U is an upper triangular matrix. All the functions store the result as the matrix L without main diagonal combined with U. All the functions return pivot indices as an array of integers (it should support at least `msize()` elements) pointed to by `nPivots` so i-th row was interchanged with `nPivots[i]`-th row. The first version sets a calling matrix to be equal to the `m`'s LU factorization and the second one creates an object of type `scbmatrix` as the calling band matrix's LU factorization. The functions throw exception of type `cvmexception` in case of inappropriate sizes of the operands or when the matrix to be factorized is close to cingular. The first version also changes numbers of super-diagonals to be equal to $k_l + k_u$ in order to keep the result of factorization. It is recommended to use `iarray` for pivot values. This function is provided mostly for solving multiple systems of linear equations using `scmatrix::solve_lu` function, See also `scbmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
                  10., 11., 12.};
    scbmatrix ma((std::complex<double>*)a,3,1,0);
    scbmatrix mLU(3,1,0);
    cmatrix  mb1(3,2); cvector vb1(3);
    cmatrix  mb2(3,2); cvector vb2(3);
    cmatrix  mx1(3,2); cvector vx1(3);
    cmatrix  mx2(3,2); cvector vx2(3);
    iarray   nPivots(3);
    double   dErr = 0.;
    mb1.randomize_real(-1.,3.); mb1.randomize_imag(1.,5.);
    mb2.randomize_real(-2.,5.); mb2.randomize_imag(-3.,0.);
    vb1.randomize_real(-2.,4.); vb1.randomize_imag(-4.,1.);
    vb2.randomize_real(-3.,1.); vb2.randomize_imag(4.,5.);
```

```
    mLU.low_up(ma, nPivots);
    mx1 = ma.solve_lu (mLU, nPivots, mb1, dErr);
    std::cout << mx1 << dErr << std::endl << std::endl;
    mx2 = ma.solve_lu (mLU, nPivots, mb2);
    std::cout << mx2 << std::endl;;
    std::cout << ma * mx1 - mb1 << std::endl << ma * mx2 - mb2;

    vx1 = ma.solve_lu (mLU, nPivots, vb1, dErr);
    std::cout << vx1 << dErr << std::endl;
    vx2 = ma.solve_lu (mLU, nPivots, vb2);
    std::cout << vx2 << std::endl;;
    std::cout << ma * vx1 - vb1 << std::endl << ma * vx2 - vb2;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(1.20e+000,4.02e-002) (1.82e+000,1.23e+000)
(-6.55e-001,1.37e-001) (-6.41e-001,-8.72e-001)
(7.75e-001,4.70e-002) (5.35e-001,8.11e-001)
1.45e-015

(-4.52e-001,-2.68e-002) (-1.09e+000,2.01e-001)
(6.08e-001,-4.76e-001) (5.48e-001,-1.95e-001)
(-3.46e-001,1.57e-001) (-3.38e-001,-7.54e-002)

(0.00e+000,4.44e-016) (-2.22e-016,8.88e-016)
(-2.22e-016,2.22e-016) (0.00e+000,0.00e+000)
(-1.11e-016,0.00e+000) (-3.33e-016,-6.66e-016)

(0.00e+000,0.00e+000) (2.22e-016,2.22e-016)
(0.00e+000,0.00e+000) (4.44e-016,-2.22e-016)
(8.88e-016,5.55e-016) (0.00e+000,0.00e+000)
(-1.28e+000,-5.12e-001) (8.22e-001,1.59e-001) (-6.45e-001,-3.74e-001)
1.31e-015
(1.26e+000,1.50e+000) (-5.13e-001,-4.66e-001) (5.97e-001,7.01e-001)

(0.00e+000,8.88e-016) (-4.44e-016,4.44e-016) (-8.88e-016,0.00e+000)

(2.22e-016,-8.88e-016) (4.44e-016,-8.88e-016) (-2.22e-016,8.88e-016)
```

2.12.51 identity

Function

```
scbmatrix& scbmatrix::identity();
```

sets a calling band matrix to be equal to identity matrix and returns a reference to the matrix changed. The function doesn't change numbers of sub- and super-diagonals. See also scbmatrix. Example:

```
using namespace cvm;

srbmatrix m(4);
m.randomize(0.,1.);
std::cout << m << std::endl;
std::cout << m.identity();
```

prints

```
(0.576128,1.42595) (0,0) (0,0) (0,0)
(0.956359,-0.919523) (0.869716,-0.704093) (0,0) (0,0)
(0,0) (0.0959807,0.0616779) (0.632618,1.1793) (0,0)
(0,0) (0,0) (0.532182,-0.870724) (0.338023,1.22892)

(1,0) (0,0) (0,0) (0,0)
(0,0) (1,0) (0,0) (0,0)
(0,0) (0,0) (1,0) (0,0)
(0,0) (0,0) (0,0) (1,0)
```

2.12.52 vanish

Function

```
scbmatrix& scbmatrix::vanish();
```

sets every element of a calling band matrix to be equal to zero and returns a reference to the matrix changed. This function is faster than `scbmatrix::set(TR)` with zero operand passed. See also `scbmatrix`. Example:

```
using namespace cvm;

scbmatrix m(4,1,0);
m.randomize_real(0.,1.);
m.randomize_imag(-1.,2.);
std::cout << m << std::endl;
std::cout << m.vanish();
```

prints

```
(0.584094,0.985931) (0,0) (0,0) (0,0)
(0.197546,0.0150761) (0.483413,-0.733848) (0,0) (0,0)
(0,0) (0.844356,1.97848) (0.814692,1.50194) (0,0)
(0,0) (0,0) (0.118931,-0.720756) (0.936796,-0.582232)

(0,0) (0,0) (0,0) (0,0)
(0,0) (0,0) (0,0) (0,0)
(0,0) (0,0) (0,0) (0,0)
(0,0) (0,0) (0,0) (0,0)
```

2.12.53 randomize_real

Function

```
scbmatrix& scbmatrix::randomize_real (TR dFrom, TR dTo);
```

fills a real part of a calling band matrix with pseudo-random numbers distributed between dFrom and dTo. The function returns a reference to the matrix changed. See also scbmatrix. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
scbmatrix m(3,0,1);
m.randomize_real(0.,3.);
std::cout << m;
```

prints

```
(1.78e+000,0.00e+000) (1.17e+000,0.00e+000) (0.00e+000,0.00e+000)
(0.00e+000,0.00e+000) (1.09e-002,0.00e+000) (6.05e-001,0.00e+000)
(0.00e+000,0.00e+000) (0.00e+000,0.00e+000) (2.49e+000,0.00e+000)
```

2.12.54 randomize_imag

Function

```
scbmatrix& scbmatrix::randomize_imag (TR dFrom, TR dTo);
```

fills an imaginary part of a calling band matrix with pseudo-random numbers distributed between dFrom and dTo. The function returns a reference to the matrix changed. See also scbmatrix. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
scbmatrix m(3,0,1);
m.randomize_imag(0.,3.);
std::cout << m;
```

prints

```
(0.00e+000,1.80e+000) (0.00e+000,1.68e-001) (0.00e+000,0.00e+000)
(0.00e+000,0.00e+000) (0.00e+000,1.05e+000) (0.00e+000,1.40e+000)
(0.00e+000,0.00e+000) (0.00e+000,0.00e+000) (0.00e+000,1.98e+000)
```

2.13 srsmatrix

This is end-user class encapsulating a symmetric matrix in Euclidean space of real numbers.

```
template <typename TR>
class srsmatrix : public srmatrix <TR> {
public:
    srsmatrix ();
    explicit srsmatrix (int nMN);
    srsmatrix (TR* pD, int nMN, TR tol = cvmMachSp());
    srsmatrix (const srsmatrix& m);
    explicit srsmatrix (const rmatrix& m, TR tol = cvmMachSp());
    explicit srsmatrix (const rvector& v);
    srsmatrix (srsmatrix& m, int nRowCol, int nSize);
    TR operator () (int im, int in) const throw (cvmexception);
    const rvector operator () (int i) const throw (cvmexception);
    const rvector operator [] (int i) const throw (cvmexception);
    const rvector diag (int i) const throw (cvmexception);
    srsmatrix& operator = (const srsmatrix& m) throw (cvmexception);
    srsmatrix& assign (const rvector& v, TR tol = cvmMachSp())
                       throw (cvmexception);
    srsmatrix& assign (const TR* pD, TR tol = cvmMachSp())
                       throw (cvmexception);
    srsmatrix& assign (int nRowCol, const srsmatrix& m)
                       throw (cvmexception);
    srsmatrix& set (TR x);
    srsmatrix& set (int nRow, int nCol, TR x);
    srsmatrix& set_diag (int i, const rvector& v)
                         throw (cvmexception);
    srsmatrix& resize (int nNewMN) throw (cvmexception);
    bool operator == (const srsmatrix& m) const;
    bool operator != (const srsmatrix& m) const;
    srsmatrix& operator << (const srsmatrix& m) throw (cvmexception);
    srsmatrix operator + (const srsmatrix& m) const
                          throw (cvmexception);
    srsmatrix operator - (const srsmatrix& m) const
                          throw (cvmexception);
    srsmatrix& sum (const srsmatrix& m1,
                    const srsmatrix& m2) throw (cvmexception);
    srsmatrix& diff (const srsmatrix& m1,
                     const srsmatrix& m2) throw (cvmexception);
    srsmatrix& operator += (const srsmatrix& m) throw (cvmexception);
```

```
    srsmatrix& operator -= (const srsmatrix& m) throw (cvmexception);
    srsmatrix operator - () const;
    srsmatrix& operator ++ ();
    srsmatrix& operator ++ (int);
    srsmatrix& operator -- ();
    srsmatrix& operator -- (int);
    srsmatrix operator * (TR d) const;
    srsmatrix operator / (TR d) const throw (cvmexception);
    srsmatrix& operator *= (TR d);
    srsmatrix& operator /= (TR d) throw (cvmexception);
    srsmatrix& normalize ();
    srsmatrix operator ~ () const throw (cvmexception);
    srsmatrix& transpose (const srsmatrix& m) throw (cvmexception);
    srsmatrix& transpose ();
    rvector operator * (const rvector& v) const throw (cvmexception);
    rmatrix operator * (const rmatrix& m) const throw (cvmexception);
    srmatrix operator * (const srmatrix& m) const throw (cvmexception);
    srsmatrix& syrk (TR alpha,
                     const rvector& v, TR beta) throw (cvmexception);
    srsmatrix& syrk (bool bTransp, TR alpha,
                     const rmatrix& m, TR beta) throw (cvmexception);
    srsmatrix& syr2k (TR alpha,
                     const rvector& v1, const rvector& v2, TR beta)
                     throw (cvmexception);
    srsmatrix& syr2k (bool bTransp, TR alpha,
                     const rmatrix& m1, const rmatrix& m2, TR beta)
                     throw (cvmexception);
    srsmatrix& inv (const srsmatrix& mArg) throw (cvmexception);
    srsmatrix inv () const throw (cvmexception);
    srsmatrix& exp (const srsmatrix& m,
                   TR tol = cvmMachSp()) throw (cvmexception);
    srsmatrix exp (TR tol = cvmMachSp()) const throw (cvmexception);
    srsmatrix& polynom (const srsmatrix& m, const rvector& v)
                        throw (cvmexception);
    srsmatrix polynom (const rvector& v) const throw (cvmexception);
    rvector eig (srmatrix& mEigVect) const throw (cvmexception);
    rvector eig () const throw (cvmexception);
    srmatrix cholesky () const throw (cvmexception);
    srmatrix bunch_kaufman () const throw (cvmexception);
    srsmatrix& identity ();
    srsmatrix& vanish ();
    srsmatrix& randomize (TR dFrom, TR dTo);
};
```

2.13.1 srsmatrix ()

Constructor

```
srsmatrix::srsmatrix ();
```

creates an empty `srsmatrix` object. See also `srsmatrix`. Example:

```
using namespace cvm;

srsmatrix m;
std::cout << m.msize() << " " << m.nsize() << " " << m.size()
          << std::endl << std::endl;
m.resize (3);
m.set(1.);
std::cout << m;
```

prints

```
0 0 0

1 1 1
1 1 1
1 1 1
```

2.13.2 srsmatrix (int)

Constructor

```
explicit srsmatrix::srsmatrix (int nMN);
```

creates an $n \times n$ `srsmatrix` object where n is passed in `nMN` parameter. The constructor throws an exception of type `cvmexception` in case of non-positive size passed or memory allocation failure. See also `srsmatrix`. Example:

```
using namespace cvm;

srsmatrix m(4);
std::cout << m.msize() << " " << m.nsize() << " " << m.size()
          << std::endl << std::endl;
m.set(1.);
std::cout << m;
```

prints

```
4 4 16

1 1 1 1
1 1 1 1
1 1 1 1
1 1 1 1
```

2.13.3 srsmatrix (TR*,int)

Constructor

```
srsmatrix::srsmatrix (TR* pD, int nMN, TR tol = cvmMachSp());
```

creates an $n \times n$ `srsmatrix` object where n is passed in `nMN` parameter. Unlike others, this constructor *does not allocate a memory*. It just shares a memory with an array pointed to by `pD`. The constructor throws an exception of type `cvmexception` if the matrix created doesn't appear to be symmetric. The symmetry tolerance is set by parameter `tol`. See also `srsmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
srsmatrix m (a, 3);
std::cout << m << std::endl;
m.set(2,3,7.77);
std::cout << m << std::endl
          << a[7] << " " << a[5] << std::endl;
```

prints

```
1 2 3
2 5 6
3 6 9

1 2 3
2 5 7.77
3 7.77 9

7.77 7.77
```

2.13.4 srsmatrix (const srsmatrix&)

Copy constructor

```
srsmatrix::srsmatrix (const srsmatrix& m);
```

creates a `srsmatrix` object as a copy of `m`. The constructor throws an exception of type `cvmexception` in case of memory allocation failure. See also `srsmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
srsmatrix m (a, 3);
srmatrix mc(m);
m.set(2,3,7.77);
std::cout << m << std::endl << mc;
```

prints

```
1 2 3
2 5 7.77
3 7.77 9

1 2 3
2 5 6
3 6 9
```

2.13.5 srsmatrix (const rmatrix&)

Constructor

```
explicit srsmatrix::srsmatrix (const rmatrix& m, TR tol = cvmMachSp());
```

creates a `srsmatrix` object as a copy of matrix `m`. It's assumed that $m \times n$ matrix `m` must have equal sizes, i.e. $m = n$ is satisfied, and must be symmetric. The symmetry tolerance is set by parameter `tol`. The constructor throws an exception of type `cvmexception` if this is not true or in case of memory allocation failure. See also `srsmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
rmatrix m(a, 3, 3);
srsmatrix ms(m);
std::cout << ms;
```

prints

```
1 2 3
2 5 6
3 6 9
```

2.13.6 srsmatrix (const rvector&)

