Abstract — Continuous viscosity measurement assists in product quality procurement and maximizes the efficiency of plant operation. However, state-of-the-art commercial instruments have severe drawbacks such as using by-pass loops or being invasive. Based on a conventional electromagnetic flowmeter, a non-invasive process viscometer has been developed. Through slight modifications of a commercial flowmeter two measured quantities related to the velocity profile can be determined. Emerging from a two-parametric rheological model as a theoretical background, the complete velocity profile can be approximated. An additional differential pressure sensor provides a third rheological parameter so that the viscosity can be obtained. A prototype of the proposed viscometer has been constructed and extensively tested with various fluids. The results underline the good performance of the measuring principle.

Keywords — Electromagnetic flowmeter, rheological parameters, in-line viscometer, non-invasive measurement

I. INTRODUCTION

In many industrial processes such as manufacturing of food, detergents, paints or cosmetics [1, 2], viscosity is a key quality parameter for intermediate and final products. In contrast to conventional laboratory viscometers, that only allow offline determination of viscosity for samples, process viscometers gather in-line information on viscosity directly within the process and in real time. However, state-of-the-art commercial instruments have severe drawbacks such as using by-pass loops or being invasive [3, 4]. The fluid structure may be distorted or damaged, so that the measured sample is not representative. Therefore, the related parameters, e.g. the viscosity, are not suited to process control. It therefore hardly comes as a surprise that industrial development seeks for a non-invasive method.

Such a non-invasive method for measuring spatially distributed quantities is offered by tomography. However, for automatic control the costly and computationally expensive reconstruction of an image is not required. Instead, the raw sensor data are used directly for process monitoring [5]. Using a priori knowledge, the amount of measurements can be reduced. In particular, if a parametric process model exists, only its parameters have to be determined by measurements.

This paper presents the application of the above basic principle to an electromagnetic flowmeter and proposes a non-invasive process viscometer. The device can be integrated directly into the main pipe. Hence, it does not cause additional pressure loss and does not considerably disturb the flow.

II. RHEOLOGICAL MODELLING

The velocity profile of a fully developed, laminar flow depends on the rheological behaviour of the fluid. One of the most important material properties characterizing the flow behaviour is the shear viscosity [6]

\[
\eta(\dot{\gamma}) = \frac{\tau(\dot{\gamma})}{\dot{\gamma}},
\]

where \( \tau \) is the shear stress and \( \dot{\gamma} \) denotes the shear rate. For determining the viscosity, the material must be stressed by a deformation. In a pipe flow, the pressure difference produces a simple shear flow. The resulting shear stress is directly proportional to the radial position independent of the material property, varying over the pipe from zero at the center \( (r = 0) \) to the maximum at the wall \( (r = R) \), as shown in Fig. 1. In contrast, the flow behaviour has a strong influence on the profile of the shear rate. For a Newtonian fluid, the viscosity is constant w.r.t. shear rate \( \dot{\gamma} \). In this case, the shear rate is also proportional to the radial position (see Fig. 1a). Fluids whose viscosity is a function of the shear rate are

- a) Newtonian fluid \((N=1)\)

- b) Shear-thinning fluid (here: \(N=0.2\))

![Figure 1: Flow behaviour in a straight circular pipe](image)
called non-Newtonian. Thereby, shear-thinning, shear-thickening and yield stress behaviour are distinguished. In the vast majority of cases, fluids show shear-thinning behaviour, which means that the viscosity decreases with increasing shear rate. For all non-Newtonian fluids, the relationship between the shear rate and the radius is nonlinear. In the case of shear-thinning behaviour, the profile shows a high gradient near the wall and has low values for relatively high radii (see Fig. 1b). This means that for a comparatively large range around the pipe axis, the values of the shear rate are negligible in comparison to those near the wall.