Constructor

```
explicit srsmatrix::srsmatrix (const rvector& v);
```

creates a `srsmatrix` object of size `v.size()` by `v.size()` and assigns vector `v` to its main diagonal. The constructor throws an exception of type `cvmexception` in case of memory allocation failure. See also `srsmatrix`, `rvector`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5.};
const rvector v(a, 5);
srsmatrix m(v);
std::cout << m;
```

prints

```
1 0 0 0 0
0 2 0 0 0
0 0 3 0 0
0 0 0 4 0
0 0 0 0 5
```

2.13.7 submatrix

Submatrix constructor

```
srsmatrix::srsmatrix (srsmatrix& m, int nRowCol, int nSize);
```

creates a `srmatrix` object as a *submatrix* of symmetric matrix `m`. It means that the matrix object created shares a memory with some part of `m`. This part is defined by its upper left corner (parameter `nRowCol`, 1-based) and its size (parameter `nSize`). See also `srsmatrix`. Example:

```
using namespace cvm;

srsmatrix m(5);
srsmatrix subm(m, 2, 2);
subm.set(1.);
std::cout << m;
```

prints

```
0 0 0 0 0
0 1 1 0 0
0 1 1 0 0
0 0 0 0 0
```

2.13.8 operator (,)

Indexing operator

```
TR srsmatrix::operator () (int im, int in) const throw (cvmexception);
```

provides access to an element of a matrix. Unlike indexing operators in other classes, this operator doesn't return an *l-value* because this would make the matrix non-symmetric. The operator is 1-based. The operator throws an exception of type `cvmexception` if some of parameters passed is outside of `[1,msize()]` range. See also `srsmatrix`, `Matrix::msize()`, `Matrix::nsize()`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
srsmatrix m(a, 3);
std::cout << m(1,1) << " " << m(2,3) << std::endl;
```

prints

```
1 6
```

2.13.9 operator ()

Indexing operator

```
const rvector srsmatrix::operator () (int i) const throw (cvmexception);
```

provides access to an i-th column of a matrix. Unlike indexing operators in other classes, this operator doesn't return an *l-value* because this would make the matrix non-symmetric. The operator creates an object of class `rvector` as a *copy* of a column and therefore it's *not an l-value*. The operator is 1-based. The operator throws an exception of type `cvmexception` if the parameter `i` is outside of `[1,nsize()]` range. See also `srsmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
srsmatrix m(a, 3);
std::cout << m(1) << m(2) << m(3);
```

prints

```
1 2 3
2 5 6
3 6 9
```

2.13.10 operator []

Indexing operator

```
const rvector srsmatrix::operator [] (int i) const throw (cvmexception);
```

provides access to an i-th row of a matrix. Unlike indexing operators in other classes, this operator doesn't return an *l-value* because this would make the matrix non-symmetric. The operator creates an object of class `rvector` as a *copy* of a row and therefore it's *not an l-value*. The operator is 1-based. The operator throws an exception of type `cvmexception` if the parameter `i` is outside of `[1,msize()]` range. See also `srsmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
srsmatrix m(a, 3);
std::cout << m[1] << m[2] << m[3];
```

prints

```
1 2 3
2 5 6
3 6 9
```

2.13.11 diag

Function

```
const rvector srsmatrix::diag (int i) const throw (cvmexception);
```

provides access to an i-th diagonal of a matrix, where $i = 0$ for main diagonal, $i < 0$ for lower diagonals and $i > 0$ for upper ones. Unlike `diag` function in other classes, this one doesn't return an *l-value* because this would make the matrix non-symmetric. The function creates an object of class `rvector` as a *copy* of a diagonal and therefore it's *not an l-value*. The function is 1-based. The function throws an exception of type `cvmexception` if the parameter `i` is outside of `[-msize()+1,nsize()-1]` range. See also `srsmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
srsmatrix m(a, 3);
std::cout << m << std::endl
          << m.diag(-2) << m.diag(-1) << m.diag(0)
          << m.diag(1) << m.diag(2);
```

prints

```
1 2 3
2 5 6
3 6 9

3
2 6
1 5 9
2 6
3
```

2.13.12 operator = (const srsmatrix&)

Operator

```
srsmatrix& srsmatrix::operator = (const srsmatrix& m)
throw (cvmexception);
```

sets an every element of a calling symmetric matrix to a value of appropriate element of symmetric matrix `m` and returns a reference to the matrix changed. The operator throws an exception of type `cvmexception` in case of different sizes of the operands. See also `srsmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
const srsmatrix m1(a, 3);
srsmatrix m2(3);

m2 = m1;
std::cout << m2;
```

prints

```
1 2 3
2 5 6
3 6 9
```

2.13.13 assign (const TR*)

Function

```
srsmatrix& srsmatrix::assign (const rvector& v, TR tol = cvmMachSp())
throw (cvmexception);

srsmatrix& srsmatrix::assign (const TR* pD, TR tol = cvmMachSp())
throw (cvmexception);
```

sets every element of a calling matrix to a value of appropriate element of a vector `v` or an array pointed to by `pD` and returns a reference to the matrix changed. The function throws an exception of type `cvmexception` if the matrix changed doesn't appear to be symmetric. The symmetry tolerance is set by parameter `tol`. See also `srsmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
srsmatrix m(3);
m.assign(a);
std::cout << m;
```

prints

```
1 2 3
2 5 6
3 6 9
```

2.13.14 assign (int, int, const srsmatrix&)

Function

```
srsmatrix& srsmatrix::assign (int nRowCol, const srsmatrix& m)
throw (cvmexception);
```

sets main sub-matrix of a calling symmetric matrix beginning with 1-based row `nRowCol` to a symmetric matrix `m` and returns a reference to the matrix changed. The function throws an exception of type `cvmexception` if `nRowCol` is not positive or matrix `m` doesn't fit. See also `srsmatrix`. Example:

```
using namespace cvm;

srsmatrix m1(5);
srsmatrix m2(2);
m1.set(1.);
m2.set(2.);
m1.assign(2,m2);
std::cout << m1;
```

prints

```
1 1 1 1 1
1 2 2 1 1
1 2 2 1 1
1 1 1 1 1
1 1 1 1 1
```

2.13.15 set (TR)

Function

```
srsmatrix& srsmatrix::set (TR x);
```

sets every element of a calling matrix to a value of parameter `x` and returns a reference to the matrix changed. See also `srsmatrix`. Example:

```
using namespace cvm;

srsmatrix m(3);
m.set(3.);
std::cout << m;
```

prints

```
3 3 3
3 3 3
3 3 3
```

2.13.16 set (int,int,TR)

Function

```
srsmatrix& srsmatrix::set (int nRow, int nCol, TR x);
```

sets both elements located on nRow's row and nCol's column and on nCol's row and nRow's column to a value of parameter x and returns a reference to the matrix changed (thus the matrix remains symmetric). The parameters passed are 1-based. The function throws an exception of type cvmexception if anyone of the parameters passed is outside of [1,msize()] range. See also srsmatrix. Example:

```
using namespace cvm;

srsmatrix m(3);
m.set(3.);
m.set(1,3,7.);
std::cout << m;
```

prints

```
3 3 7
3 3 3
7 3 3
```

2.13.17 set_diag (int,rvector)

Function

```
srsmatrix& srsmatrix::set_diag (int i, const rvector& v)
throw (cvmexception);
```

assigns vector v to an i-th diagonal of a matrix, where $i = 0$ for main diagonal, $i < 0$ for lower diagonals and $i > 0$ for upper ones. If $i \neq 0$, then the function assigns the vector to both i-th and $-i$-th diagonals (thus the matrix remains symmetric). The function returns a reference to the matrix changed. The function is 1-based. The function throws an exception of type cvmexception if the parameter i is outside of [-msize()+1,nsize()-1] range or if the vector v passed has a size not equal to msize()-abs(i). See also srsmatrix. Example:

```
using namespace cvm;

srsmatrix m(3);
rvector v(2);
m.set(3.);
v.set(1.);
m.set_diag(1,v);
std::cout << m;
```

prints

```
3 1 3
1 3 1
3 1 3
```

2.13.18 resize

Function

```
srsmatrix& srsmatrix::resize (int nNewMN) throw (cvmexception);
```

changes a size of a calling matrix to `nNewMN` by `nNewMN` and returns a reference to the matrix changed. In case of increasing of its size, the matrix is filled up with zeroes. The function throws an exception of type `cvmexception` in case of negative size passed or memory allocation failure. See also `srsmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
srsmatrix m(a,3);
std::cout << m << std::endl;
m.resize(4);
std::cout << m;
```

prints

```
1 2 3
2 5 6
3 6 9

1 2 3 0
2 5 6 0
3 6 9 0
0 0 0 0
```

2.13.19 operator ==

Operator

```
bool srsmatrix::operator == (const srsmatrix& m) const;
```

compares a calling symmetric matrix with symmetric matrix `m` and returns `true` if they have the same sizes and their appropriate elements differ by not more than the smallest normalized positive number. Returns `false` otherwise. See also `srsmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 2., 3.};
srsmatrix m1(a, 2);
srsmatrix m2(2);
m2.set(1,1,1.);
m2.set(1,2,2.);
m2.set(2,2,3.);

std::cout << (m1 == m2) << std::endl;
```

prints

```
1
```

2.13.20 operator !=

Operator

```
bool srsmatrix::operator != (const srsmatrix& m) const;
```

compares a calling matrix with a matrix `m` and returns `true` if they have different sizes or at least one of their appropriate elements differs by more than the smallest normalized positive number. Returns `false` otherwise. See also `srsmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 2., 3.};
srsmatrix m1(a, 2);
srsmatrix m2(2);
m2.set(1,1,1.0001);
m2.set(1,2,2.);
m2.set(2,2,3.);

std::cout << (m1 != m2) << std::endl;
```

prints

```
1
```

2.13.21 operator <<

Operator

```
srsmatrix& srsmatrix::operator << (const srsmatrix& m)
throw (cvmexception);
```

destroys a calling matrix, creates a new one as a copy of `m` and returns a reference to the matrix changed. The operator throws an exception of type `cvmexception` in case of memory allocation failure. See also `srsmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
srsmatrix m(a, 3);
srsmatrix mc(1);
std::cout << m << std::endl << mc << std::endl;
mc << m;
std::cout << mc;
```

prints

```
1 2 3
2 5 6
3 6 9

0

1 2 3
2 5 6
3 6 9
```

2.13.22 operator +

Operator

```
srsmatrix srsmatrix::operator + (const srsmatrix& m) const
throw (cvmexception);
```

creates an object of type `srsmatrix` as a sum of a calling symmetric matrix and symmetric matrix `m`. It throws an exception of type `cvmexception` in case of different sizes of the operands. See also `srsmatrix::sum`, `srsmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
srsmatrix m1(a, 3);
srsmatrix m2(3);
m2.set(1.);
std::cout << m1 + m2 << std::endl << m1 + m1;
```

prints

```
2 3 4
3 6 7
4 7 10

2 4 6
4 10 12
6 12 18
```

2.13.23 operator -

Operator

```
srsmatrix srsmatrix::operator - (const srsmatrix& m) const
throw (cvmexception);
```

creates an object of type `srsmatrix` as a difference of a calling symmetric matrix and symmetric matrix `m`. It throws an exception of type `cvmexception` in case of different sizes of the operands. See also `srsmatrix::diff`, `srsmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
srsmatrix m1(a, 3);
srsmatrix m2(3);
m2.set(1.);
std::cout << m1 - m2 << std::endl << m1 - m1;
```

prints

```
0 1 2
1 4 5
2 5 8

0 0 0
0 0 0
0 0 0
```

2.13.24 sum

Function

```
srsmatrix& srsmatrix::sum (const srsmatrix& m1, const srsmatrix& m2)
throw (cvmexception);
```

assigns a result of addition of symmetric matrices `m1` and `m2` to a calling symmetric matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes of the operands. See also `srsmatrix::operator +` , `srsmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
const srsmatrix m1(a, 3);
srsmatrix m2(3);
srsmatrix m(3);
m2.set(1.);

std::cout << m.sum(m1, m2) << std::endl;
std::cout << m.sum(m, m2);
```

prints

```
2 3 4
3 6 7
4 7 10

3 4 5
4 7 8
5 8 11
```

2.13.25 diff

Function

```
srsmatrix& srsmatrix::diff (const srsmatrix& m1, const srsmatrix& m2)
throw (cvmexception);
```

assigns a result of subtraction of symmetric matrices `m1` and `m2` to a calling symmetric matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes of the operands. See also `srsmatrix::operator -` , `srsmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
const srsmatrix m1(a, 3);
srsmatrix m2(3);
srsmatrix m(3);
m2.set(1.);

std::cout << m.diff(m1, m2) << std::endl;
std::cout << m.diff(m, m2);
```

prints

```
0 1 2
1 4 5
2 5 8

-1 0 1
0 3 4
1 4 7
```

2.13.26 operator +=

Operator

```
srsmatrix& srsmatrix::operator += (const srsmatrix& m)
throw (cvmexception);
```

adds symmetric matrix `m` to a calling symmetric matrix and returns a reference to the matrix changed. It throws an exception of type `cvmexception` in case of different sizes of the operands. See also `srsmatrix::operator +` , `srsmatrix::sum`, `srsmatrix`. Example:

```
using namespace cvm;

srsmatrix m1(3);
srsmatrix m2(3);
m1.set(1.);
m2.set(2.);

m1 += m2;
std::cout << m1 << std::endl;

// well, you can do this too, but temporary object would be created
m2 += m2;
std::cout << m2;
```

prints

```
3 3 3
3 3 3
3 3 3

4 4 4
4 4 4
4 4 4
```

2.13.27 operator -=

Operator

```
srsmatrix& srsmatrix::operator -= (const srsmatrix& m)
throw (cvmexception);
```

subtracts symmetric matrix m from a calling symmetric matrix and returns a reference to the matrix changed. It throws an exception of type cvmexception in case of different sizes of the operands. See also srsmatrix::operator - , srsmatrix::diff, srsmatrix. Example:

```
using namespace cvm;

srsmatrix m1(3);
srsmatrix m2(3);
m1.set(1.);
m2.set(2.);

m1 -= m2;
std::cout << m1 << std::endl;

// well, you can do this too, but temporary object would be created
m2 -= m2;
std::cout << m2;
```

prints

```
-1 -1 -1
-1 -1 -1
-1 -1 -1

0 0 0
0 0 0
0 0 0
```

2.13.28 operator - ()

Operator

```
srsmatrix srsmatrix::operator - () const throw (cvmexception);
```

creates an object of type `srsmatrix` as a calling symmetric matrix multiplied by -1. See also `srsmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
srsmatrix m(a, 3);
std::cout << -m;
```

prints

```
-1 -2 -3
-2 -5 -6
-3 -6 -9
```

2.13.29 operator ++

Operator

```
srsmatrix& srsmatrix::operator ++ ();
srsmatrix& srsmatrix::operator ++ (int);
```

adds identity matrix to a calling symmetric matrix and returns a reference to the matrix changed. See also srsmatrix. Example:

```
using namespace cvm;

srsmatrix m(4);
m.set(4.);
m++;
std::cout << m << std::endl;
std::cout << ++m;
```

prints

```
5 4 4 4
4 5 4 4
4 4 5 4
4 4 4 5

6 4 4 4
4 6 4 4
4 4 6 4
4 4 4 6
```

2.13.30 operator --

Operator

```
srsmatrix& srsmatrix::operator -- ();
srsmatrix& srsmatrix::operator -- (int);
```

subtracts identity matrix from a calling symmetric matrix and returns a reference to the matrix changed. See also srsmatrix. Example:

```
using namespace cvm;

srsmatrix m(4);
m.set(4.);
m--;
std::cout << m << std::endl;
std::cout << --m;
```

prints

```
3 4 4 4
4 3 4 4
4 4 3 4
4 4 4 3

2 4 4 4
4 2 4 4
4 4 2 4
4 4 4 2
```

2.13.31 operator * (TR)

Operator

```
srsmatrix srsmatrix::operator * (TR d) const;
```

creates an object of type `srsmatrix` as a product of a calling symmetric matrix and a number `d`. See also `srsmatrix::operator *=` , `srsmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
srsmatrix m(a,3);
std::cout << m * 5.;
```

prints

```
5 10 15
10 25 30
15 30 45
```

2.13.32 operator / (TR)

Operator

```
srsmatrix srsmatrix::operator / (TR d) const throw (cvmexception);
```

creates an object of type `srsmatrix` as a quotient of a calling symmetric matrix and a number `d`. It throws an exception of type `cvmexception` if `d` has an absolute value equal or less than the smallest normalized positive number. See also `srsmatrix::operator /=` , `srsmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
srsmatrix m(a,3);
std::cout << m / 4.;
```

prints

```
0.25 0.5 0.75
0.5 1.25 1.5
0.75 1.5 2.25
```

2.13.33 operator *= (TR)

Operator

```
srsmatrix& srsmatrix::operator *= (TR d);
```

multiplies a calling symmetric matrix by a number d and returns a reference to the matrix changed. See also srsmatrix::operator * , srsmatrix. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
srsmatrix m(a,3);
m *= 2.;
std::cout << m;
```

prints

```
2 4 6
4 10 12
6 12 18
```

2.13.34 operator /= (TR)

Operator

```
srsmatrix& srsmatrix::operator /= (TR d) throw (cvmexception);
```

divides a calling symmetric matrix by a number `d` and returns a reference to the matrix changed. It throws an exception of type `cvmexception` if `d` has an absolute value equal or less than the smallest normalized positive number. See also `srsmatrix::operator /`, `srsmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
srsmatrix m(a,3);
m /= 2.;
std::cout << m;
```

prints

```
0.5 1 1.5
1 2.5 3
1.5 3 4.5
```

2.13.35 normalize

Function

```
srsmatrix& srsmatrix::normalize ();
```

normalizes a calling symmetric matrix so its Euclidean norm becomes equal to 1 if it was greater than the smallest normalized positive number before the call (otherwise the function does nothing). See also `srsmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (3);
double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
srsmatrix m(a,3);
m.normalize();
std::cout << m << m.norm() << std::endl;
```

prints

```
6.984e-002 1.397e-001 2.095e-001
1.397e-001 3.492e-001 4.191e-001
2.095e-001 4.191e-001 6.286e-001
1.000e+000
```

2.13.36 transposition

Operator and functions

```
srsmatrix srsmatrix::operator ~ () const throw (cvmexception);
srsmatrix& srsmatrix::transpose (const srsmatrix& m) throw (cvmexception);
srsmatrix& srsmatrix::transpose ();
```

do nothing since a matrix calling is symmetric. They are provided to override the same member functions and operator of the class `srmatrix`. See also `srsmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
srsmatrix m(a,3);
std::cout << m - ~m;
```

prints

```
0 0 0
0 0 0
0 0 0
```

2.13.37 operator * (const rvector&)

Operator

```
rvector srsmatrix::operator * (const rvector& v) const
throw (cvmexception);
```

creates an object of type `rvector` as a product of a calling symmetric matrix and a vector `v`. It throws an exception of type `cvmexception` if the number of columns of the calling matrix differs from the size of the vector `v`. Use `rvector::mult` in order to get rid of a new object creation. See also `srsmatrix` and `rvector`. Example:

```
using namespace cvm;

try {
    srsmatrix m (4);
    rvector v(4);
    m.set(1.);
    v.set(1.);

    std::cout << m * v;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
4 4 4 4
```

2.13.38 operator * (const rmatrix&)

Operator

```
rmatrix srsmatrix::operator * (const rmatrix& m) const
throw (cvmexception);
```

creates an object of type `rmatrix` as a product of a calling symmetric matrix and a matrix `m`. It throws an exception of type `cvmexception` if the number of columns of the calling matrix differs from the number of rows of the matrix `m`. Use `rmatrix::mult` in order to get rid of a new object creation. See also `rmatrix` and `srsmatrix`. Example:

```
using namespace cvm;

try {
    srsmatrix ms(4);
    rmatrix m(4,2);
    ms.set(1.);
    m.set(2.);
    std::cout << ms * m;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
8 8
8 8
8 8
8 8
```

2.13.39 operator * (const srmatrix&)

Operator

```
srmatrix srsmatrix::operator * (const srmatrix& m) const
throw (cvmexception);
```

creates an object of type `srmatrix` as a product of a calling symmetric matrix and a matrix `m`. It throws an exception of type `cvmexception` if the operands have different sizes. Use `rmatrix::mult` in order to get rid of a new object creation. See also `srmatrix` and `srsmatrix`. Example:

```
using namespace cvm;

try {
    srsmatrix ms(3);
    srmatrix m(3);
    ms.set(1.);
    m.set(2.);
    std::cout << ms * m << std::endl;

    double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
    const srsmatrix ms2(a, 3);
    std::cout << ms2 * ms;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
6 6 6
6 6 6
6 6 6

6 6 6
13 13 13
18 18 18
```

2.13.40 syrk

Functions

```
srsmatrix&
srsmatrix::syrk (TR alpha, const rvector& v, TR beta)
throw (cvmexception);

srsmatrix&
srsmatrix::syrk (bool bTransp, TR alpha, const rmatrix& m, TR beta)
throw (cvmexception);
```

call one of ?SYRK routines of the BLAS library performing a matrix-vector operation defined for the first version as rank-1 update operation

$$C = \alpha\, v \cdot v' + \beta\, C,$$

and for the second version as

$$C = \alpha\, M \cdot M^T + \beta\, C \quad \text{or} \quad c = \alpha\, M^T \cdot M + \beta\, C.$$

Here α and β are real numbers (parameters `alpha` and `beta`), M is a matrix (parameter `m`), C is a calling symmetric matrix and v is a vector (parameter `v`). First operation for the second version of the function is performed if `bTransp` passed is `false` and second one otherwise. The function returns a reference to the matrix changed and throws an exception of type `cvmexception` in case of inappropriate sizes of the operands. See also `rvector`, `rmatrix` and `srsmatrix` Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4.};
rvector v(a,4);
srsmatrix ms(4);
ms.set(1.);
ms.syrk (2., v, 1.);
std::cout << ms << std::endl;

rmatrix m(4,2);
m(1) = v;
m(2).set(1.);
ms.syrk (false, 2., m, 0.);
std::cout << ms << std::endl;

srsmatrix ms2(2);
ms2.syrk (true, 1., m, 0.);
std::cout << ms2;
```

prints

```
3 5 7 9
5 9 13 17
7 13 19 25
9 17 25 33

4 6 8 10
6 10 14 18
8 14 20 26
10 18 26 34

30 10
10 4
```

2.13.41 syr2k

Functions

```
srsmatrix&
srsmatrix::syr2k (TR alpha, const rvector& v1,
                  const rvector& v2, TR beta) throw (cvmexception);

srsmatrix&
srsmatrix::syr2k (bool bTransp, TR alpha, const rmatrix& m1,
                  const rmatrix& m2, TR beta) throw (cvmexception);
```

call one of ?SYR2K routines of the BLAS library performing a matrix-vector operation defined for the first version as rank-1 update operation

$$C = \alpha v_1 \cdot v_2' + \alpha v_2 \cdot v_1' + \beta C,$$

and for the second version as

$$C = \alpha M_1 \cdot M_2' + \alpha M_2 \cdot M_1' + \beta C \quad \text{or} \quad C = \alpha M_1' \cdot M_2 + \alpha M_2' \cdot M_1 + \beta C.$$

Here α and β are real numbers (parameters `alpha` and `beta`), M_1 and M_2 are matrices (parameters `m1` and `m2`), C is a calling symmetric matrix and v_1 and v_2 are vectors (parameters `v1` and `v2`). First operation for the second version of the function is performed if `bTransp` passed is `false` and second one otherwise. The function returns a reference to the matrix changed and throws an exception of type `cvmexception` in case of inappropriate sizes of the operands. See also `rvector`, `rmatrix` and `srsmatrix` Example:

```
using namespace cvm;

double a1[] = {1., 2., 3., 4.};
double a2[] = {1., 2., 3., 4.};
rvector v1(a1,4);
rvector v2(a2,4);
srsmatrix ms(4);
ms.set(1.);
ms.syr2k (2., v1, v2, 1.);
std::cout << ms << std::endl;

rmatrix m1(4,2);
rmatrix m2(4,2);
m1.set(1.);
m2.set(2.);
ms.syr2k (false, 2., m1, m2, 0.);
std::cout << ms << std::endl;
```

```
srsmatrix ms2(2);
ms2.syr2k (true, 1., m1, m2, 0.);
std::cout << ms2;
```

prints

```
5 9 13 17
9 17 25 33
13 25 37 49
17 33 49 65

16 16 16 16
16 16 16 16
16 16 16 16
16 16 16 16

16 16
16 16
```

2.13.42 inv

Functions

```
srsmatrix& srsmatrix::inv (const srsmatrix& m) throw (cvmexception);
srsmatrix srsmatrix::inv () const throw (cvmexception);
```

implement symmetric matrix inversion. The first version sets a calling symmetric matrix to be equal to a symmetric matrix `m` inverted and the second one creates an object of type `srsmatrix` as inverted calling matrix. The functions throw exception of type `cvmexception` in case of inappropriate sizes of the operands or when the matrix to be inverted is close to cingular. See also `srsmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (5);

double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.05};
const srsmatrix m(a, 3);
const srsmatrix mi = m.inv();

std::cout << mi << std::endl;
std::cout << mi * m - eye_real(3);
```

prints

```
1.85000e+002 -2.00000e+000 -6.00000e+001
-2.00000e+000 1.00000e+000 0.00000e+000
-6.00000e+001 0.00000e+000 2.00000e+001

0.00000e+000 0.00000e+000 0.00000e+000
0.00000e+000 0.00000e+000 0.00000e+000
0.00000e+000 0.00000e+000 0.00000e+000
```

2.13.43 exp

Functions

```
srsmatrix& srsmatrix::exp (const srsmatrix& m, TR tol = cvmMachSp())
throw (cvmexception);

srsmatrix srsmatrix::exp (TR tol = cvmMachSp()) const
throw (cvmexception);
```

compute an exponent of a calling symmetric matrix using Padé approximation defined as

$$R_{pq}(z) = D_{pq}(z)^{-1} N_{pq}(z) = 1 + z + \cdots + z^p/p!\,,$$

where

$$N_{pq}(z) = \sum_{k=0}^{p} \frac{(p+q-k)!p!}{(p+q)!k!(q-k)!} z^k,$$
$$D_{pq}(z) = \sum_{k=0}^{q} \frac{(p+q-k)!p!}{(p+q)!k!(q-k)!} (-z)^k$$

along with the matrix normalizing as described in [2], p. 572. The functions use `DMEXP` (or `SMEXP` for `float` version) FORTRAN subroutine implementing the algorithm. The first version sets the calling symmetric matrix to be equal to the exponent of a symmetric matrix `m` and returns a reference to the matrix changed. The second version creates an object of type `srsmatrix` as the exponent of the calling matrix. The algorithm uses parameter `tol` as $\varepsilon(p, q)$ in order to choose constants p and q so that

$$\varepsilon(p, q) \geqslant 2^{3-(p+q)} \frac{p!q!}{(p+q)!(p+q+1)!}.$$

This parameter is equal to the largest relative spacing by default. The functions throw an exception of type `cvmexception` in case of inappropriate sizes of the operands or when LAPACK subroutine fails. See also `srsmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (15);

double a[] = {1., 2., 1., 2., 0., -1., 1., -1., 2.};
const srsmatrix m(a, 3);
std::cout << m.exp();
```

prints

```
9.198262499129184e+000 5.558586002658855e+000 3.852443363622591e+000
5.558586002658857e+000 5.345819135506593e+000 -1.706142639036265e+000
3.852443363622590e+000 -1.706142639036266e+000 1.090440513816545e+001
```

Matlab output:

```
  Columns 1 through 2

    9.198262499129212e+000    5.558586002658862e+000
    5.558586002658865e+000    5.345819135506588e+000
    3.852443363622600e+000   -1.706142639036258e+000

  Column 3

    3.852443363622601e+000
   -1.706142639036260e+000
    1.090440513816545e+001
```

2.13.44 polynomial

Functions

```
srsmatrix& srsmatrix::polynom (const srsmatrix& m, const rvector& v)
throw (cvmexception);

srsmatrix srsmatrix::polynom (const rvector& v) const
throw (cvmexception);
```

compute a symmetric matrix polynomial defined as

$$p(A) = b_0 I + b_1 A + \cdots + b_q A^q$$

using the Horner's rule:

$$p(A) = \sum_{k=0}^{r} B_k (A^s)^k, \quad s = \text{floor}(\sqrt{q}\,),\ r = \text{floor}(q/s)$$

where

$$B_k = \begin{cases} \sum\limits_{i=0}^{s-1} b_{sk+i} A^i, & k = 0, 1, \ldots, r-1 \\ \sum\limits_{i=0}^{q-sr} b_{sr+i} A^i, & k = r. \end{cases}$$

See also [2], p. 568. The coefficients $b_0, b_1, \ldots, b_q$ are passed in the parameter v, where q is equal to `v.size()-1`, so the functions compute matrix polynomial equal to

$$\mathtt{v}[1] * \mathtt{I} + \mathtt{v}[2] * \mathtt{m} + \cdots + \mathtt{v}[\mathtt{v.size}()] * \mathtt{m}^{\mathtt{v.size}()-1}$$

The first version sets a calling symmetric matrix to be equal to the polynomial of a symmetric matrix `m` and the second one creates an object of type `srsmatrix` as the polynomial of a calling symmetric matrix. The functions use `DPOLY` (or `SPOLY` for `float` version) FORTRAN subroutine implementing the Horner's algorithm. The functions throw an exception of type `cvmexception` in case of inappropriate sizes of the operands. See also `srsmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (7);
double a[] = {1., 2., 1., 2., 0., -1., 1., -1., 2.};
double av[] = {2.2, 1.3, 1.1, -0.9, 0.2,
               -0.45, 45, -30, 10, 3, 3.2};
const rvector v(av, 11);
const srsmatrix m(a, 3);

std::cout << m.polynom (v);
```

prints

```
6.2127400e+004 2.3998000e+004 3.4100550e+004
2.3998000e+004 2.8026850e+004 1.0102550e+004
3.4100550e+004 1.0102550e+004 5.2024850e+004
```