As the shear rate equals the velocity gradient
\[
\dot{\gamma} = -\frac{dv}{dr},
\]
the shape of the velocity profile also depends on the flow behaviour of the fluid. Figure 1 depicts the laminar velocity profiles of both a Newtonian and a shear-thinning fluid for the same volumetric flow rates. Whereas the profile is parabolic for the Newtonian behaviour, the shear-thinning fluid shows a flatter profile.

\[
\eta = K \cdot \dot{\gamma}^{N-1},
\]
where \(N\) denotes the power-law index and \(K\) is termed consistency. Newtonian materials are characterized by \(N = 1\), whereas shear-thinning materials are described by \(N < 1\). The power-law model is widely used to describe non-Newtonian flow properties of fluids in theoretical analysis as well as in practical engineering applications [6]. In Fig. 2 the viscosity function for a fluid with shear-thinning behaviour is shown. Obviously, the power-law model fits the experimental results only over a limited range of the shear rate. Thus, the equation has to be applied to the relevant segment which is given by the test conditions. As mentioned above, the values of the shear rate are negligible small in the inner part of the pipe, such that the relevant values have to be considered near the pipe wall.

Describing the flow behaviour by the power-law model, the velocity profile is given by
\[
v(r) = \frac{1 + 3N}{1 + N} \cdot \bar{v} \left( 1 - \left( \frac{r}{R} \right)^{1+1/N} \right),
\]
where \(\bar{v}\) denotes the mean velocity and \(R\) is the pipe radius. Thus, the velocity profile can be completely described by the two parameters \(N\) and \(\bar{v}\). As will be derived in the following section, the modified flowmeter provides – in addition to the measurement of the mean velocity \(\bar{v}\) – a second measured quantity related to the flow profile. Hence, the approach is able to determine the power-law index \(N\) and consequently the complete flow profile \(v(r)\).

The consistency \(K\) depends not only on the velocity, but also on the shear stress. Thus, it cannot be obtained with the modified flowmeter by itself. Instead, an additional pressure sensor is necessary measuring the pressure loss \(\Delta p\) in the pipe.
From the relationships
\[
\tau (r = R) = \tau _w = \frac{\Delta p \cdot R}{2 \cdot L} \quad \text{and} \quad \dot{\gamma}_w = \left( \frac{1 + 3N}{N} \right) \frac{\bar{v}}{R},
\]
the consistency \( K \) results in
\[
K = \frac{\tau _w}{\dot{\gamma}_w}. \quad (6)
\]
\( \tau _w \) and \( \dot{\gamma}_w \) are the values at the pipe wall and \( L \) is the distance between the pressure tapping points.

Inserting the consistency (cf. Eq. (6)) and the power-law index in Eq. (3), the viscosity can be calculated. In contrast to a laboratory rheometer, where the viscosity function is determined step by step by varying the shear rate, the viscosity is here calculated for the complete shear rate range in the pipe.

III. SENSOR SET-UP
A) Modification of an electromagnetic flowmeter

Conventional electromagnetic flowmeters are widely used in industrial processes for flow measurement of electrically conducting fluids. A coil system mounted on the outside of the pipe generates a homogeneous magnetic field perpendicular to the pipe axis. The voltage induced between two electrodes mounted diametrically at an electrically insulated pipe wall is proportional to the mean flow velocity. In order to obtain information on the spatial distribution of the velocity, our approach is to generate a magnetic field of spatially varying magnitude. This inhomogeneous field is applied in alternation to the homogeneous field inducing a voltage depending on the shape of the flow profile. Consequently, a flowmeter with two alternately generated magnetic fields measures two quantities describing the velocity profile. The realization in a commercial device requires only slight modifications of the sensor (see Fig. 3). Firstly, the direction of the current in one of the coils have to be alternately switched. Secondly, the electrodes, which are usually placed perpendicular to the coil axis, have to be placed at angles of \( \pm 45^\circ \).

In general, for a laminar, fully-developed and axially symmetric pipe flow, the induced voltage in the electrodes is given by [8]
\[
U_n = \int_{r=0}^{R} v(r) \cdot g_n(r) \cdot r \, dr, \quad (7)
\]
where \( v(r) \) denotes the flow velocity as a function of radial...
position $r$, and $g_n$ denotes a weight function, respectively. The subscript $n$ refers to the position of the switch in Fig. 3. For $n = 0$, the coil currents have the same direction generating a homogenous field (Fig. 3a), whereas for $n = 1$, the coil currents have opposite directions and the magnetic field is significantly inhomogeneous (Fig. 3b).