Matlab output:

```
  Columns 1 through 2

    6.212740000000001e+004    2.399800000000000e+004
    2.399800000000000e+004    2.802685000000000e+004
    3.410055000000000e+004    1.010255000000000e+004

  Column 3

    3.410055000000000e+004
    1.010255000000000e+004
    5.202485000000000e+004
```

2.13.45 eig

Functions

```
rvector srsmatrix::eig (srmatrix& mEigVect) const throw (cvmexception);
rvector srsmatrix::eig () const throw (cvmexception);
```

solve a symmetric eigenvalue problem and return a real vector with eigenvalues of a calling symmetric matrix. The first version sets the output parameter `mEigVect` to be equal to the square matrix containing orthogonal eigenvectors as columns. All the functions throw an exception of type `cvmexception` in case of inappropriate sizes of the operands or in case of convergence error. See also `rvector`, `srmatrix` and `srsmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (10);
double a[] = {1., 2., 1., 2., 0., -1., 1., -1., 2.};
const srsmatrix m(a, 3);
srmatrix me(3);
rvector v(3);

v = m.eig(me);
std::cout << v << std::endl;

std::cout << m * me(1) - me(1) * v(1);
std::cout << m * me(2) - me(2) * v(2);
std::cout << m * me(3) - me(3) * v(3);
```

prints

```
-2.0489173395e+000 2.3568958679e+000 2.6920214716e+000

4.4408920985e-016 0.0000000000e+000 5.5511151231e-016
-1.1102230246e-016 2.2204460493e-016 2.2204460493e-016
0.0000000000e+000 -1.1102230246e-016 -4.4408920985e-016
```

2.13.46 Cholesky

Function

```
srmatrix srsmatrix::cholesky () const throw (cvmexception);
```

forms the Cholesky factorization of a symmetric positive-definite matrix A defined as

$$A = U^T U,$$

where U is upper triangular matrix. It utilizes one of ?POTRF routines of the LAPACK library. The function creates an object of type srmatrix as the factorization of a calling matrix. The function throws an exception of type cvmexception in case of convergence error. See also srmatrix and srsmatrix. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 1., 2., 5., -1., 1., -1., 20.};
    const srsmatrix m(a, 3);

    srmatrix h = m.cholesky();
    std::cout << h << std::endl;
    std::cout << ~h * h - m;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
1 2 1
0 1 -3
0 0 3.16228

0 0 0
0 0 0
0 0 0
```

2.13.47 Bunch-Kaufman

Function

```
srmatrix srsmatrix::bunch_kaufman () throw (cvmexception);
```

forms the Bunch-Kaufman factorization of a calling symmetric matrix (cited from the MKL library documentation):

$$A = PUDU^T P^T,$$

where A is the calling matrix, P is a permutation matrix, U and L are upper and lower triangular matrices with unit diagonal, and D is a symmetric block-diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks. U and L have 2-by-2 unit diagonal blocks corresponding to the 2-by-2 blocks of D. It utilizes one of `?SYTRF` routines of the LAPACK library. The function creates an object of type `srmatrix` as the factorization of a calling matrix. The function throws an exception of type `cvmexception` in case of convergence error. See also `srmatrix` and `srsmatrix`. The function is mostly designed to be used for subsequent calls of `?SYTRS`, `?SYCON` and `?SYTRI` routines of the LAPACK library. Currently it's used internally in `srmatrix::det` flow when argument is symmetric but not positive-definite.

2.13.48 identity

Function

```
srsmatrix& srsmatrix::identity();
```

sets a calling symmetric matrix to be equal to identity matrix and returns a reference to the matrix changed. See also srsmatrix. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (3);
srsmatrix m(3);
m.randomize(0.,1.);

std::cout << m << std::endl;
std::cout << m.identity();
```

prints

```
1.329e-001 8.527e-001 3.110e-001
8.527e-001 6.152e-001 3.247e-001
3.110e-001 3.247e-001 9.145e-001

1.000e+000 0.000e+000 0.000e+000
0.000e+000 1.000e+000 0.000e+000
0.000e+000 0.000e+000 1.000e+000
```

2.13.49 vanish

Function

```
srsmatrix& srsmatrix::vanish();
```

sets every element of a calling symmetric matrix to be equal to zero and returns a reference to the matrix changed. See also srsmatrix. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (3);
srsmatrix m(3);
m.randomize(0.,1.);

std::cout << m << std::endl;
std::cout << m.vanish();
```

prints

```
1.422e-001 1.477e-001 1.445e-001
1.477e-001 8.893e-001 1.669e-002
1.445e-001 1.669e-002 7.766e-001

0.000e+000 0.000e+000 0.000e+000
0.000e+000 0.000e+000 0.000e+000
0.000e+000 0.000e+000 0.000e+000
```

2.13.50 randomize

Function

```
srsmatrix& srsmatrix::randomize (TR dFrom, TR dTo);
```

fills a calling symmetric matrix with pseudo-random numbers distributed between `dFrom` and `dTo` keeping it to be symmetric. The function returns a reference to the matrix changed. See also `srsmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (7);

srsmatrix m(3);
m.randomize(-2.,3.);
std::cout << m;
```

prints

```
-1.2277291e+000 3.6610004e-001 2.1380047e+000
3.6610004e-001 1.0336924e+000 -1.8565630e+000
2.1380047e+000 -1.8565630e+000 1.7774285e+000
```

2.14 schmatrix

This is end-user class encapsulating a hermitian matrix in Euclidean space of complex numbers.

```
template <typename TR, typename TC>
class schmatrix : public scmatrix <TR,TC> {
public:
    schmatrix ();
    explicit schmatrix (int nMN);
    schmatrix (TC* pD, int nMN, TR tol = cvmMachSp());
    schmatrix (const schmatrix& m);
    explicit schmatrix (const cmatrix& m, TR tol = cvmMachSp());
    explicit schmatrix (const rvector& v);
    explicit schmatrix (const srsmatrix& m);
    schmatrix (const TR* pRe, const TR* pIm, int nMN,
               TR tol = cvmMachSp());
    schmatrix (const srmatrix& mRe, const srmatrix& mIm,
               TR tol = cvmMachSp());
    schmatrix (schmatrix& m, int nRowCol, int nSize);
    TC operator () (int im, int in) const throw (cvmexception);
    const cvector operator () (int i) const throw (cvmexception);
    const cvector operator [] (int i) const throw (cvmexception);
    const cvector diag (int i) const throw (cvmexception);
    const srsmatrix real () const;
    const srmatrix imag () const;
    schmatrix& operator = (const schmatrix& m) throw (cvmexception);
    schmatrix& assign (const cvector& v, TR tol = cvmMachSp())
                       throw (cvmexception);
    schmatrix& assign (const TC* pD, TR tol = cvmMachSp())
                       throw (cvmexception);
    schmatrix& assign (int nRowCol, const schmatrix& m)
                       throw (cvmexception);
    schmatrix& set (int nRow, int nCol, TC z);
    schmatrix& set_diag (int i, const cvector& v) throw (cvmexception);
    schmatrix& set_main_diag (const rvector& v) throw (cvmexception);
    schmatrix& assign_real (const srsmatrix& m) throw (cvmexception);
    schmatrix& set_real (TR d) throw (cvmexception);
    schmatrix& resize (int nNewMN) throw (cvmexception);
    bool operator == (const schmatrix& v) const;
    bool operator != (const schmatrix& v) const;
    schmatrix& operator << (const schmatrix& m) throw (cvmexception);
    schmatrix operator + (const schmatrix& m) const
```

```
                      throw (cvmexception);
schmatrix operator - (const schmatrix& m) const
                      throw (cvmexception);
schmatrix& sum (const schmatrix& m1,
                const schmatrix& m2) throw (cvmexception);
schmatrix& diff (const schmatrix& m1,
                 const schmatrix& m2) throw (cvmexception);
schmatrix& operator += (const schmatrix& m) throw (cvmexception);
schmatrix& operator -= (const schmatrix& m) throw (cvmexception);
schmatrix operator - () const;
schmatrix& operator ++ ();
schmatrix& operator ++ (int);
schmatrix& operator -- ();
schmatrix& operator -- (int);
schmatrix operator * (TR d) const;
schmatrix operator / (TR d) const throw (cvmexception);
scmatrix operator * (TC z) const;
scmatrix operator / (TC z) const throw (cvmexception);
schmatrix& operator *= (TR d);
schmatrix& operator /= (TR d) throw (cvmexception);
schmatrix& normalize ();
schmatrix operator ~ () const;
schmatrix& conj (const schmatrix& m) throw (cvmexception);
schmatrix& conj ();
cvector operator * (const cvector& v) const throw (cvmexception);
cmatrix operator * (const cmatrix& m) const throw (cvmexception);
scmatrix operator * (const scmatrix& m) const throw (cvmexception);
schmatrix& herk (TC alpha,
                 const cvector& v, TC beta) throw (cvmexception);
schmatrix& herk (bool bTransp, TC alpha,
                 const cmatrix& m, TC beta) throw (cvmexception);
schmatrix& her2k (TC alpha,
                 const cvector& v1, const cvector& v2, TC beta)
                 throw (cvmexception);
schmatrix& her2k (bool bTransp, TC alpha,
                 const cmatrix& m1, const cmatrix& m2, TC beta)
                 throw (cvmexception);
schmatrix& inv (const schmatrix& mArg) throw (cvmexception);
schmatrix inv () const throw (cvmexception);
schmatrix& exp (const schmatrix& m,
                TR tol = cvmMachSp ()) throw (cvmexception);
schmatrix exp (TR tol = cvmMachSp ()) const throw (cvmexception);
schmatrix& polynom (const schmatrix& m, const rvector& v)
```

```
                        throw (cvmexception);
    schmatrix polynom (const rvector& v) const throw (cvmexception);
    rvector eig (scmatrix& mEigVect) const throw (cvmexception);
    rvector eig () const throw (cvmexception);
    scmatrix cholesky () const throw (cvmexception);
    scmatrix bunch_kaufman () const throw (cvmexception);
    schmatrix& identity ();
    schmatrix& vanish ();
    schmatrix& randomize_real (TR dFrom, TR dTo);
    schmatrix& randomize_imag (TR dFrom, TR dTo);
};
```

2.14.1 schmatrix ()

Constructor

```
schmatrix::schmatrix ();
```

creates an empty schmatrix object. See also schmatrix. Example:

```
using namespace cvm;

schmatrix m;
std::cout << m.msize() << " " << m.nsize() << " "
          << m.size() << std::endl;

m.resize (3);
std::cout << m;
```

prints

```
0 0 0
(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
```

2.14.2 schmatrix (int)

Constructor

```
explicit schmatrix::schmatrix (int nMN);
```

creates an $n \times n$ `schmatrix` object where n is passed in `nMN` parameter. The constructor throws an exception of type `cvmexception` in case of non-positive size passed or memory allocation failure. See also `schmatrix`. Example:

```
using namespace cvm;

schmatrix m (4);
std::cout << m.msize() << std::endl
          << m.nsize() << std::endl
          << m.size() << std::endl << m;
```

prints

```
4
4
16
(0,0) (0,0) (0,0) (0,0)
(0,0) (0,0) (0,0) (0,0)
(0,0) (0,0) (0,0) (0,0)
(0,0) (0,0) (0,0) (0,0)
```

2.14.3 schmatrix (TC*,int)

Constructor

```
schmatrix::schmatrix (TC* pD, int nMN, TR tol = cvmMachSp());
```

creates an $n \times n$ scmatrix object where n is passed in nMN parameter. Unlike others, this constructor *does not allocate a memory*. It just shares a memory with an array pointed to by pD. The constructor throws an exception of type cvmexception if the matrix created doesn't appear to be hermitian (tolerance is set by parameter tol). If subsequent application flow would change the array passed so it becomes not a hermitian matrix anymore, results are not predictable. See also schmatrix. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
const schmatrix m ((std::complex<double>*)a, 3);
std::cout << m;
```

prints

```
(1,0) (2,-1) (-1,-2)
(2,1) (2,0) (0,-3)
(-1,2) (0,3) (3,0)
```

2.14.4 schmatrix (const schmatrix&)

Copy constructor

```
schmatrix::schmatrix (const schmatrix& m)
```

creates a `schmatrix` object as a copy of `m`. The constructor throws an exception of type `cvmexception` in case of memory allocation failure. See also `schmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left |
                std::ios::showpos);
std::cout.precision (1);
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m ((std::complex<double>*)a, 3);
scmatrix mc(m);

m.set(1,2, std::complex<double>(7.7,7.7));
std::cout << m << std::endl << mc;
```

prints

```
(+1.0e+000,+0.0e+000) (+7.7e+000,+7.7e+000) (-1.0e+000,-2.0e+000)
(+7.7e+000,-7.7e+000) (+2.0e+000,+0.0e+000) (+0.0e+000,-3.0e+000)
(-1.0e+000,+2.0e+000) (+0.0e+000,+3.0e+000) (+3.0e+000,+0.0e+000)

(+1.0e+000,+0.0e+000) (+2.0e+000,-1.0e+000) (-1.0e+000,-2.0e+000)
(+2.0e+000,+1.0e+000) (+2.0e+000,+0.0e+000) (+0.0e+000,-3.0e+000)
(-1.0e+000,+2.0e+000) (+0.0e+000,+3.0e+000) (+3.0e+000,+0.0e+000)
```

2.14.5 schmatrix (const cmatrix&)

Constructor

```
explicit schmatrix::schmatrix (const cmatrix& m, TR tol = cvmMachSp())
```

creates a schmatrix object as a copy of matrix m. It's assumed that $m \times n$ matrix m must have equal sizes, i.e. $m = n$ is satisfied and it has to be a hermitian one (tolerance is set by parameter tol). The constructor throws an exception of type cvmexception if this is not true or in case of memory allocation failure. See also schmatrix and cmatrix. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
cmatrix m ((std::complex<double>*)a, 3, 3);
scmatrix mch(m);
std::cout << mch;
```

prints

```
(1,0) (2,-1) (-1,-2)
(2,1) (2,0) (0,-3)
(-1,2) (0,3) (3,0)
```

2.14.6 schmatrix (const rvector&)

Constructor

```
explicit schmatrix::schmatrix (const rvector& v);
```

creates a `schmatrix` object of size `v.size()` by `v.size()` and assigns vector `v` to its main diagonal. The constructor throws an exception of type `cvmexception` in case of memory allocation failure. See also `schmatrix` and `rvector`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5.};
const rvector v (a, 5);
schmatrix m(v);
std::cout << m.msize() << " " << m.nsize() << " "
          << m.size() << std::endl << m;
```

prints

```
5 5 25
(1,0) (0,0) (0,0) (0,0) (0,0)
(0,0) (2,0) (0,0) (0,0) (0,0)
(0,0) (0,0) (3,0) (0,0) (0,0)
(0,0) (0,0) (0,0) (4,0) (0,0)
(0,0) (0,0) (0,0) (0,0) (5,0)
```

2.14.7 schmatrix (const srsmatrix&)

Constructor

```
explicit schmatrix::schmatrix (const srsmatrix& m);
```

creates a `schmatrix` object having the same dimension as real symmetric matrix `m` and copies the matrix `m` to its real part. See also `schmatrix` and `srsmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
srsmatrix m(a, 3);
schmatrix mch(m);
std::cout << mch;
```

prints

```
(1,0) (2,0) (3,0)
(2,0) (5,0) (6,0)
(3,0) (6,0) (9,0)
```

2.14.8 schmatrix (const TR*,const TR*,int)

Constructor

```
schmatrix::schmatrix (const TR* pRe, const TR1* pIm, int nMN,
                      TR tol = cvmMachSp());
```

creates a `schmatrix` object of size `nMN` by `nMN` and copies every element of arrays pointed to by `pRe` and `pIm` to a real and imaginary part of the matrix created respectively. Use `NULL` pointer to fill up appropriate part with zero values. The constructor throws an exception of type `cvmexception` if the matrix created doesn't appear to be hermitian (tolerance is et by parameter `tol`) or in case of memory allocation failure. See also `schmatrix`. Example:

```
using namespace cvm;

double re[] = {1., 2., -1., 2., 2., 0., -1., 0., 3.};
double im[] = {0., 1., 2., -1., 0., 3., -2., -3., 0.};
schmatrix m(re, im, 3);
std::cout << m;
```

prints

```
(1,0) (2,-1) (-1,-2)
(2,1) (2,0) (0,-3)
(-1,2) (0,3) (3,0)
```

2.14.9 schmatrix (const srmatrix&, const srmatrix&)

Constructor

```
schmatrix::schmatrix (const srmatrix& mRe, const srmatrix& mIm,
                      TR tol = cvmMachSp());
```

creates a `schmatrix` object of the same size as `mRe` and `mIm` has (the constructor throws an exception of type `cvmexception` if `mRe` and `mIm` have different sizes) and copies matrices `mRe` and `mIm` to a real and imaginary part of the matrix created respectively. The constructor throws an exception of type `cvmexception` if the matrix created doesn't appear to be hermitian (tolerance is et by parameter `tol`) or in case of memory allocation failure. See also `schmatrix`, `srmatrix`. Example:

```
using namespace cvm;

double re[] = {1., 2., -1., 2., 2., 0., -1., 0., 3.};
double im[] = {0., 1., 2., -1., 0., 3., -2., -3., 0.};
srmatrix mr(re, 3);
srmatrix mi(im, 3);
schmatrix m(mr, mi);
std::cout << m;
```

prints

```
(1,0) (2,-1) (-1,-2)
(2,1) (2,0) (0,-3)
(-1,2) (0,3) (3,0)
```

2.14.10 submatrix

Submatrix constructor

```
schmatrix::schmatrix (schmatrix& m, int nRowCol, int nSize);
```

creates a `scmatrix` object as a *submatrix* of `m`. It means that the matrix object created shares a memory with some part of `m`. This part is defined by its upper left corner (parameter `nRowCol`, 1-based) and its size (parameter `nSize`). See also `schmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m ((std::complex<double>*)a, 3);
schmatrix m2 (m, 1, 2);

std::cout << m2 << std::endl;
m2.set_real(7.7);
std::cout << m;
```

prints

```
(1,0) (2,-1)
(2,1) (2,0)

(7.7,0) (7.7,-1) (-1,-2)
(7.7,1) (7.7,0) (0,-3)
(-1,2) (0,3) (3,0)
```

2.14.11 operator (,)

Indexing operator

```
TC schmatrix::operator () (int im, int in) const throw (cvmexception);
```

returns an element of a hermitian matrix located on im-row and in-th column. The operator is 1-based. The operator throws an exception of type cvmexception if some of parameters passed is outside of [1,msize()] range. See also schmatrix. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m ((std::complex<double>*)a, 3);
std::cout << m << std::endl;
std::cout << m(3,2) << " " << m(1,3) << std::endl;
```

prints

```
(1,0) (2,-1) (-1,-2)
(2,1) (2,0) (0,-3)
(-1,2) (0,3) (3,0)

(0,3) (-1,-2)
```

2.14.12 operator ()

Indexing operator

`const cvector schmatrix::operator () (int i) const throw (cvmexception);`

provides access to an i-th column of a hermitian matrix. Unlike scmatrix::operator (), this operator creates only a *copy* of a column and therefore it returns *not an l-value*. The operator is 1-based. The operator throws an exception of type `cvmexception` if the parameter `i` is outside of `[1,nsize()]` range. See also `schmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m ((std::complex<double>*)a, 3);
std::cout << m << std::endl;
std::cout << m(1) << std::endl << m(3);
```

prints

```
(1,0) (2,-1) (-1,-2)
(2,1) (2,0) (0,-3)
(-1,2) (0,3) (3,0)

(1,0) (2,1) (-1,2)

(-1,-2) (0,-3) (3,0)
```

2.14.13 operator []

Indexing operator

```
const cvector schmatrix::operator [] (int i) const throw (cvmexception);
```

provides access to an i-th row of a hermitian matrix. Unlike scmatrix::operator [], this operator creates only a *copy* of a column and therefore it returns *not an l-value*. The operator is 1-based. The operator throws an exception of type `cvmexception` if the parameter `i` is outside of `[1,nsize()]` range. See also `schmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m ((std::complex<double>*)a, 3);
std::cout << m << std::endl;
std::cout << m[1] << std::endl << m[3];
```

prints

```
(1,0) (2,-1) (-1,-2)
(2,1) (2,0) (0,-3)
(-1,2) (0,3) (3,0)

(1,0) (2,-1) (-1,-2)

(-1,2) (0,3) (3,0)
```

2.14.14 diag

Functions

```
const cvector schmatrix::diag (int i) const throw (cvmexception);
```

provide access to an i-th diagonal of a matrix, where $i = 0$ for main diagonal, $i < 0$ for lower diagonals and $i > 0$ for upper ones. Unlike cmatrix::diag, this operator creates only a *copy* of a diagonal and therefore it returns *not an l-value*. The function throws an exception of type `cvmexception` if the parameter `i` is outside of `[-msize()+1,nsize()-1]` range. See also `schmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m ((std::complex<double>*)a, 3);
std::cout << m << std::endl;
std::cout << m.diag(-2)
          << m.diag(-1)
          << m.diag(0)
          << m.diag(1)
          << m.diag(2);
```

prints

```
(1,0) (2,-1) (-1,-2)
(2,1) (2,0) (0,-3)
(-1,2) (0,3) (3,0)

(-1,2)
(2,1) (0,3)
(1,0) (2,0) (3,0)
(2,-1) (0,-3)
(-1,-2)
```

2.14.15 real

Function

```
const srsmatrix schmatrix::real () const;
```

creates an object of type `const srsmatrix` as a real part of a calling hermitian matrix. Please note that, unlike cvector::real, this function creates new object *not sharing* a memory with a real part of the calling matrix, i.e. the matrix returned is *not an l-value*. See also `srsmatrix`, `schmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m ((std::complex<double>*)a, 3);
std::cout << m << std::endl;
std::cout << m.real();
```

prints

```
(1,0) (2,-1) (-1,-2)
(2,1) (2,0) (0,-3)
(-1,2) (0,3) (3,0)

1 2 -1
2 2 0
-1 0 3
```

2.14.16 imag

Function

```
const srmatrix schmatrix::imag () const;
```

creates an object of type `const srmatrix` as an imaginary part of a calling matrix. Please note that, unlike cvector::imag, this function creates new object *not sharing* a memory with an imaginary part of the calling matrix, i.e. the matrix returned is *not an l-value*. See also `srmatrix`, `schmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m ((std::complex<double>*)a, 3);
std::cout << m << std::endl;
std::cout << m.imag();
```

prints

```
(1,0) (2,-1) (-1,-2)
(2,1) (2,0) (0,-3)
(-1,2) (0,3) (3,0)

0 -1 -2
1 0 -3
2 3 0
```

2.14.17 operator = (const schmatrix&)

Operator

```
schmatrix& schmatrix::operator = (const schmatrix& m)
throw (cvmexception);
```

sets an every element of a calling hermitian matrix to a value of appropriate element of a hermitian matrix m and returns a reference to the matrix changed. The operator throws an exception of type cvmexception in case of different matrix sizes. See also schmatrix. Example:

```
using namespace cvm;

try {
    double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
                  0., 3., -1., -2., 0., -3., 3., 0.};
    schmatrix m1((std::complex<double>*)a, 3);
    schmatrix m2(3);

    m2 = m1;
    std::cout << m2;
}
catch (exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(1,0) (2,-1) (-1,-2)
(2,1) (2,0) (0,-3)
(-1,2) (0,3) (3,0)
```

2.14.18 assign (const TC*)

Function

```
schmatrix& schmatrix::assign (const cvector& v, TR tol = cvmMachSp())
throw (cvmexception);

schmatrix& schmatrix::assign (const TC* pD, TR tol = cvmMachSp())
throw (cvmexception);
```

sets every element of a calling hermitian matrix to a value of appropriate element of a vector `v` or an array pointed to by `pD` and returns a reference to the matrix changed. The function throws an exception of type `cvmexception` if the matrix changed doesn't appear to be hermitian (tolerance is set by parameter `tol`). See also `schmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m(3);
m.assign((std::complex<double>*)a);
std::cout << m;
```

prints

```
(1,0) (2,-1) (-1,-2)
(2,1) (2,0) (0,-3)
(-1,2) (0,3) (3,0)
```

2.14.19 assign (int, int, const schmatrix&)

Function

```
schmatrix& schmatrix::assign (int nRowCol, const schmatrix& m)
throw (cvmexception);
```

sets main sub-matrix of a calling hermitian matrix beginning with 1-based row `nRowCol` to a hermitian matrix `m` and returns a reference to the matrix changed. The function throws an exception of type `cvmexception` if `nRowCol` is not positive or matrix `m` doesn't fit. See also `schmatrix`. Example:

```
using namespace cvm;

schmatrix m1(5);
schmatrix m2(2);
m2.set_main_diag(rvector(2,2.));
m2.set(1,2,std::complex<double>(2.,2.));
m1.assign(2,m2);
std::cout << m1;
```

prints

```
(0,0) (0,0) (0,0) (0,0) (0,0)
(0,0) (2,0) (2,2) (0,0) (0,0)
(0,0) (2,-2) (2,0) (0,0) (0,0)
(0,0) (0,0) (0,0) (0,0) (0,0)
(0,0) (0,0) (0,0) (0,0) (0,0)
```

2.14.20 set (int,int,TC)

Function

```
schmatrix& schmatrix::set (int nRow, int nCol, TC z) throw (cvmexception);
```

sets an element located on nRow's row and nCol's column to a value of parameter z and an element located on nCol's row and nRow's column to a value of parameter z conjugated (keeping a calling matrix to be hermitian). The parameters passed are 1-based. The function returns a reference to the matrix changed. The function throws an exception of type cvmexception if anyone of the parameters passed is outside of [1,msize()] range. It also throws the exception in case of assigning a complex number with non-zero imaginary part to any element of the main diagonal. See also schmatrix. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);

m.set(1,2,std::complex<double>(7.7,7.7));
m.set(3,3,11.11);
std::cout << m;
```

prints

```
(1,0) (7.7,7.7) (-1,-2)
(7.7,-7.7) (2,0) (0,-3)
(-1,2) (0,3) (11.11,0)
```

2.14.21 set_diag

Function

```
schmatrix& schmatrix::set_diag (int i, const cvector& v)
throw (cvmexception);
```

sets an i-th diagonal of a calling hermitian matrix, where $i = 0$ for main diagonal, $i < 0$ for lower diagonals and $i > 0$ for upper ones, to be equal to a complex vector passed in parameter v. The function also sets $-i$-th diagonal to be equal to the vector v conjugated, thus keeping the matrix to be hermitian. The parameter i passed is 1-based. The function returns a reference to the matrix changed. The function throws an exception of type cvmexception if the parameter i passed is outside of [-msize()+1,nsize()-1] range or equal to zero. It also throws the exception if the vector v passed has a size not equal to msize()-abs(i). Use schmatrix::set_main_diag to set the main diagonal. See also schmatrix. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);
cvector v(2);
v.set(std::complex<double>(7.7,7.7));
m.set_diag(1, v);
std::cout << m;
```

prints

```
(1,0) (7.7,7.7) (-1,-2)
(7.7,-7.7) (2,0) (7.7,7.7)
(-1,2) (7.7,-7.7) (3,0)
```

2.14.22 set_main_diag

Function

```
schmatrix& schmatrix::set_main_diag (const rvector& v)
throw (cvmexception);
```

sets the main diagonal of a calling hermitian matrix to be equal to a real vector passed in parameter v. The function returns a reference to the matrix changed. The function throws an exception of type cvmexception if the vector v passed has a size not equal to msize(). See also schmatrix. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);
rvector v(3);
v.set(7.7);
m.set_main_diag(v);
std::cout << m;
```

prints

```
(7.7,0) (2,-1) (-1,-2)
(2,1) (7.7,0) (0,-3)
(-1,2) (0,3) (7.7,0)
```

2.14.23 assign_real

Function

```
schmatrix& schmatrix::assign_real (const srsmatrix& m)
throw (cvmexception);
```

sets real part of a calling hermitian matrix to be equal to a real symmetric matrix `m`. The function returns a reference to the matrix changed. The function throws an exception of type `cvmexception` in case of different sizes of the operands. See also `schmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);
srsmatrix ms(3);
ms.set(7.);
m.assign_real(ms);
std::cout << m;
```

prints

```
(7,0) (7,-1) (7,-2)
(7,1) (7,0) (7,-3)
(7,2) (7,3) (7,0)
```

2.14.24 set_real

Function

```
schmatrix& schmatrix::set_real (TR d);
```

sets every element of a real part of a calling hermitian matrix to be equal to a real number d. The function returns a reference to the matrix changed. See also schmatrix. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);
m.set_real(7.);
std::cout << m;
```

prints