The radial weight function is given as

$$g_n(r) = \int \int_{\phi z} \left( B_{n,x} \frac{\partial G}{\partial y} - B_{n,y} \frac{\partial G}{\partial x} \right) \, d\phi \, dz \tag{8}$$

where $G$ is the Green’s function of the pipe describing position and design of the electrodes, whereas the magnetic fields are considered by the magnetic induction $B_n$. For an ideal flowmeter with point electrodes and an ideal homogeneous or inhomogeneous magnetic field, the weight functions can be calculated analytically. In the case of a real flowmeter, however, no closed formula for the weight functions exists, hence a numerical calculation is required. The magnetic fields as well as the Green’s function can be calculated with an FEM-program considering the geometry of the real sensor.

B) Implementation

In Fig. 4 the normalized weight functions $g_0$ and $g_1$ calculated for the used modified flowmeter are shown. Using these functions and describing the velocity profile $v(r)$ with the power-law model of Eq. (4), both voltages $U_0$ and $U_1$ can be determined as a function of the power-law index $N$ from Eq. (7). Varying the power-law index, a characteristic curve between the quantity of interest, the power-law index $N$, and the ratio of the measured signals $U_1/U_0$ is calculated (see Fig. 4). This characteristic curve $N(U_1/U_0)$ is stored as look-up table in the signal processing unit of the modified flowmeter.

In a process line, the power-law $N$ index is determined by interpolation from the look-up table. Additionally, a differential pressure sensor is mounted delivering the second rheological parameter, the consistency $K$. The combination of both instruments represents the in-line process viscometer which allows the continuous measurement of the viscosity $\eta$ (see Fig. 5).

C) Model verification

The presented modified flowmeter is based on two models:
On the one hand, the modelling of the sensor and the calculation of the weight functions with an FEM program, and on the other hand, the description of the velocity profile in the pipe by the rheological model. Since satisfactory measuring results depend on an accurate description of the real conditions by the models, verification of both models is essential. Measurements with a Hall sensor show tight correspondence between the measured magnetic induction $B_m$ and numerically calculated values [8]. The flow model has also been verified by measurements. Using a laser-doppler-velocimeter (LDV), a reference for the velocity profile in the pipe is available. These measurements show that the power-law index derived from this profile differs only slightly from the result obtained by the modified flowmeter [8].

IV. EXPERIMENTAL RESULTS

The presented measuring principle is realized on the base of an electromagnetic flowmeter by Endress + Hauser (Promag 53 F, nominal width 50 mm). For experimental tests in the laboratory, the modified flowmeter and a differential pressure sensor have been installed in a recirculating pipe system (see Fig. 6). The fluid is conveyed by means of a progressing cavity pump, temperature is held constant by a cooling system with heat exchanger.

A) Accuracy

Using the experimental setup, a broad variety of fluids with very different flow behaviour have been studied (sugar sirup, xanthan solution, luviskol solution, shampoo, tomato ketchup, mayonnaise, mustard). The viscosity curves of sugar sirup ($c = 10\%$), 12 % aqueous solution of luviskol (a thickening agent) and ketchup are plotted in Fig. 7. The solid lines represent the measurements of the in-line device with a flow rate of 1.0 l/s, which leads to a mean velocity of approximately 0.5 m/s. In a diagram with double-logarithmic scales, the two-parametric power-law model is represented by straight lines, the slopes of which indicate the power-law index $N$. The viscosity has been calculated for the relevant shear rate range of $0.16 \cdot \dot{\gamma}_W \leq \dot{\gamma} \leq \dot{\gamma}_W$. As mentioned in Section II, the decisive values for the flow behaviour are the high shear rates near the wall, whereas the shear rates near the pipe axis are negligible. In the case of the Newtonian sugar sirup and the strongly shear-thinning ketchup, the power-law index is constant over a wide shear rate range, i.e. the value is independent of the flow rate. For the interesting shear rate range, the luviskol solution changes from Newtonian to shear-thinning flow, which means the power-law index depends on the flow rate. Using a cone-plate geometry of a commercial laboratory rheometer, the viscosity functions of the three fluids have been measured for reference. These curves have been approximated by the power-law model in the same relevant shear rate region, whereby reference values of the power-law index are determined. Table 1 compares the flow indices obtained by the modified flowmeter with the reference values. For all these materials the values indicate the accurate performance of the in-line system.