```
(7,0) (7,-1) (7,-2)
(7,1) (7,0) (7,-3)
(7,2) (7,3) (7,0)
```

2.14.25 resize

Function

```
schmatrix& schmatrix::resize (int nNewMN) throw (cvmexception);
```

changes a size of a calling hermitian matrix to `nNewMN` by `nNewMN` and returns a reference to the matrix changed. In case of increasing of its size, the matrix is filled up with zeroes. The function throws an exception of type `cvmexception` in case of non-positive size passed or memory allocation failure. See also `schmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);
std::cout << m << std::endl;
m.resize(4);
std::cout << m;
```

prints

```
(1,0) (2,-1) (-1,-2)
(2,1) (2,0) (0,-3)
(-1,2) (0,3) (3,0)

(1,0) (2,-1) (-1,-2) (0,0)
(2,1) (2,0) (0,-3) (0,0)
(-1,2) (0,3) (3,0) (0,0)
(0,0) (0,0) (0,0) (0,0)
```

2.14.26 operator ==

Operator

```
bool schmatrix::operator == (const schmatrix& m) const;
```

compares a calling hermitian matrix with a hermitian matrix `m` and returns `true` if they have the same sizes and their appropriate elements differ by not more than the smallest normalized positive number. Returns `false` otherwise. See also `schmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m1((std::complex<double>*)a,3);
schmatrix m2(3);
m2.assign((std::complex<double>*)a);
std::cout << (m1 == m2) << std::endl;
```

prints

```
1
```

2.14.27 operator !=

Operator

```
bool schmatrix::operator != (const schmatrix& m) const;
```

compares a calling hermitian matrix with a hermitian matrix `m` and returns `true` if they have different sizes or at least one of their appropriate elements differs by more than the smallest normalized positive number. Returns `false` otherwise. See also `schmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m1((std::complex<double>*)a,3);
schmatrix m2(3);
m2.assign((std::complex<double>*)a);
m2.set(2,1,std::complex<double>(2.,1.000001));
std::cout << (m1 != m2) << std::endl;
```

prints

```
1
```

2.14.28 operator <<

Operator

```
schmatrix& schmatrix::operator << (const schmatrix& m)
throw (cvmexception);
```

destroys a calling hermitian matrix, creates a new one as a copy of `m` and returns a reference to the matrix changed. The operator throws an exception of type `cvmexception` in case of memory allocation failure. See also `schmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);
schmatrix mc(1);
std::cout << m << std::endl;
std::cout << mc << std::endl;
mc << m;
std::cout << mc;
```

prints

```
(1,0) (2,-1) (-1,-2)
(2,1) (2,0) (0,-3)
(-1,2) (0,3) (3,0)

(0,0)

(1,0) (2,-1) (-1,-2)
(2,1) (2,0) (0,-3)
(-1,2) (0,3) (3,0)
```

2.14.29 operator +

Operator

```
schmatrix schmatrix::operator + (const schmatrix& m) const
throw (cvmexception);
```

creates an object of type `schmatrix` as a sum of a calling hermitian matrix and a hermitian matrix `m`. It throws an exception of type `cvmexception` in case of different sizes of the operands. See also `schmatrix::sum`, `schmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
double b[] = {1., 0., 1., 1., 1., 1., 1., -1., 1., 0.,
              1., 1., 1., -1., 1., -1., 1., 0.};
schmatrix m1((std::complex<double>*)a,3);
schmatrix m2((std::complex<double>*)b,3);
std::cout << m1 + m2;
```

prints

```
(2,0) (3,-2) (0,-3)
(3,2) (3,0) (1,-4)
(0,3) (1,4) (4,0)
```

2.14.30 operator -

Operator

```
schmatrix schmatrix::operator - (const schmatrix& m) const
throw (cvmexception);
```

creates an object of type `schmatrix` as a difference of a calling hermitian matrix and a hermitian matrix `m`. It throws an exception of type `cvmexception` in case of different sizes of the operands. See also `schmatrix::diff`, `schmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
double b[] = {1., 0., 1., 1., 1., 1., 1., -1., 1., 0.,
              1., 1., 1., -1., 1., -1., 1., 0.};
schmatrix m1((std::complex<double>*)a,3);
schmatrix m2((std::complex<double>*)b,3);
std::cout << m1 - m2;
```

prints

```
(0,0) (1,0) (-2,-1)
(1,0) (1,0) (-1,-2)
(-2,1) (-1,2) (2,0)
```

2.14.31 sum

Function

```
schmatrix& schmatrix::sum (const schmatrix& m1, const schmatrix& m2)
throw (cvmexception);
```

assigns a result of addition of hermitian matrices m1 and m2 to a calling hermitian matrix and returns a reference to the matrix changed. It throws an exception of type cvmexception in case of different sizes of the operands. See also schmatrix::operator + , schmatrix. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
double b[] = {1., 0., 1., 1., 1., 1., 1., -1., 1., 0.,
              1., 1., 1., -1., 1., -1., 1., 0.};
schmatrix m1((std::complex<double>*)a,3);
schmatrix m2((std::complex<double>*)b,3);
schmatrix m(3);
std::cout << m.sum(m1, m2);
```

prints

```
(2,0) (3,-2) (0,-3)
(3,2) (3,0) (1,-4)
(0,3) (1,4) (4,0)
```

2.14.32 diff

Function

```
schmatrix& schmatrix::diff (const schmatrix& m1, const schmatrix& m2)
throw (cvmexception);
```

assigns a result of subtraction of hermitian matrices m1 and m2 to a calling hermitian matrix and returns a reference to the matrix changed. It throws an exception of type cvmexception in case of different sizes of the operands. See also schmatrix::operator - , schmatrix. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
double b[] = {1., 0., 1., 1., 1., 1., 1., -1., 1., 0.,
              1., 1., 1., -1., 1., -1., 1., 0.};
schmatrix m1((std::complex<double>*)a,3);
schmatrix m2((std::complex<double>*)b,3);
schmatrix m(3);
std::cout << m.diff(m1, m2);
```

prints

```
(0,0) (1,0) (-2,-1)
(1,0) (1,0) (-1,-2)
(-2,1) (-1,2) (2,0)
```

2.14.33 operator +=

Operator

```
schmatrix& schmatrix::operator += (const schmatrix& m)
throw (cvmexception);
```

adds a hermitian matrix m to a calling hermitian matrix and returns a reference to the matrix changed. It throws an exception of type cvmexception in case of different sizes of the operands. See also schmatrix::operator + , schmatrix::sum, schmatrix. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
double b[] = {1., 0., 1., 1., 1., 1., 1., -1., 1., 0.,
              1., 1., 1., -1., 1., -1., 1., 0.};
schmatrix m1((std::complex<double>*)a,3);
schmatrix m2((std::complex<double>*)b,3);
m1 += m2;
m2 += m2;
std::cout << m1 << std::endl;
std::cout << m2;
```

prints

```
(2,0) (3,-2) (0,-3)
(3,2) (3,0) (1,-4)
(0,3) (1,4) (4,0)

(2,0) (2,-2) (2,-2)
(2,2) (2,0) (2,-2)
(2,2) (2,2) (2,0)
```

2.14.34 operator -=

Operator

```
schmatrix& schmatrix::operator -= (const schmatrix& m)
throw (cvmexception);
```

subtracts a hermitian matrix m from a calling hermitian matrix and returns a reference to the matrix changed. It throws an exception of type cvmexception in case of different sizes of the operands. See also schmatrix::operator - , schmatrix::diff, schmatrix. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
double b[] = {1., 0., 1., 1., 1., 1., 1., -1., 1., 0.,
              1., 1., 1., -1., 1., -1., 1., 0.};
schmatrix m1((std::complex<double>*)a,3);
schmatrix m2((std::complex<double>*)b,3);
m1 -= m2;
m2 -= m2;
std::cout << m1 << std::endl;
std::cout << m2;
```

prints

```
(0,0) (1,0) (-2,-1)
(1,0) (1,0) (-1,-2)
(-2,1) (-1,2) (2,0)

(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
```

2.14.35 operator - ()

Operator

```
schmatrix schmatrix::operator - () const throw (cvmexception);
```

creates an object of type schmatrix as a calling hermitian matrix multiplied by -1. See also schmatrix. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);
std::cout << -m;
```

prints

```
(-1,0) (-2,1) (1,2)
(-2,-1) (-2,0) (0,3)
(1,-2) (0,-3) (-3,0)
```

2.14.36 operator ++

Operator

```
schmatrix& schmatrix::operator ++ ();
schmatrix& schmatrix::operator ++ (int);
```

adds identity matrix to a calling hermitian matrix and returns a reference to the matrix changed. See also schmatrix. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);
m++;
std::cout << m << std::endl;
std::cout << ++m;
```

prints

```
(2,0) (2,-1) (-1,-2)
(2,1) (3,0) (0,-3)
(-1,2) (0,3) (4,0)

(3,0) (2,-1) (-1,-2)
(2,1) (4,0) (0,-3)
(-1,2) (0,3) (5,0)
```

2.14.37 operator --

Operator

```
schmatrix& schmatrix::operator -- ();
schmatrix& schmatrix::operator -- (int);
```

subtracts identity matrix from a calling hermitian matrix and returns a reference to the matrix changed. See also schmatrix. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);
m--;
std::cout << m << std::endl;
std::cout << --m;
```

prints

```
(0,0) (2,-1) (-1,-2)
(2,1) (1,0) (0,-3)
(-1,2) (0,3) (2,0)

(-1,0) (2,-1) (-1,-2)
(2,1) (0,0) (0,-3)
(-1,2) (0,3) (1,0)
```

2.14.38 operator * (TR)

Operator

```
schmatrix schmatrix::operator * (TR d) const;
```

creates an object of type schmatrix as a product of a calling hermitian matrix and a real number d. See also schmatrix::operator *= , schmatrix. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);
std::cout << m * 5.;
```

prints

```
(5,0) (10,-5) (-5,-10)
(10,5) (10,0) (0,-15)
(-5,10) (0,15) (15,0)
```

2.14.39 operator / (TR)

Operator

```
schmatrix schmatrix::operator / (TR d) const throw (cvmexception);
```

creates an object of type `schmatrix` as a quotient of a calling hermitian matrix and a real number `d`. It throws an exception of type `cvmexception` if `d` has an absolute value equal or less than the smallest normalized positive number. See also `schmatrix::operator /=` , `schmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);
std::cout << m / 2.;
```

prints

```
(0.5,0) (1,-0.5) (-0.5,-1)
(1,0.5) (1,0) (0,-1.5)
(-0.5,1) (0,1.5) (1.5,0)
```

2.14.40 operator * (TC)

Operator

```
scmatrix schmatrix::operator * (TC z) const;
```

creates an object of type `scmatrix` as a product of a calling hermitian matrix and a complex number z. See also `schmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);
std::cout << m * std::complex<double>(1.,1.);
```

prints

```
(1,1) (3,1) (1,-3)
(1,3) (2,2) (3,-3)
(-3,1) (-3,3) (3,3)
```

2.14.41 operator / (TC)

Operator

```
scmatrix schmatrix::operator / (TC z) const throw (cvmexception);
```

creates an object of type `scmatrix` as a quotient of a calling hermitian matrix and a complex number `z`. It throws an exception of type `cvmexception` if `z` has an absolute value equal or less than the smallest normalized positive number. See also `schmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);
std::cout << m / std::complex<double>(1.,1.);
```

prints

```
(0.5,-0.5) (0.5,-1.5) (-1.5,-0.5)
(1.5,-0.5) (1,-1) (-1.5,-1.5)
(0.5,1.5) (1.5,1.5) (1.5,-1.5)
```

2.14.42 operator *= (TR)

Operator

```
schmatrix& schmatrix::operator *= (TR d);
```

multiplies a calling hermitian matrix by a real number `d` and returns a reference to the matrix changed. See also `schmatrix::operator *` , `schmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);
m *= 5.;
std::cout << m;
```

prints

```
(5,0) (10,-5) (-5,-10)
(10,5) (10,0) (0,-15)
(-5,10) (0,15) (15,0)
```

2.14.43 operator /= (TR)

Operator

```
schmatrix& schmatrix::operator /= (TR d) throw (cvmexception);
```

divides a calling hermitian matrix by a real number d and returns a reference to the matrix changed. It throws an exception of type cvmexception if d has an absolute value equal or less than the smallest normalized positive number. See also schmatrix::operator / , schmatrix. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);
m /= 2.;
std::cout << m;
```

prints

```
(0.5,0) (1,-0.5) (-0.5,-1)
(1,0.5) (1,0) (0,-1.5)
(-0.5,1) (0,1.5) (1.5,0)
```

2.14.44 normalize

Function

```
schmatrix& schmatrix::normalize ();
```

normalizes a calling hermitian matrix so its Euclidean norm becomes equal to 1 if it was greater than the smallest normalized positive number before the call (otherwise the function does nothing). See also `schmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);

m.normalize();
std::cout << m << m.norm() << std::endl;
```

prints

```
(1.39e-001,0.00e+000) (2.77e-001,-1.39e-001) (-1.39e-001,-2.77e-001)
(2.77e-001,1.39e-001) (2.77e-001,0.00e+000) (0.00e+000,-4.16e-001)
(-1.39e-001,2.77e-001) (0.00e+000,4.16e-001) (4.16e-001,0.00e+000)
1.00e+000
```

2.14.45 conjugation

Operator and functions

```
schmatrix schmatrix::operator ~ () const throw (cvmexception);
schmatrix& schmatrix::conj (const schmatrix& m) throw (cvmexception);
schmatrix& schmatrix::conj () throw (cvmexception);
```

do nothing since a matrix calling is hermitian. They are provided to override the same member functions and operator of the class scmatrix. See also schmatrix. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);

std::cout << m - ~m;
```

prints

```
(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
```

2.14.46 operator * (const cvector&)

Operator

```
cvector schmatrix::operator * (const cvector& v) const
throw (cvmexception);
```

creates an object of type `cvector` as a product of a calling hermitian matrix and a vector `v`. It throws an exception of type `cvmexception` if the number of columns of the calling matrix differs from the size of the vector `v`. Use `cvector::mult` in order to get rid of a new object creation. See also `schmatrix` and `cvector`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
try {
    schmatrix m((std::complex<double>*)a,3);
    cvector v(3);
    v.set(std::complex<double>(1.,1.));

    std::cout << m * v;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(5,-1) (6,2) (-3,7)
```

2.14.47 operator * (const cmatrix&)

Operator

```
cmatrix schmatrix::operator * (const cmatrix& m) const
throw (cvmexception);
```

creates an object of type `cmatrix` as a product of a calling hermitian matrix and a matrix `m`. It throws an exception of type `cvmexception` if the number of columns of the calling matrix differs from the number of rows of the matrix `m`. Use `cmatrix::mult` in order to get rid of a new object creation. See also `cmatrix` and `schmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
try {
    schmatrix ms((std::complex<double>*)a,3);
    cmatrix m(3,2);
    m.set(std::complex<double>(1.,1.));

    std::cout << ms * m;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(5,-1) (5,-1)
(6,2) (6,2)
(-3,7) (-3,7)
```

2.14.48 operator * (const scmatrix&)

Operator

```
scmatrix schmatrix::operator * (const scmatrix& m) const
throw (cvmexception);
```

creates an object of type `scmatrix` as a product of a calling hermitian matrix and a matrix `m`. It throws an exception of type `cvmexception` if the operands have different sizes. Use `cmatrix::mult` in order to get rid of a new object creation. See also `scmatrix` and `schmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
try {
    schmatrix ms((std::complex<double>*)a,3);
    scmatrix m(3);
    m.set(std::complex<double>(1.,1.));

    std::cout << ms * m;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(5,-1) (5,-1) (5,-1)
(6,2) (6,2) (6,2)
(-3,7) (-3,7) (-3,7)
```

2.14.49 herk

Functions

```
schmatrix&
schmatrix::herk (TC alpha, const cvector& v, TC beta)
throw (cvmexception);

schmatrix&
schmatrix::herk (bool bTransp, TC alpha, const cmatrix& m, TC beta)
throw (cvmexception);
```

call one of ?HERK routines of the BLAS library performing a matrix-vector operation defined for the first version as rank-1 update operation

$$C = \alpha v \cdot v' + \beta C,$$

and for the second version as

$$C = \alpha M \cdot M^H + \beta C \quad \text{or} \quad C = \alpha M^H \cdot M + \beta C.$$

Here α and β are complex numbers (parameters `alpha` and `beta`), M is a complex matrix (parameter `m`), C is a calling hermitian matrix and v is a complex vector (parameter `v`). First operation for the second version of the function is performed if `bTransp` passed is `false` and second one otherwise. The function returns a reference to the matrix changed and throws an exception of type `cvmexception` in case of inappropriate sizes of the operands. See also `cvector`, `cmatrix` and `schmatrix` Example:

```
using namespace cvm;

double a[] = {1., -1., 2., 2., 3., -3.};
cvector v((std::complex<double>*)a, 3);
schmatrix mh(3);
mh.set_real(1.);
mh.herk (std::complex<double>(2.,-2.), v,
         std::complex<double>(1.,1.));
std::cout << mh << std::endl;

cmatrix m(3,2);
m(1) = v;
m(2).set(std::complex<double>(-1.,1.));
mh.herk (false, std::complex<double>(2.,-1.), m, 0.);
std::cout << mh << std::endl;

schmatrix mh2(2);
```

```
mh2.herk (true, std::complex<double>(1.,1.), m,
          std::complex<double>(0.,1.));
std::cout << mh2;
```

prints

```
(5,0) (1,-8) (13,0)
(1,8) (17,0) (1,24)
(13,0) (1,-24) (37,0)

(8,0) (4,-8) (16,0)
(4,8) (20,0) (4,24)
(16,0) (4,-24) (40,0)

(28,0) (-8,4)
(-8,-4) (6,0)
```

2.14.50 her2k

Functions

```
schmatrix&
schmatrix::her2k (TC alpha, const cvector& v1,
                  const cvector& v2, TC beta) throw (cvmexception);

schmatrix&
schmatrix::her2k (bool bTransp, TC alpha, const cmatrix& m1,
                  const cmatrix& m2, TC beta) throw (cvmexception);
```

call one of ?HER2K routines of the BLAS library performing a matrix-vector operation defined for the first version as rank-1 update operation

$$C = \alpha v_1 \cdot v_2' + \alpha v_2 \cdot v_1' + \beta C,$$

and for the second version as

$$C = \alpha M_1 \cdot M_2^H + \alpha M_2 \cdot M_1^H + \beta C \quad \text{or} \quad C = \alpha M_1^H \cdot M_2 + \alpha M_2^H \cdot M_1 + \beta C.$$

Here α and β are complex numbers (parameters alpha and beta), M_1 and M_2 are complex matrices (parameters m1 and m2), C is a calling hermitian matrix and v_1 and v_2 are vectors (parameters v1 and v2). First operation for the second version of the function is performed if bTransp passed is false and second one otherwise. The function returns a reference to the matrix changed and throws an exception of type cvmexception in case of inappropriate sizes of the operands. See also cvector, cmatrix and schmatrix Example:

```
using namespace cvm;

double a1[] = {1., -1., 2., 2., 3., -3.};
double a2[] = {-2., 1., 1., -2., 1., -3.};
cvector v1((std::complex<double>*)a1, 3);
cvector v2((std::complex<double>*)a2, 3);
schmatrix mh(3);
mh.set_real(1.); mh.set(1,3,std::complex<double>(4.,1.));
mh.her2k (std::complex<double>(2.,-1.), v1, v2,
          std::complex<double>(-1.,-1.));
std::cout << mh << std::endl;

cmatrix m1(3,2);
cmatrix m2(3,2);
m1.set_real(2.); m1.set_imag(-1.);
m2.set_real(-3.); m2.set_imag(1.);
mh.her2k (false, std::complex<double>(2.,1.), m1, m2,
```

```
                 std::complex<double>(3.,-2.));
std::cout << mh << std::endl;

schmatrix mh2(2);
mh2.her2k (true, std::complex<double>(1.,1.), m1, m2,
                 std::complex<double>(2.,-3.));
std::cout << mh2;
```

prints

```
(-11,0) (-4,9) (-9,-16)
(-4,-9) (3,0) (20,23)
(-9,16) (20,-23) (59,0)

(-93,0) (-72,27) (-87,-48)
(-72,-27) (-51,0) (0,69)
(-87,48) (0,-69) (117,0)

(-36,0) (-36,0)
(-36,0) (-36,0)
```

2.14.51 inv

Functions

```
schmatrix& schmatrix::inv (const schmatrix& m) throw (cvmexception);
schmatrix schmatrix::inv () const throw (cvmexception);
```

implement hermitian matrix inversion. The first version sets a calling hermitian matrix to be equal to a hermitian matrix `m` inverted and the second one creates an object of type `schmatrix` as inverted calling matrix. The functions throw exception of type `cvmexception` in case of inappropriate sizes of the operands or when the matrix to be inverted is close to cingular. See also `schmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);
const schmatrix mi = m.inv();

std::cout << mi << std::endl;
std::cout << mi * m - eye_complex(3);
```

prints

```
(-1.50e+000,0.00e+000) (-1.67e-016,8.33e-017) (-5.00e-001,-1.00e+000)
(-1.67e-016,-8.33e-017) (-1.00e+000,0.00e+000) (0.00e+000,-1.00e+000)
(-5.00e-001,1.00e+000) (0.00e+000,1.00e+000) (-1.50e+000,0.00e+000)

(2.22e-016,0.00e+000) (1.11e-016,0.00e+000) (2.78e-017,0.00e+000)
(2.78e-016,2.22e-016) (4.44e-016,0.00e+000) (0.00e+000,-4.44e-016)
(2.22e-016,-4.44e-016) (0.00e+000,-8.88e-016) (-2.22e-016,0.00e+000)
```

2.14.52 exp

Functions

```
schmatrix& schmatrix::exp (const schmatrix& m, TR tol = cvmMachSp ())
throw (cvmexception);

schmatrix schmatrix::exp (TR tol = cvmMachSp ()) const
throw (cvmexception);
```

compute an exponent of a calling hermitian matrix using Padé approximation defined as

$$R_{pq}(z) = D_{pq}(z)^{-1} N_{pq}(z) = 1 + z + \cdots + z^p/p!\,,$$

where

$$N_{pq}(z) = \sum_{k=0}^{p} \frac{(p+q-k)!p!}{(p+q)!k!(q-k)!} z^k,$$
$$D_{pq}(z) = \sum_{k=0}^{q} \frac{(p+q-k)!p!}{(p+q)!k!(q-k)!} (-z)^k$$

along with the matrix normalizing as described in [2], p. 572. The functions use ZMEXP (or CMEXP for float version) FORTRAN subroutine implementing the algorithm. The first version sets the calling hermitian matrix to be equal to the exponent of a hermitian matrix m and returns a reference to the matrix changed. The second version creates an object of type schmatrix as the exponent of the calling matrix. The algorithm uses parameter tol as $\varepsilon(p,q)$ in order to choose constants p and q so that

$$\varepsilon(p,q) \geqslant 2^{3-(p+q)} \frac{p!q!}{(p+q)!(p+q+1)!}.$$

This parameter is equal to the largest relative spacing by default. The functions throw an exception of type cvmexception in case of inappropriate sizes of the operands or when LAPACK subroutine fails. See also schmatrix. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left |
                std::ios::showpos);
std::cout.precision (15);
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);
schmatrix me(3);
me.exp(m);
```

```
std::cout << "Column 1" << std::endl
<< me(1,1) << std::endl << me(2,1) << std::endl << me(3,1) << std::endl
          << "Column 2" << std::endl
<< me(1,2) << std::endl << me(2,2) << std::endl << me(3,2) << std::endl
          << "Column 3" << std::endl
<< me(1,3) << std::endl << me(2,3) << std::endl << me(3,3) << std::endl;
```

prints

```
Column 1
(+2.673228708372002e+002,+1.091141066389412e-014)
(+3.071187567026803e+002,+1.535593783513402e+002)
(-1.749365628720766e+002,+3.498731257441531e+002)
Column 2
(+3.071187567026803e+002,-1.535593783513401e+002)
(+4.422594337092766e+002,+1.919736460939478e-015)
(-9.600094996571151e-015,+5.034325040954932e+002)
Column 3
(-1.749365628720765e+002,-3.498731257441531e+002)
(+6.184072406183948e-015,-5.034325040954932e+002)
(+5.744416275398805e+002,+1.540673883337074e-014)
```

Matlab output:

```
  Column 1

     2.673228708371998e+002 -7.105427357601002e-015i
     3.071187567026802e+002 +1.535593783513401e+002i
    -1.749365628720764e+002 +3.498731257441527e+002i

  Column 2

     3.071187567026802e+002 -1.535593783513401e+002i
     4.422594337092769e+002 -5.489286670342458e-016i
     3.549798266275454e-015 +5.034325040954932e+002i

  Column 3

    -1.749365628720763e+002 -3.498731257441526e+002i
    -1.776065298147746e-014 -5.034325040954931e+002i
     5.744416275398801e+002 -2.096383162906490e-014i
```

2.14.53 polynomial

Functions

```
schmatrix& schmatrix::polynom (const schmatrix& m, const rvector& v)
throw (cvmexception);

schmatrix schmatrix::polynom (const rvector& v) const
throw (cvmexception);
```

compute a hermitian matrix polynomial defined as

$$p(A) = b_0 I + b_1 A + \cdots + b_q A^q$$

using the Horner's rule:

$$p(A) = \sum_{k=0}^{r} B_k (A^s)^k, \quad s = \mathrm{floor}(\sqrt{q}\,),\ r = \mathrm{floor}(q/s)$$

where

$$B_k = \begin{cases} \sum_{i=0}^{s-1} b_{sk+i} A^i, & k = 0, 1, \ldots, r-1 \\ \sum_{i=0}^{q-sr} b_{sr+i} A^i, & k = r. \end{cases}$$

See also [2], p. 568. The *real* coefficients $b_0, b_1, \ldots, b_q$ are passed in the parameter v, where q is equal to `v.size()-1`, so the functions compute matrix polynomial equal to

$$\mathtt{v}[1] * \mathtt{I} + \mathtt{v}[2] * \mathtt{m} + \cdots + \mathtt{v}[\mathtt{v.size}()] * \mathtt{m}^{\mathtt{v.size}()-1}$$

The first version sets a calling hermitian matrix to be equal to the polynomial of a hermitian matrix `m` and the second one creates an object of type `schmatrix` as the polynomial of a calling symmetric matrix. The functions use `ZPOLY` (or `CPOLY` for `float` version) FORTRAN subroutine implementing the Horner's algorithm. The functions throw an exception of type `cvmexception` in case of inappropriate sizes of the operands. See also `schmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left |
                std::ios::showpos);
std::cout.precision (10);
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
            0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);
double re[]={2.2,1.3,1.1,-0.9,0.2,-0.45,45.,-30.,10.,3.,1.13};
const rvector vr(re, 11);
```

```
schmatrix mp(3);

mp.polynom (m, vr);

std::cout << "Column 1" << std::endl
<< mp(1,1) << std::endl << mp(2,1) << std::endl << mp(3,1) << std::endl
          << "Column 2" << std::endl
<< mp(1,2) << std::endl << mp(2,2) << std::endl << mp(3,2) << std::endl
          << "Column 3" << std::endl
<< mp(1,3) << std::endl << mp(2,3) << std::endl << mp(3,3) << std::endl;
```

prints

```
Column 1
(+1.2319548758e+008,+0.0000000000e+000)
(+1.4179323916e+008,+7.0896619580e+007)
(-8.0802738460e+007,+1.6160547692e+008)
Column 2
(+1.4179323916e+008,-7.0896619580e+007)
(+2.0399822604e+008,+0.0000000000e+000)
(+0.0000000000e+000,+2.3250209650e+008)
Column 3
(-8.0802738460e+007,-1.6160547692e+008)
(+0.0000000000e+000,-2.3250209650e+008)
(+2.6498872674e+008,+0.0000000000e+000)
```

Matlab output:

```
  Column 1

     1.231954875800000e+008
     1.417932391600000e+008 +7.089661958000000e+007i
    -8.080273845999999e+007 +1.616054769200000e+008i

  Column 2

     1.417932391600000e+008 -7.089661958000000e+007i
     2.039982260400000e+008
                          0 +2.325020965000000e+008i

  Column 3

    -8.080273845999999e+007 -1.616054769200000e+008i
                          0 -2.325020965000000e+008i
     2.649887267400000e+008
```

2.14.54 eig

Functions

```
rvector schmatrix::eig (scmatrix& mEigVect) const throw (cvmexception);
rvector schmatrix::eig () const throw (cvmexception);
```

solve a symmetric eigenvalue problem and return a real vector with eigenvalues of a calling hermitian matrix. The first version sets the output parameter `mEigVect` to be equal to the square matrix containing orthogonal eigenvectors as columns. All the functions throw an exception of type `cvmexception` in case of inappropriate sizes of the operands or in case of convergence error. See also `rvector`, `scmatrix` and `schmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);
scmatrix me(3);
rvector v(3);

v = m.eig(me);
std::cout << v << std::endl;
cvector vc(v);

std::cout << m * me(1) - me(1) * vc(1);
std::cout << m * me(2) - me(2) * vc(2);
std::cout << m * me(3) - me(3) * vc(3);
// orthogonality check:
std::cout << me(1) % me(2) << std::endl;
std::cout << me(2) % me(3) << std::endl;
std::cout << me(1) % me(3) << std::endl;
```

prints