<table>
<thead>
<tr>
<th>Material</th>
<th>$N_{\text{flowmeter}}$</th>
<th>$N_{\text{reference}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>sugar sirup</td>
<td>1.01</td>
<td>1.00</td>
</tr>
<tr>
<td>luviskol solution</td>
<td>0.78</td>
<td>0.83</td>
</tr>
<tr>
<td>ketchup</td>
<td>0.16</td>
<td>0.23</td>
</tr>
</tbody>
</table>

Table 1: Comparison of the power-law indices obtained from the flowmeter with reference values

B) Repeatability

The modified flowmeter delivers values for the power-law index close to those obtained with the laboratory rheometer. For evaluating the measured data of the modified flowmeter, the repeatability is discussed in addition to the accuracy. For this purpose, the standard deviation of the ratio of the measured voltages as a function of the volumetric flow rate is
analysed. Figure 8 shows the standard deviation related to the absolute values. For flow rates larger than 0.5 l/s the deviation is smaller than 1 %, whereas for smaller flow rates, the uncertainty increases. This curve coincides with repeatability curves of a conventional electromagnetic flowmeter [9]. According to the operation principle, the absolute value of the induced voltage is directly correlated with the flow velocity; see Eq. (7). The measuring sensitivity of a conventional electromagnetic flowmeter amounts to 300 µV per m/s. Due to the different position of the electrodes in the modified flowmeter (±45° instead of ±90°), the sensitivity is reduced by the factor √2 for the homogeneous field. In the inhomogeneous case, the magnetic field is generally attenuated, therefore the sensitivity is approximately a quarter of the homogeneous value, i.e. around 50 µV per m/s. Consequently, the signal-to-noise-ratio is relatively small for flow rates smaller than 0.5 l/s which means nearly 0.25 m/s with the pipe diameter of 50 mm. For comparison, typical flow rates of many food products in process lines like mayonnaise and ketchup are between 0.2 m/s and 2 m/s.

![Figure 9: Repeatability of the power-law index](image)

By transforming the standard deviation of the ratio of both measured voltages with the characteristic curve \( N(U_1/U_0) \), the repeatability of the measured power-law index is estimated. Because of the nonlinear relationship between the power-law index and the ratio of measured voltages (cf. Fig. 4), the deviation of the power-law index depends on the ratio \( U_1/U_0 \), i.e. it depends on the flow behaviour. The curve plotted in Fig. 9 represents the results for the food system mustard with a power-law index of 0.22. For flow rates larger than 0.5 l/s, a deviation below 10 % is achieved. This precision is acceptable to determine the rheological parameter power-law index for most engineering purposes. Measuring this parameter with a laboratory instrument is prone to numerous sources of error like taking a representative sample, the risk of damaging the often complex structure during introduction of the sample into the viscometer, the different filling levels of the shear gap and the inaccuracy of calibration especially for non-Newtonian fluids [10].

V. CONCLUSIONS

This paper has presented a measurement system that provides a non-contact method to determine rheological parameters. Its main advantages are in-line and real-time operation, non-invasivity and a simple hardware realization. Emerging from a slightly modified electromagnetic flowmeter, it has been shown that two parameters of the flow profile can be determined. Using the rheological power-law model, these two parameters have been combined with a third parameter whereby viscosity functions have been modelled. A differential pressure sensor provides this third parameter. The accuracy of the proposed in-line system has been verified in experiments for a broad variety of materials. The system supplies results comparable to those obtained using a laboratory rheometer, which proves appropriate for most applications. The repeatability of the measured voltages of the modified flowmeter resembles with the properties of a conventional flowmeter. As far as the additional rheological parameter is concerned, acceptable repeatability is achieved.

ACKNOWLEDGEMENT

The author would like to thank Endress+Hauser Flowtec AG (Reinach, Switzerland) for supporting this work, and Mr. Thomas Budmiger for helpful discussions.

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