```
-8.13e-001 -3.44e-001 7.16e+000

(1.39e-016,2.22e-016) (5.25e-017,-1.11e-016) (1.94e-016,1.67e-016)
(4.86e-016,4.44e-016) (7.63e-017,0.00e+000) (3.33e-016,2.22e-016)
(-2.22e-016,-8.88e-016) (-5.55e-017,-8.88e-016) (8.88e-016,-5.55e-017)
(-5.17e-017,-9.74e-017)
(-5.81e-017,-5.40e-017)
(2.37e-018,-3.56e-017)
```

2.14.55 Cholesky

Function

```
scmatrix schmatrix::cholesky () const throw (cvmexception);
```

forms the Cholesky factorization of a hermitian positive-definite matrix A defined as

$$A = U^H U,$$

where U is upper triangular matrix. It utilizes one of ?POTRF routines of the LAPACK library. The function creates an object of type scmatrix as the factorization of a calling matrix. The function throws an exception of type cvmexception in case of convergence error. See also scmatrix and schmatrix. Example:

```
using namespace cvm;

try {
    double a[] = {3., 0., 2., 1., -1., 2., 2., -1., 3., 0.,
                  0., 3., -1., -2., 0., -3., 5., 0.};
    const schmatrix m((std::complex<double>*)a,3);

    scmatrix h = m.cholesky();
    std::cout << h << std::endl;
    std::cout << ~h * h - m;
}
catch (exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(1.73205,0) (1.1547,-0.57735) (-0.57735,-1.1547)
(0,0) (1.1547,0) (0,-1.1547)
(0,0) (0,0) (1.41421,0)

(-4.44089e-016,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
```

2.14.56 Bunch-Kaufman

Function

```
scmatrix schmatrix::bunch_kaufman () throw (cvmexception);
```

forms the Bunch-Kaufman factorization of a calling hermitian matrix (cited from the MKL library documentation):

$$A = PUDU^T P^T,$$

where A is the calling matrix, P is a permutation matrix, U and L are upper and lower triangular matrices with unit diagonal, and D is a symmetric block-diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks. U and L have 2-by-2 unit diagonal blocks corresponding to the 2-by-2 blocks of D. It utilizes one of `?HETRF` routines of the LAPACK library. The function creates an object of type `scmatrix` as the factorization of a calling matrix. The function throws an exception of type `cvmexception` in case of convergence error. See also `scmatrix` and `schmatrix`. The function is mostly designed to be used for subsequent calls of `?HETRS`, `?HECON` and `?HETRI` routines of the LAPACK library. Currently it's used internally in `scmatrix::det` flow when argument is hermitian but not positive-definite matrix.

2.14.57 identity

Function

```
schmatrix& schmatrix::identity();
```

sets a calling hermitian matrix to be equal to identity matrix and returns a reference to the matrix changed. See also schmatrix. Example:

```
using namespace cvm;

schmatrix m(3);
m.randomize_real(0.,3.);
m.randomize_imag(-1.,2.);

std::cout << m << std::endl;
std::cout << m.identity();
```

prints

```
(1.93548,0) (1.84027,1.08353) (0.429579,-0.614093)
(1.84027,-1.08353) (1.76922,0) (1.71364,1.82788)
(0.429579,0.614093) (1.71364,-1.82788) (0.824915,0)

(1,0) (0,0) (0,0)
(0,0) (1,0) (0,0)
(0,0) (0,0) (1,0)
```

2.14.58 vanish

Function

```
schmatrix& schmatrix::vanish();
```

sets every element of a calling hermitian matrix to be equal to zero and returns a reference to the matrix changed. See also schmatrix. Example:

```
using namespace cvm;

schmatrix m(3);
m.randomize_real(0.,3.);
m.randomize_imag(-1.,2.);

std::cout << m << std::endl;
std::cout << m.vanish();
```

prints

```
(1.95499,0) (1.03925,0.830378) (0.951628,0.563677)
(1.03925,-0.830378) (0.150426,0) (2.29365,-0.580218)
(0.951628,-0.563677) (2.29365,0.580218) (0.0841395,0)

(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
```

2.14.59 randomize_real

Function

```
schmatrix& schmatrix::randomize_real (TR dFrom, TR dTo);
```

fills a real part of a calling hermitian matrix with pseudo-random numbers distributed between `dFrom` and `dTo` keeping it to be hermitian. The function returns a reference to the matrix changed. See also `schmatrix`. Example:

```
using namespace cvm;

schmatrix m(3);
m.randomize_real(0.,3.);
std::cout << m << std::endl;
```

prints

```
(1.98245,0) (2.72103,0) (0.167272,0)
(2.72103,0) (0.0285653,0) (1.63765,0)
(0.167272,0) (1.63765,0) (1.15882,0)
```

2.14.60 randomize_imag

Function

```
schmatrix& schmatrix::randomize_imag (TR dFrom, TR dTo);
```

fills an imaginary part of a calling hermitian matrix with pseudo-random numbers distributed between `dFrom` and `dTo` keeping it to be hermitian. The function returns a reference to the matrix changed. See also `schmatrix`. Example:

```
using namespace cvm;

schmatrix m(3);
m.randomize_imag(0.,3.);
std::cout << m << std::endl;
```

prints

```
(0,0) (0,0.13834) (0,2.39903)
(0,-0.13834) (0,0) (0,0.609577)
(0,-2.39903) (0,-0.609577) (0,0)
```

2.15 cvmexception

The CVM library exceptions class.

```
class cvmexception : public std::exception
{
public:
    explicit cvmexception (int nCause, ...);
    int cause () const;
    virtual const char* what () const;
    static int getNextCause ();
    static bool add (int nNewCause, const char* szNewMessage);
};
```

2.15.1 cause

Function

```
int cvmexception::cause () const;
```

returns a numeric code of an exception thrown. Possible codes can be found in `cvm.h` file. See also `cvmexception`. Example:

```
using namespace cvm;

try {
    rvector v(10);
    v[11] = 1.;
}
catch (cvmexception& e) {
    std::cout << "Exception " << e.cause () << ": "
              << e.what () << std::endl;
}
```

prints

```
Exception 2: Out of range
```

2.15.2 what

Function

```
virtual const char* cvmexception::what () const throw();
```

returns a string describing an exception happened. This function overrides `std::exception::what()`. This allows you to catch just one type of exception in your application. See also `cvmexception`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 1., 2.};
    const srsmatrix m(a, 2);
    std::cout << m;
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
Exception: The matrix passed doesn't appear to be symmetric
```

2.15.3 Customization

Constructor and functions

```
explicit cvmexception (int nCause, ...);
static bool cvmexception::add (int nNewCause, const char* szNewMessage);
static int cvmexception::getNextCause ();
```

allow to add and use customized exception codes and messages. See also cvmexception. Example:

```
using namespace cvm;

const int nNextCause = cvmexception::getNextCause();
cvmexception::add (nNextCause,
                   "My first exception with %d parameter");
cvmexception::add (nNextCause + 1,
                   "My second exception with %s parameter");

try {
    throw cvmexception (nNextCause, 1234);
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}

try {
    throw cvmexception (nNextCause + 1, "Hi!");
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
Exception: My first exception with 1234 parameter
Exception: My second exception with Hi! parameter
```

2.16 Utilities

These functions have cvm namespace scope and can be used for different purposes.

```
template <typename T>
T* cvmMalloc (size_t nEls) throw (cvmexception);
template <typename T>
T* cvmAddRef (const T* pD);
template <typename T>
int cvmFree (T*& pD);
void cvmExit ();
treal cvmMachMin ();
treal cvmMachSp ();
srmatrix eye_real (int nM);
scmatrix eye_complex (int nM);
operator * (,);
```

2.16.1 cvmMalloc

Function

```
template <typename T>
T* cvmMalloc (size_t nEls) throw (cvmexception);
```

allocates `nEls` units of type `T` from the CVM library's memory pool and returns a pointer to the memory allocated. It can throw an exception of type `cvmexception` if there is not enough memory in the global pool. This is the preferable way to allocate memory in case of using the CVM library because it is faster and more robust. See also `cvmAddRef` and `cvmFree`. Example:

```
using namespace cvm;

double* p = cvmMalloc<double> (10);
p[0] = 1.;
p[1] = 2.;
p[2] = 3.;

rvector v(3);
v.assign(p);
std::cout << v;

cvmFree (p);
```

prints

```
1 2 3
```

2.16.2 cvmAddRef

Function

```
template <typename T>
void cvmAddRef (const T* pD);
```

increments a reference counter for a memory block pointed to by `pD` if this block was allocated from the CVM library's memory pool (using `cvmMalloc` function). If `pD` points to a foreign memory block then the function does nothing. See also `cvmAddRef`. Example:

```
using namespace cvm;

double* p = cvmMalloc<double> (10);
p[0] = 1.;
p[1] = 2.;
p[2] = 3.;

cvmAddRef (p);
cvmFree (p);    // this call doesn't allocate a memory
cvmFree (p);    // this one does
```

2.16.3 cvmFree

Function

```
template <typename T>
int cvmFree (T*& pD);
```

decrements a reference counter for a memory block pointed to by `pD` if this block was allocated from the CVM library's memory pool (using `cvmMalloc` function) and returns the reference counter it changed. If the function returns zero then it sets the pointer `pD` to be equal to `NULL` and "frees" the memory, i.e. returns the memory block to a list of free ones (see CVM memory management for details). If `pD` points to a foreign memory block then the function does nothing and returns -1. See also `cvmAddRef`. Example:

```
using namespace cvm;

double* pf = new double[10];
double* p  = cvmMalloc<double> (10);

cvmAddRef (p);

std::cout << cvmFree (p) << " ";
std::cout << p << std::endl;

std::cout << cvmFree (p) << " ";
std::cout << p << std::endl;

std::cout << cvmFree (pf) << " ";
std::cout << pf << std::endl;

delete[] pf;
```

prints

```
1 003C66B0
0 00000000
-1 003C7A40
```

2.16.4 cvmExit

Function

```
void cvmExit ();
```

destroys the CVM library's memory pool if CVM_USE_POOL_MANAGER is defined. Otherwise does nothing. All objects created using this pool are not accessible after calling of this function. Call this function in the last expression only if you have problems with memory deallocation while finishing execution of your program (I was not able to experience such problems, this function is provided just in case). See also cvmMalloc. Example:

```
using namespace cvm;

int main(int argc, char* argv[])
    try {
        rvector v(3);
        v[1] = v[2] = v[3] = 1.;
        std::cout << v.norm2() << std::endl;
    }
    catch (cvmexception& e) {
        std::cout << "Exception " << e.cause ()
                  << ": " << e.what () << std::endl;
    }

    cvmExit ();
    return 1;
}
```

prints

```
1.73205
```

2.16.5 cvmMachMin

Function

`treal cvmMachMin ();`

returns the smallest normalized positive number or, i.e. `numeric_limits<treal>::min()` where `treal` is `typedef`'ed as `double` by default or as `float` for float version of the library. See also cvmMachSp. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (15);
std::cout << cvmMachMin() << std::endl;
```

for Intel Pentium® III machine prints

```
+2.225073858507201e-308
```

2.16.6 cvmMachSp

Function

```
treal cvmMachSp ();
```

returns the largest relative spacing or, in other words, the difference between 1 and the least value greater than 1 that is representable, i.e. `numeric_limits<treal>::epsilon()` where `treal` is `typedef`'ed as `double` by default or as `float` for float version of the library. See also `cvmMachMin`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (15);
std::cout << cvmMachSp() << std::endl;
```

for Intel Pentium® III machine prints

```
+2.220446049250313e-016
```

2.16.7 eye_real

Function

```
srmatrix eye_real (int nM);
```

creates a `nM` by `nM` object of type `srmatrix` equal to identity matrix. See also `srmatrix`.
Example:

```
using namespace cvm;

std::cout << eye_real (4);
```

prints

```
1 0 0 0
0 1 0 0
0 0 1 0
0 0 0 1
```

2.16.8 eye_complex

Function

```
scmatrix eye_complex (int nM);
```

creates a nM by nM object of type scmatrix equal to identity matrix. See also scmatrix.
Example:

```
using namespace cvm;

std::cout << eye_complex (4);
```

prints

```
(1,0) (0,0) (0,0) (0,0)
(0,0) (1,0) (0,0) (0,0)
(0,0) (0,0) (1,0) (0,0)
(0,0) (0,0) (0,0) (1,0)
```

2.16.9 operator *

Operators

```
inline rvector operator * (TR d, const rvector& v);
inline rmatrix operator * (TR d, const rmatrix& m);
inline srmatrix operator * (TR d, const srmatrix& m);
inline srbmatrix operator * (TR d, const srbmatrix& m);
inline srsmatrix operator * (TR d, const srsmatrix& m);
inline cvector operator * (TR d, const cvector& v);
inline cmatrix operator * (TR d, const cmatrix& m);
inline scmatrix operator * (TR d, const scmatrix& m);
inline scbmatrix operator * (TR d, const scbmatrix& m);
inline schmatrix operator * (TR d, const schmatrix& m);
inline cvector operator * (std::complex<TR> c, const cvector& v);
inline cmatrix operator * (std::complex<TR> c, const cmatrix& m);
inline scmatrix operator * (std::complex<TR> c, const scmatrix& m);
inline scbmatrix operator * (std::complex<TR> c, const scbmatrix& m);
inline schmatrix operator * (std::complex<TR> c, const schmatrix& m);
inline rvector operator * (CVM_LONGEST_INT d, const rvector& v);
inline rmatrix operator * (CVM_LONGEST_INT d, const rmatrix& m);
inline srmatrix operator * (CVM_LONGEST_INT d, const srmatrix& m);
inline srbmatrix operator * (CVM_LONGEST_INT d, const srbmatrix& m);
inline srsmatrix operator * (CVM_LONGEST_INT d, const srsmatrix& m);
inline cvector operator * (CVM_LONGEST_INT d, const cvector& v);
inline cmatrix operator * (CVM_LONGEST_INT d, const cmatrix& m);
inline scmatrix operator * (CVM_LONGEST_INT d, const scmatrix& m);
inline scbmatrix operator * (CVM_LONGEST_INT d, const scbmatrix& m);
inline schmatrix operator * (CVM_LONGEST_INT d, const schmatrix& m);
```

provide an ability to make left-sided multiplication of numbers by different CVM objects. Example:

```
using namespace cvm;

const schmatrix scm = eye_complex (4);
std::cout << std::complex<double>(2.,1.) * scm << std::endl;

rvector v(3);
v(1) = 1.;
v(2) = 2.;
v(3) = 3.;

std::cout << 3. * v;
```

prints

```
(2,1) (0,0) (0,0) (0,0)
(0,0) (2,1) (0,0) (0,0)
(0,0) (0,0) (2,1) (0,0)
(0,0) (0,0) (0,0) (2,1)

3 6 9
```

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